Q0-The inequalities become $-1 \le x < 2$ or $-1 < x < 1$, so **A=3**.

The line has equation $y = \frac{2}{n-2}x + 4$, so $-4 = \frac{2n}{n-2} \Rightarrow -4n + 8 = 2n \Rightarrow 8 = 6n \Rightarrow n = \frac{4}{3}$ **=B**

$\frac{x^3 - y^3}{x^4 + x^2y^2 + y^4} \bullet \frac{x^3 + y^3}{x^2 - y^2} = \frac{x^6 - y^6}{x^6 - y^6} = 1$ **=C**

Multiplying both sides of the equation by $(x-1)(x+1)(x+3)$ yields $(x-2)(x+3) - 3(x-1)$

$= (2x-1)(x+1) \Rightarrow x^2 - 2x - 3 = 2x^2 + x - 1 \Rightarrow 0 = x^2 + 3x + 2 = (x+1)(x+2) \Rightarrow x = -2$ since $x = -1$ does not work. **D=-2**

ABCD= $3 \cdot \frac{4}{3} \cdot 1 \cdot -2 = -8$

$$r = \frac{3}{\sin\theta} - \frac{5r}{\sin\theta} \rightarrow r\sin\theta = 3 - 5r \rightarrow y = 3 - 5\sqrt{x^2 + y^2} \rightarrow y^2 - 6y + 9 = 25x^2 + 25y^2$$

$$25x^2 + 24y^2 + 6y - 9 = 0 \rightarrow 25 + 24 + 6 + 9 = 64$$

Q1- $6r - r\sin\theta = 2 \rightarrow 6r - 2 = y \rightarrow 6\sqrt{x^2 + y^2} = y + 2 \rightarrow y^2 + 4y + 4 = 36x^2 + 36y^2$

$$36x^2 + 35y^2 - 4y - 4 = 0 \rightarrow 36 + 35 + 4 + 4 = 79$$

$$64 + 79 = 143$$

Q2-A cube has 6 faces and each would have an area of 4, so the edge would be 2. The diameter of the inner sphere is the space diagonal for the cube, so:

$$3L^2 = 2^2 \rightarrow L^2 = \frac{4}{3} \rightarrow 6\left(\frac{4}{3}\right) = 8$$

The surface area of the top pyramid to the bottom pyramid is 1:2. Therefore the scale factor is:

$$\frac{\sqrt{2}}{2} = \frac{h}{h+2} \rightarrow 2h = h\sqrt{2} + 2\sqrt{2} \rightarrow h\sqrt{2} - 2h = -2\sqrt{2} \rightarrow h\left(\sqrt{2} - 2\right) = -2\sqrt{2}$$

$$h = \frac{-2\sqrt{2}}{\left(\sqrt{2} - 2\right)} \bullet \frac{\sqrt{2} + 2}{\sqrt{2} + 2} = \frac{-4 - 4\sqrt{2}}{-2} = 2 + 2\sqrt{2} \rightarrow 2 + 2\sqrt{2} + 2 = 4 + 2\sqrt{2}$$

$$8 + 4 + 2\sqrt{2} = 12 + 2\sqrt{2}$$

$$\left(\frac{2}{3}\right)^4+4\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)=\frac{16}{81}+\frac{32}{81}=\frac{48}{81}=\frac{16}{27}$$

Q3- $n\frac{1}{8}=8\to n=64$

$$\frac{16}{27}\bullet\frac{1}{64}=\frac{1}{108}$$

$$y^2=12(x+4)\to A=\frac{1}{2}(4p)(p)\to p=3\to A=18$$

Q4-

$$4\left(x^2-2x+1\right)+y^2+6y+9=4\to 4(x-1)^2+(y+3)^2=4$$

$$\frac{(x-1)^2}{1}+\frac{(y+3)^2}{4}=1\to A=\frac{2b^2}{a}\bullet 2c=2c=2\sqrt{3}$$

$$18+2\sqrt{3}$$

$$\lim_{x\to 0}\frac{2-\sqrt{4-x}}{x}\bullet\frac{2+\sqrt{4-x}}{2+\sqrt{4-x}}=\frac{4-4+x}{2+\sqrt{4-x}}=\frac{1}{4}$$

$$\lim_{x\to\infty}\left(\sqrt{x^2+2x}-x\right)=\sqrt{x^2+2x+1}-x=\sqrt{(x+1)^2}-x=x+1-x=1$$

