

For each question, "E) NOTA" indicates that none of the above answers is correct.

1. Evaluate:  $\begin{bmatrix} 3 & 9 & 3 \\ 4 & 6 & 2 \\ 5 & 1 & 5 \end{bmatrix} + \begin{bmatrix} 5 & -2 & 8 \\ 3 & -9 & 5 \\ 3 & 1 & -6 \end{bmatrix}$

A)  $\begin{bmatrix} 9 & 7 & 10 \\ 7 & -3 & 7 \\ 8 & 1 & 2 \end{bmatrix}$  B)  $\begin{bmatrix} 11 & 7 & 8 \\ 7 & -3 & 7 \\ 8 & 2 & 8 \end{bmatrix}$  C)  $\begin{bmatrix} 8 & 7 & 11 \\ 7 & -3 & 7 \\ 8 & 2 & -1 \end{bmatrix}$  D)  $\begin{bmatrix} 8 & 2 & 8 \\ 2 & -3 & 7 \\ -1 & 7 & 11 \end{bmatrix}$  E) NOTA

2. If  $A = \begin{bmatrix} 3 & 11 & -8 \\ 1 & -5 & -9 \end{bmatrix}$  and  $B = \begin{bmatrix} 7 & 4 & 5 \\ 4 & -2 & 10 \end{bmatrix}$ , Find  $5A - 2B$

A)  $\begin{bmatrix} 1 & 47 & -50 \\ -3 & -21 & -65 \end{bmatrix}$  B)  $\begin{bmatrix} -1 & -47 & 50 \\ 3 & 29 & 69 \end{bmatrix}$  C)  $\begin{bmatrix} 15 & 55 & -40 \\ 5 & -25 & -49 \end{bmatrix}$  D)  $\begin{bmatrix} 6 & 9 & -26 \\ -50 & -29 & -18 \end{bmatrix}$

E) NOTA

3. Evaluate:  $\begin{vmatrix} 6 & 6 & 6 & 9 & 9 \\ 9 & 9 & 9 & 6 & 6 \\ 1 & 2 & 3 & 4 & 5 \\ -9 & -9 & -6 & -6 & -6 \\ -6 & -6 & -9 & -9 & -9 \end{vmatrix}$

A) 69 B) -69 C) 0 D) 96 E) NOTA

4. Given non-coincident points D and S in two dimensions, what shape is the locus of points P where  $\vec{DP} = t \vec{DS}$ ?

A) a line B) a circle C) a parabola D) a hyperbola E) NOTA

5. A particle travels along the line written in parametric form:  $x = 2 + 2t$ ;  $y = 4 - t$ ;  $z = 9 - 2t$  starting at the point (2, 4, 9), in a direction such that its x-coordinate is increasing. What is its position after 8 seconds?

A) (-18, 4, 7) B) (6, -8, 9) C) (-7, -4, 18) D) (18, -4, -7) E) NOTA

6. What is the secant of the angle between the vectors  $\langle 5, 12 \rangle$  and  $\langle -8, 6 \rangle$ ?

- A)  $\frac{-65}{16}$       B)  $\frac{65}{16}$       C)  $\frac{16}{65}$       D)  $\frac{-16}{65}$       E) NOTA

7. Evaluate:  $\begin{vmatrix} x & y & 4 \\ 1 & 6 & 3 \\ 3 & -5 & 9 \end{vmatrix}$

- A)  $69x - 76$       B)  $23x - 11y - 76$       C)  $52x - 19y + 76$       D)  $88y - 76$       E) NOTA

8. What values of  $x$  will satisfy the inequality  $\left| \begin{vmatrix} -3 & x-11 \\ 2x & 6 \end{vmatrix} \right| > 2$

- A)  $x > 9$     B)  $x < -1, x > 9$     C)  $x < -1, x > 10$     D)  $1 < x < 10$     E) NOTA

9. What is the trace of the matrix  $\begin{bmatrix} 1 & 2 & 1 & 3 & 7 \\ 2 & 7 & 3 & 7 & 3 \\ 1 & 3 & 8 & 3 & 1 \\ 3 & 7 & 3 & 7 & 2 \\ 7 & 3 & 1 & 2 & 1 \end{bmatrix}$ ?

- A) 22      B) 23      C) 24      D) 25      E) NOTA

10. Suppose matrix  $D = \begin{bmatrix} 6 & 9 \\ 6 & 9 \end{bmatrix}$ . If  $x$  is the larger of  $D$ 's eigenvalues and  $y$  is the smaller of  $D$ 's eigenvalues, what is  $28x - 10y$ ?

