#0 Alpha Bowl
MAΘ National Convention 2015

Consider when $f(x) = 6x^7 - 10x^6 + x^5 - 7x^4 + 7x^2 + x + 5$ is divided by $g(x) = 3x - 2$.

If $q(x)$ and $r(x)$ are the quotient and remainder functions, respectively, that exist according to the Division Algorithm when $f(x)$ is divided by $g(x)$, find the value of $q(1) + r(2015)$.

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#1 Alpha Bowl
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$x^{\log x^4} = \frac{x^8}{1000}$ has two positive solutions. Let A = the ratio of the larger solution to the smaller solution.

Let $B = \log_5 625^{10}$.

Let $C = -3\log_8 4$.

Find $\frac{B \cdot C}{A}$.

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#2 Alpha Bowl
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Use $M = \begin{bmatrix} 1 & -1 & 1 \\ 0 & -2 & 1 \\ -2 & -3 & 0 \end{bmatrix}$ for the following problems.

Let A = the element in row 2, column 3, of M^{-1}.

Let B = the element in row 2, column 3, of M^T. (T represents the transpose matrix.)

Let C = the element in row 2, column 3, of $adjM$. (*adj* represents the adjoint matrix.)

Let D = the element in row 2, column 3, of M^2.

Find the value of $A+B+C+D$.

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#3 Alpha Bowl
MAΘ National Convention 2015

Given the equation of the ellipse $9(x-1)^2+4(y+3)^2=36$, find the value of $A\cdot B\cdot C$ given the following:

A = the distance between the foci.

B = the focal width.

C = the distance from one focus to the closer directrix.

Leave your answer as an improper fraction.

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#4 Alpha Bowl
MAΘ National Convention 2015

A = the sum of the first 60 positive even integers.

B = the sum of the squares of the first 60 positive integers.

C = the sum $1+2+2^2+2^3+...+2^{n-1}$ for $n=14$.

D = the sum of the numerator and denominator in the simplified sum of

$$\frac{1}{2\cdot3}+\frac{1}{3\cdot4}+\frac{1}{4\cdot5}+...+\frac{1}{(n+1)\cdot(n+2)} \text{ for } n=10.$$

Find the sum of the values of *A*, *B*, *C*, and *D* which are multiples of 3.

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#5 Alpha Bowl
MAΘ National Convention 2015

DeBrie has two bags. Bag 1 contains 3 red and 2 green balls, and Bag 2 contains 4 red and 5 green balls. DeBrie will toss a fair coin and if a head turns up, she will select a ball from Bag 1. If a tail turns up, she will select a ball from Bag 2.

A = the probability that a green ball is selected.

B = the probability that a green ball from Bag 2 is selected.

Suppose DeBrie will select Bag 1 or Bag 2 based on the outcome of a standard, fair, six-sided die. If a 5 or 6 turns up, she will choose a ball from Bag 1; otherwise, she will choose from Bag 2.

C = the probability of selecting a green ball.

If all drawings were random, find the value of $A + B - C$.

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#6 Alpha Bowl
MAΘ National Convention 2015

Over a long period of time it has been observed that Cowboy Curtis can hit a target with probability 0.8.

A = the probability that in four shots, he hits the target exactly two times.

B = the probability that in four shots, he hits the target at least two times.

When written in reduced form, $A+B=\frac{4^x 11^y}{5^z}$. Find $x+y+z$.

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#7 Alpha Bowl
MAΘ National Convention 2015

Use the fact that $e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$ to evaluate each limit, then find the product *ABCD*.

$A = \lim_{n\to\infty} 2\left(1+\frac{1}{n}\right)^n$ $B = \lim_{n\to\infty}\left(1+\frac{1}{2n}\right)^{6n}$ $C = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{2n}$ $D = \lim_{n\to 0}\left(1+3n\right)^{\frac{1}{3n}}$

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#8 Alpha Bowl
MAΘ National Convention 2015

Let (A, B) be the solution to the system $\begin{cases} 2^A \cdot 2^B = 8 \\ \dfrac{2^A}{2^B} = 32 \end{cases}$.

