Answers:

1. C
2. A
3. C
4. A
5. D
6. B
7. E
8. D
9. C
10. B
11. C
12. D
13. C
14. A
15. D
16. E
17. C
18. A
19. C
20. D
21. B
22. C
23. A
24. C
25. A
26. D
27. B
28. D
29. E
30. C

Solutions:

1. $\begin{bmatrix}3 & 9 & 3\\4 & 6 & 2\\5 & 1 & 5\end{bmatrix}+\begin{bmatrix}5 & -2 & 8\\3 & -9 & 5\\3 & 1 & -6\end{bmatrix}=\begin{bmatrix}3+5 & 9-2 & 3+8\\4+3 & 6-9 & 2+5\\5+3 & 1+1 & 5-6\end{bmatrix}=\begin{bmatrix}8 & 7 & 11\\7 & -3 & 7\\8 & 2 & -1\end{bmatrix}$ C

2. $5\begin{bmatrix}3 & 11 & -8\\1 & -5 & -9\end{bmatrix}=\begin{bmatrix}15 & 55 & -40\\5 & -25 & -45\end{bmatrix}$, $2\begin{bmatrix}7 & 4 & 5\\4 & -2 & 10\end{bmatrix}=\begin{bmatrix}14 & 8 & 10\\8 & -4 & 20\end{bmatrix}$, then
$\begin{bmatrix}15 & 55 & -40\\5 & -25 & -45\end{bmatrix}-\begin{bmatrix}14 & 8 & 10\\8 & -4 & 20\end{bmatrix}=\begin{bmatrix}1 & 47 & -50\\-3 & -21 & -65\end{bmatrix}$ A

3. Since top 2 rows and bottom rows are opposite they reverse each other and the determinant is 0

4. Look for all points P such that the vector $\overrightarrow{AP}$ is a multiple vector of $\overrightarrow{AB}$. So $\overrightarrow{AP}$ is therefore on the same of opposite direction as $\overrightarrow{AB}$ this causes P to line on the line connecting A and B.

A

5. multiple 8 (for the amount of seconds it travels) and 3 (the rate of travel) = 24
Then 24= $u\sqrt{2^2+(-1)^2+(-2)^2}$, so 24=3u so u=8. Then [2, 4, 9] + 8[2, -1, -2] = [x, y, z]. Then [2, 4, 9] + [16, -8, -16] = [18, -4, -7] D

6. Formula: if θ is the angle between the vectors, then Cosθ= $\frac{[a,b]\cdot[c,d]}{\sqrt{a^2+b^2}\cdot\sqrt{c^2+d^2}}$ so
$\frac{[5,12]\cdot[-8,6]}{\sqrt{5^2+12^2}\cdot\sqrt{(-8)^2+6^2}}=\frac{-40+72}{13\cdot 10}=\frac{32}{130}=\frac{16}{65}$ then secθ=$\frac{65}{16}$ B

7. $\begin{vmatrix}x & y & 4\\1 & 6 & 3\\3 & -5 & 9\end{vmatrix}$ is 69x + 0y – 76 = 69x – 92 E

8. $\begin{vmatrix}-3 & x-11\\2x & 6\end{vmatrix}$ = $-2x^2+22x-18>2$ $2x^2-22x+20>0$ $2(x^2-11x+10)$ (x-10)(x-1) <0
1<X<10 D

9. The trace is the sum of the main diagonal of the matrix so the trace is 1+7+8+7+1=24 C

10. To find the eigenvalues of D, we take $\begin{vmatrix} 6-\lambda & 9 \\ 6 & 9-\lambda \end{vmatrix}$ and set equal to zero and solve. The determinant leaves us with the quadratic $\lambda^2 - 15\lambda$, which has the solutions λ = 15 and λ = 0. The larger of these two is 15, and the smaller is 0. Therefore, 28x + 10y = 420 – 0 = 420. B

11. By the standard matrix formula, the area is equal to $\frac{1}{2}\begin{vmatrix} 4 & 2 & 1 \\ 0 & 6 & 1 \\ 9 & 0 & 1 \end{vmatrix} = 6.$ C

12. Det(AB) = Det(A)Det(B). Since Det(AB) = 420, and Det(A) = 6+54 = 60, it must be that Det(B) = 7. The only choice that has a determinant of 7 is D. D

13. A=4, B=-324, C=123, D=45. So C has the largest determinant. C

14. 6S=24i-120j+6k, 9A= 9i+54j-81k, and 8E=16i+32j+40k. So 6S+9A-8E =

(24i-120j+6k) + (9i+54j-81k) - (16i+32j+40k) = 17i-98j-115k. A

15. 6(9) + (-4)(20) = -26. D

16. By putting the three vectors into a 3x3 matrix and finding the determinant, we can see the volume of the parallepiped is 66. E

17. The shortest time will be when Becky heads due east and allows the current to sweep them southward along the way. This time will be the shortest because all of her speed is directed in an east-west direction (if she was to angle her direction, they would reduce the east-west component of their speed, and thus it take them longer to cross the east-west 6 mile span of the river). Using vector addition, one can find the speed and distance of their resultant journey:

$$\theta \tan^{-1}\frac{3}{3\sqrt{3}} = 30^o \qquad \frac{6}{\cos 30^o} = \frac{6}{\frac{\sqrt{3}}{2}} = \frac{12\sqrt{3}}{3} = 4\sqrt{3}. \qquad \text{C}$$