- A) -96      B) 420      C) 690      D) 240      E) NOTA

11. Find the area of the triangle having vertices  $(4,2)$ ,  $(0,6)$ ,  $(9,0)$

- A) 4      B) 20      C) 6      D) 9      E) NOTA

12. If  $A = \begin{bmatrix} 2 & 9 \\ -6 & 3 \end{bmatrix}$  and the determinant of the product  $AB$  is 420, which of the following matrices could be the matrix  $B$ ?

- A)  $\begin{bmatrix} 2 & 9 \\ -6 & 3 \end{bmatrix}$       B)  $\begin{bmatrix} -2 & 4 \\ 5 & 3 \end{bmatrix}$       C)  $\begin{bmatrix} 2 & 2 \\ -4 & 4 \end{bmatrix}$       D)  $\begin{bmatrix} 3 & 2 \\ 1 & 3 \end{bmatrix}$       E) NOTA

13. Which of the following matrices has the greatest determinant?

A)  $\begin{bmatrix} 4 & 9 & 0 \\ 6 & 2 & 4 \\ 1 & 2 & 0 \end{bmatrix}$       B)  $\begin{bmatrix} 1 & 4 & 2 & 2 \\ 1 & 5 & 0 & 4 \\ 0 & 6 & 0 & 0 \\ 9 & 5 & 9 & 0 \end{bmatrix}$       C)  $\begin{bmatrix} 7 & 9 \\ -9 & 6 \end{bmatrix}$       D)  $\begin{bmatrix} 4 & 2 & 0 & 1 & 6 \\ 2 & 0 & 1 & 6 & 9 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 6 & 9 & 0 & 0 & 9 \end{bmatrix}$       E) NOTA

14. If  $S=4i-20j+k$ ,  $A=i+6j-9k$ , and  $E=2i+4j+5k$ . Then calculate  $6S+9A-8E$

A)  $17i-98j-115k$     B)  $42i-24k-96k$     C)  $69i+84K-20k$     D)  $-17i+98j+115k$     E) NOTA

15. What is the dot product of the vectors  $[6, -4]$  and  $[9, 20]$ ?

A) -62                  B) 26                  C) 62                  D) -26                  E) NOTA

16. What is the volume of a parallelepiped with the following three dimensional vectors given:  $\langle 5, 1, 5 \rangle$ ,  $\langle 0, 6, -9 \rangle$ , and  $\langle 4, 2, 0 \rangle$ ?

A)  $\frac{11}{2}\sqrt{2}$               B)  $\sqrt{69}$               C) 2016              D)  $\frac{5}{9}\sqrt{7}$               E) NOTA

17. Standing on the western bank of a river that spans 6 miles across, Becky wishes to swim to the eastern bank. She swims at a speed of  $3\sqrt{3}$  miles per hour, and the river flows due south with a current of 3 miles per hour. What distance, in miles, will Becky traverse if they cross the river in as little time as possible?

A) 7                      B) 5                      C)  $4\sqrt{3}$                       D)  $6\sqrt{3}$                       E) NOTA

18. Assume that a plane always flies at the same airspeed. The plane starts at the origin and flies for 5 hours at  $150^\circ$  clockwise of north and then promptly switches its direction to  $270^\circ$  clockwise of north, how many hours will it take the plane to reach the y-axis again?

A)  $5/2$                   B) 3                      C) 4                      D)  $\sqrt{2}$                       E) NOTA

19. Suppose  $c = 3$  is the determinant of  $A$  and the adjoint of  $A$  is  $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 9 \\ 1 & 9 & 6 \end{bmatrix}$  and  $B$  is  $\begin{bmatrix} 6 & 9 & 1 \\ 9 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$

Find  $A^{-1} + B$

- A)  $\begin{bmatrix} 2 & 3 & 1 \\ 3 & 1 & 9 \\ 1 & 9 & 4 \end{bmatrix}$     B)  $\begin{bmatrix} \frac{2}{3} & 9 & \frac{1}{3} \\ 9 & \frac{1}{3} & 3 \\ \frac{1}{3} & 3 & 6 \end{bmatrix}$     C)  $\begin{bmatrix} \frac{20}{3} & 9 & \frac{4}{3} \\ 9 & \frac{4}{3} & 3 \\ \frac{4}{3} & 3 & 4 \end{bmatrix}$     D)  $\begin{bmatrix} 6 & 9 & 1 \\ 9 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$     E) NOTA