The solution to $3^{2x-1} = 2^{3x}$ is $\log_{\frac{C}{D}} 3$, where C and D are positive, relatively prime integers.

$E = e^{\ln 3} + e^{\ln 2}$.

F is the solution to $\log_2(4x+4) - 2\log_2 x = 3$.

Find the value of $A+B+C+D+E+F$.

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#9 Alpha Bowl
MAΘ National Convention 2015

Let A = the 2015th digit after the decimal in the decimal expansion of $\frac{1}{7}$.

Let B = the smallest positive integer greater than 2 which leaves a remainder of 2 when divided by 3, 4, 5, 6, 7, or 8.

In a Fibonacci-type sequence of increasing positive integers, each number after the first two is equal to the sum of the two numbers that immediately precedes it. If the 10th number in the sequence is 322, compute the 5th number, C.

Find $A+B-C$. (Hint: The answer is a palindrome.)

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#10 Alpha Bowl
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Find the value of $\frac{B}{A \cdot C}$, given:

$A = \sin\left[\mathrm{Cos}^{-1}\left(\frac{3}{5}\right) + \frac{\pi}{2}\right].$ $B = \sin\left[\mathrm{Tan}^{-1}\left(-\frac{3}{4}\right) + \mathrm{Cos}^{-1}\left(-\frac{4}{5}\right)\right].$ $C = \tan\left[\frac{1}{2}\mathrm{Arcsin}\left(\frac{15}{17}\right)\right].$

Leave your answer as an improper fraction.

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Leave your answer as an improper fraction.

#11 Alpha Bowl
MAΘ National Convention 2015

Let A = the square of the area of the parallelogram with sides $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ with $P(1,4,6)$, $Q(-2,5,-1)$, and $R(1,-1,1)$.

Let B = the scalar triple product of vectors $a=\langle 1,4,-7\rangle$, $b=\langle 2,-1,4\rangle$, and $c=\langle 0,-9,18\rangle$.

Let C = the volume of the parallelepiped with adjacent edges $\overline{GH}$, $\overline{GI}$, and $\overline{GJ}$ with $G(2,0,-1)$, $H(4,1,0)$, $I(3,-1,1)$, and $J(2,-2,2)$.

Find the value of $A+B+C$.

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Find the value of $A+B+C$.

#12 Alpha Bowl
MAΘ National Convention 2015

Find the value of $A+B+C+D$ given that:

$A=$ the area enclosed by the curve generated by the polar equation $r=-2$.

$B=$ the area enclosed by the curve generated by the polar equation $r=2\cos\theta$.

$C=$ the area enclosed by the curve generated by the polar equation $r=\dfrac{10}{3+2\cos\theta}$.

$D=$ the area enclosed by the curve generated by the parametric equations $\begin{cases} x=4\cos t \\ y=3\sin t \end{cases}, t\in\mathbb{R}$.

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#13 Alpha Bowl
MAΘ National Convention 2015

Let A, B, C be the roots of $2x^3 - 7x^2 + kx - 2 = 0$, given that the roots are in geometric progression.

The equation $x^3 + Dx^2 + Ex + F = 0$ has the root $\sqrt[3]{\sqrt{28}+6} - \sqrt[3]{\sqrt{28}-6}$ with D, E, and F as integers.

Find the value of $A \cdot B \cdot C + D + E + F$.

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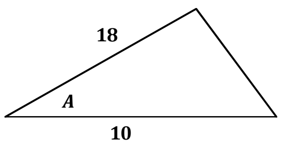
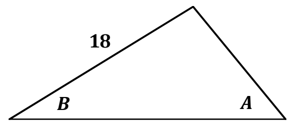
Find the value of $A \cdot B \cdot C + D + E + F$.

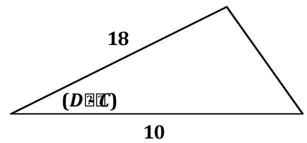
#14 Alpha Bowl
MAΘ National Convention 2015

When solved over $0^\circ \le x < 360^\circ$, $\tan(2x) = \sqrt{3}$ has four solutions, A, B, C, and D, where $A < B < C < D$.

Using the diagrams below and the angle measurements found above, find the sum of the areas enclosed by the three triangles.







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