Q5- $\lim_{x\to 0}\frac{4^x-4^{-x}}{4^x+4^{-x}}=\frac{1-1}{1+1}=0$

$$\lim_{x\to 0}\frac{\frac{1}{x+6}-\frac{1}{6}}{x}=\frac{6-(x+6)}{6x(x+6)}=\frac{-1}{6x+36}=-\frac{1}{36}$$

$$\frac{9+36-1}{36}=\frac{44}{36}\ or\ \frac{11}{9}$$

Q6-

$$S=140-A\to XA+\frac{X}{2}S=2001\to XA+70X-\frac{X}{2}A=2001\to\frac{X}{2}A+70X=2001$$

$$AX+140X=4002=X(A+140)=3\bullet 2\bullet 23\bullet 29=23\bullet 174\to X=23\ and\ A=34\to 23\bullet 34=782$$

$$\frac{d}{40}=t+3$$

$$\frac{d}{60}=t-3\to\frac{d}{120}=6\to d=720\to t=15\to\frac{720}{15}=48$$

$$782+48=830$$

Q7-Draw triangles in the proper quadrants to make this question easier!!

$A = \cos 2x = 2\cos^2 x - 1 = 2 \bullet \frac{9}{25} - 1 = \frac{-7}{25}$

$B = \sin 2x = 2\sin x\cos x = 2 \bullet \frac{5}{13} \bullet -\frac{12}{13} = -\frac{120}{169}$

$C = \cos 2x = 2\cos^2 x - 1 = 2 \bullet \frac{16}{25} - 1 = \frac{7}{25}$

$D = \tan 2x = \frac{2\sin x\cos x}{2\cos^2 x - 1} = \frac{2 \bullet \left(\frac{-12}{13}\right)\left(\frac{5}{13}\right)}{2\left(\frac{5}{13}\right)^2 - 1} = \frac{\frac{-120}{169}}{\frac{-119}{169}} = \frac{120}{119}$

$\frac{-7}{25} \bullet -\frac{120}{169} \bullet \frac{25}{7} \bullet \frac{119}{120} = \frac{119}{169}$

Q8-If you start listing the possible factors you can quickly see that there is only one factor that works to create both roots prime. You cannot have 2 odd roots and make the middle term work so 2 must be one of the roots!!

$(x-1)(x-62) \, or \, ** (x-2)(x-61) ** \, or \, (x-3)(x-60) \, or \ldots$

$(x-2)(x-61) \rightarrow only \, 122 \rightarrow ans \, 1$

$x - \frac{1}{x} = 1 \rightarrow x^2 - x - 1 = 0 \rightarrow \frac{1+\sqrt{5}}{2}$

$x - \frac{1}{x} = -1 \rightarrow x^2 + x - 1 = 0 \rightarrow \frac{-1+\sqrt{5}}{2}$

$\frac{1+\sqrt{5}}{2} + \frac{-1+\sqrt{5}}{2} = \sqrt{5}$

$\sqrt{5}$

Q9-Draw a 6 by 8 and you will quickly see what products work. All factors that have 3 and 6 will work. There are 24 such factors out of 48 so 1/2

Greater than zero means two negatives or two positives so:

$\left(\frac{2}{3}\right)\left(\frac{2}{3}\right) = \frac{4}{9}$

$\left(\frac{1}{3}\right)\left(\frac{1}{3}\right) = \frac{1}{9}$

$\frac{5}{9} + \frac{1}{2} = \frac{10+9}{18} = \frac{19}{18}$

$$x = wr = w + d$$

$$z = wr^2 = w + 3d \rightarrow wr - w = d \rightarrow wr^2 = w + 3wr - 3w \rightarrow wr^2 = 3wr - 2w$$

$$r^2 - 3r + 2 = 0 \rightarrow (r-1)(r-2) = 0 \rightarrow r = 2 \rightarrow \frac{z}{w} = r^2 = 4$$

Q10-

note: r cannot be one because the sequence is increasing!!