20. Which of the following is an eigenvector of  $\begin{bmatrix} 2 & 0 \\ 1 & 6 \end{bmatrix}$ ?

- A)  $\begin{bmatrix} 6 \\ 9 \end{bmatrix}$     B)  $\begin{bmatrix} 5 \\ -1 \end{bmatrix}$     C)  $\begin{bmatrix} 5 \\ 0 \end{bmatrix}$     D)  $\begin{bmatrix} 4 \\ -1 \end{bmatrix}$     E) NOTA

21. Which of the following matrices is symmetric?

- A)  $\begin{bmatrix} 2 & 0 & 1 \\ 6 & 5 & 1 \\ 5 & 0 & 1 \end{bmatrix}$     B)  $\begin{bmatrix} 6 & 9 & 1 \\ 9 & 1 & 0 \\ 1 & 0 & 6 \end{bmatrix}$     C)  $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$     D)  $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$     E) NOTA

22. Let  $S$ ,  $A$ , and  $E$  be 2 by 2 matrices whose  $i, j$  th elements are  $s_{i,j}$ ,  $a_{i,j}$ , and  $e_{i,j}$ , respectively. If

$e_{i,j} = \sum_{n=1}^2 s_{n,i} \cdot a_{n,j}$ , which of the following is true?

- A)  $E = (SA)^T$     B)  $E = SA^T$     C)  $E = S^T A$     D)  $E = SA$     E) NOTA

2016

23. Evaluate:  $\begin{pmatrix} \cos \frac{\pi}{6} & -\sin \frac{\pi}{6} \\ \sin \frac{\pi}{6} & \cos \frac{\pi}{6} \end{pmatrix}$

- A)  $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$     B)  $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$     C)  $\begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{-1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$     D)  $\begin{pmatrix} \frac{-1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$     E) NOTA

24. Given the vector system

$$\bar{x} + 2\bar{y} + 3\bar{z} = [5 \ 1 \ 5]$$

$$-4\bar{y} + \bar{z} = [0 \ 4 \ 2]$$

$$3\bar{y} - \bar{z} = [0 \ 6 \ 9]$$

Find the value of  $\sum_{n=1}^3 x_n y_n z_n$ , the sum of the products of the  $n$ th components of the vectors.

A) 2016

B) 7

C) 117126

D) 1

E) NOTA

25. Find the sum of all  $x$  which satisfy

$$\begin{bmatrix} 4x^2 - 20x \\ 6x^2 + 3x + 9 \\ 2x^2 - 16 \end{bmatrix} + \begin{bmatrix} 8x + 8 \\ -21x + 3 \\ 4x \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

A) 2

B) -1

C) 1

D) -9

E) NOTA

26. Solve for  $s$ ,  $a$ , and  $e$  in the matrix equation  $6 \begin{bmatrix} s & a \\ e & -1 \end{bmatrix} = 3 \begin{bmatrix} a & e \\ -s & 1 \end{bmatrix} + 3 \begin{bmatrix} 6 & -2 \\ 7 & -s \end{bmatrix}$

$$s = 20$$

$$s = 5$$

$$s = \frac{5}{3}$$

$$s = 3$$

A)  $a = 1$ B)  $a = -1$ C)  $a = 3$ D)  $a = 0$ 

E) NOTA

$$e = 6$$

$$e = 50$$

$$e = 5$$

$$e = 2$$

27. Find  $A$  given that  $(3A)^{-1} = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$

A)  $\begin{bmatrix} \frac{4}{3} & \frac{2}{3} \\ \frac{1}{3} & 1 \end{bmatrix}$

B)  $\begin{bmatrix} \frac{1}{10} & \frac{-1}{15} \\ \frac{-1}{30} & \frac{2}{15} \end{bmatrix}$

C)  $\begin{bmatrix} \frac{4}{3} & \frac{-2}{3} \\ \frac{-1}{3} & 1 \end{bmatrix}$

D)  $\begin{bmatrix} \frac{51}{50} & \frac{-69}{31} \\ \frac{-41}{20} & \frac{2}{15} \end{bmatrix}$

E) NOTA

28. If  $A$  is a  $4 \times 4$  matrix such that  $|A| = 6$ , what is the value of  $|9A|$

A) 9999

B) 216

C) 54

D) 39366

E) NOTA

29. Find the cosine of the angle  $\theta$  between  $u = 4i + 2j + k$  and  $v = i + 6j + 9k$

A)  $6\sqrt{70}$

B)  $\frac{25}{6\sqrt{70}}$

C)  $\frac{6\sqrt{70}}{25}$

D) 25

E) NOTA

30. A is an orthogonal  $n \times n$  matrix. What is  $A^T A$ ?

A)  $A^{-1}$

B)  $A^{2T}$

C)  $I_n$

D)  $-A$

E) NOTA