18. If a plane flies for 5 hours at x miles per hour at a bearing of 150 degrees, which is 30 degrees east of south, it will end up 5x units from the origin, at the point. Of it then switches its bearing to 270 degrees, which is west, we can set up the relationship sin(30)=x/5, so x= 5/2. A

19. $A^{-1}=\frac{1}{3}\begin{bmatrix}2&0&1\\0&1&9\\1&9&6\end{bmatrix}$ $A^{-1}+B=\begin{bmatrix}\frac{2}{3}&0&\frac{1}{3}\\0&\frac{1}{3}&3\\\frac{1}{3}&3&2\end{bmatrix}+\begin{bmatrix}6&9&1\\9&1&0\\1&0&2\end{bmatrix}=\begin{bmatrix}\frac{20}{3}&9&\frac{4}{3}\\9&\frac{4}{3}&3\\\frac{4}{3}&3&4\end{bmatrix}$. C

20. D is the only eigenvector of $\begin{bmatrix}2&0\\1&6\end{bmatrix}$ with a scale of 2. D

21. B is the only matrix given that is the same after being rotated making B the only symmetric matrix given. B

22. Write out the matrices and it is the definition.

23. The matrix represents a counter-clockwise rotation of $\frac{\Pi}{6}$ about the origin repeated 2016 times. Thus, it's the same as a single rotation of $\frac{\Pi}{6}(2016)=336\Pi$. This is conterminal to 2Π. So the answer is $\begin{pmatrix}\cos 2\Pi & -\sin 2\Pi\\ \sin 2\Pi & \cos 2\Pi\end{pmatrix}=\begin{pmatrix}1&0\\0&1\end{pmatrix}$. A

24. $\begin{bmatrix}1&2&3\\0&-4&1\\0&3&-1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}5&1&5\\0&4&2\\0&6&9\end{bmatrix}$ A= $\begin{bmatrix}1&2&3\\0&-4&1\\0&3&-1\end{bmatrix}$. $|A|=1$. $A^{-1}=\begin{bmatrix}1&11&14\\0&-1&-1\\0&-3&-4\end{bmatrix}$

Then $\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1&11&14\\0&-1&-1\\0&-3&-4\end{bmatrix}\begin{bmatrix}5&1&5\\0&4&2\\0&6&9\end{bmatrix}=\begin{bmatrix}5&129&153\\0&-10&-11\\0&-36&-42\end{bmatrix}$. Then $\sum_{n=1}^{3}x_n y_n z_n$ =
(0+46440+70686)=117126. C

25. $\begin{matrix}4x^2-12x+8\\6x^2-18x+12=0\\x^2+2x-8\end{matrix}$ $\begin{matrix}4(x^2-3x+2)=(x-2)(x-1)\\6(x^2-3x+2)=(x-2)(x-1)\\x^2+2x-8=(x-2)(x+4)\end{matrix}$ x=2 only solution. A

26. $\begin{bmatrix} 6s & 6a \\ 6e & -6 \end{bmatrix} = \begin{bmatrix} 3a & 3e \\ -3s & 3 \end{bmatrix} + \begin{bmatrix} 18 & -6 \\ 21 & -3s \end{bmatrix}$. $\begin{bmatrix} 6s & 6a \\ 6e & -6 \end{bmatrix} = \begin{bmatrix} 3a+18 & 3e+3s \\ -3s+21 & 3-3s \end{bmatrix}$

3-3s=-6 s=3. 6e=-3s+21 6e=12 e=2. 6a=3(2)-60 a=0. $\begin{matrix} s=3 \\ a=0 \\ e=2 \end{matrix}$ D

27. $(3A)^{-1} = \frac{1}{3}(A^{-1})$. $A^{-1} = \begin{bmatrix} 12 & 6 \\ 3 & 9 \end{bmatrix}$. $(A^{-1})^{-1} = \frac{1}{|A|}\begin{bmatrix} 9 & -6 \\ -3 & 12 \end{bmatrix}$. $|A| = 90$. So $A = \begin{bmatrix} \frac{1}{10} & -\frac{1}{15} \\ -\frac{1}{30} & \frac{2}{15} \end{bmatrix}$.

B

28. $|9A| = 9^4|A| = 9^4(6) = 39366.$ D

29. Formula: $\cos\Theta = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\cdot\|\vec{v}\|}$ $\qquad$ $\vec{u}\cdot\vec{v} = 4(1)+(2)(6)+9(1) = 25$

$\|\vec{v}\| = \sqrt{4^2+2^2+1^2} = \sqrt{21}$ $\qquad$ $\|u\| = \sqrt{1^2+6^2+9^2} = \sqrt{118}$

$\Theta = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\cdot\|\vec{v}\|} = \frac{25}{\sqrt{2478}}$ E

30. Definition. $A^T A = I_n$ for orthogonal matrices.