$$\frac{a}{1-r} = 7 \rightarrow 1 - r = \frac{a}{7}$$

$$ar + ar^3 + ar^5 + ... = 3 \rightarrow \frac{ar}{1-r^2} = 3 \rightarrow ar = 3(1-r)(1+r) = 3 \bullet \frac{a}{7}(1+r)$$

$$r = \frac{3}{7} + \frac{3}{7}r \rightarrow \frac{4}{7}r = \frac{3}{7} \rightarrow r = \frac{3}{4} \rightarrow a = \frac{7}{4} \rightarrow \frac{3}{4} + \frac{7}{4} = \frac{5}{2}$$

$$4 \bullet \frac{5}{2} = 10$$

$$\csc^2 x - 2\cot x = 1 + \cot^2 x - 2\cot x = (\cot x - 1)^2 = 0 \rightarrow \cot x = 1 \rightarrow x = \frac{\pi}{4}\, and\, \frac{5\pi}{4} \rightarrow \frac{3\pi}{2}$$

$$\tan x + \sqrt{3} = \sec x \rightarrow \tan^2 x + 3 + 2\sqrt{3}\tan x = 1 + \tan^2 x \rightarrow 2\sqrt{3}\tan x = -2 \rightarrow \tan x = \frac{-1}{\sqrt{3}}$$

$$\frac{5\pi}{6}\, and\, \frac{11\pi}{6} \rightarrow \frac{5\pi}{6}\, is\, extraneous \rightarrow \frac{11\pi}{6}$$

Q11-

$$2\cos 2x + 2\sin^2 x = 2(\cos^2 x - \sin^2 x) + 2\sin^2 x = 1 \rightarrow \cos^2 x = \frac{1}{2} \rightarrow \cos x = \frac{\pm\sqrt{2}}{2}$$

$$\frac{\pi}{4} + \frac{3\pi}{4} + \frac{5\pi}{4} + \frac{7\pi}{4} = 4\pi$$

$$\frac{3\pi}{2} + \frac{11\pi}{6} + 4\pi = \frac{44\pi}{6} = \frac{22\pi}{3}$$

$$a^2 b = 10 \rightarrow b = \frac{10}{a^2}$$

$$ab^3 = 10 \rightarrow a \bullet \left(\frac{10}{a^2}\right)^3 = 10 \rightarrow a^5 = 100 \rightarrow a = 10^{\frac{2}{5}} \rightarrow b = 10^{\frac{1}{5}}$$

Q12- $ab = 10^{\frac{3}{5}} \rightarrow \frac{3}{5}$

$$r = 2\log x \rightarrow 2r\pi = 4\log y \rightarrow 4\pi \log x = 4\log y \rightarrow \pi = \frac{\log y}{\log x} = \log_x y = \pi$$

$$\frac{3\pi}{5}$$

Q13- You will need some formulas here. $\pi rl = \frac{3}{5}\left(\pi rl + \pi r^2\right) \to l = \frac{3}{5}\left(l - r\right) \to \frac{2}{5}l = \frac{3}{5}r \to \frac{r}{l} = \frac{2}{3}$

The hexagonal pyramid creates a 6-8-10 right triangle where the hypotenuse of the right triangle is the slant height of the cone and the base edge of 6 is the radius. $\pi rl = \pi(6)(10) = 60\pi$

AB= $\frac{2}{3} \bullet 60\pi = 40\pi$

Q14-

A:

$$\left(\frac{1}{\sqrt{4}+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{6}} + \frac{1}{\sqrt{6}+\sqrt{7}} + \ldots + \frac{1}{\sqrt{63}+\sqrt{64}}\right)\left(\frac{\left(\sqrt{4}-\sqrt{5}\right)+\left(\sqrt{5}-\sqrt{6}\right)+\left(\sqrt{6}-\sqrt{7}\right)+\ldots+\left(\sqrt{63}-\sqrt{64}\right)}{-1}\right) = \frac{\sqrt{4}-\sqrt{64}}{-1} = \frac{2-8}{-1} = 6$$

B: $\frac{n}{2}(1+n) = 4950 \to n^2 + n - 9900 = 0 \to (n+100)(n-99) = 0 \to n = 99$

A-B: $6 - 99 = -93$

Answers:

0. -8

1. 143

2. $12 + 2\sqrt{2}$

3. $\frac{1}{108}$

4. $18 + 2\sqrt{3}$

5. $\frac{11}{9}$

6. 830

7. $\frac{119}{169}$

8. $\sqrt{5}$

9. $\frac{19}{18}$

10. 10

11. $\frac{22\pi}{3}$

12. $\frac{3\pi}{5}$

13. 40π

14. -93