Alpha Systems of Equations

For all answers, choice "E. NOTA" means none of the above answers is correct.

- 1. The expression $\frac{-3x-9}{(x^2+2)(x^2-1)}$ can be decomposed into partial fractions as $\frac{Ax+B}{x^2+2} + \frac{C}{x-1} + \frac{D}{x+1}$. Find the value of A + B + C + D. A. -1 B. 1 C. -3 D. 3 E. NOTA 2. Evaluate $\bigotimes_{n=1}^{20} \frac{1}{(3n-1)(3n+2)}$. A. $\frac{5}{31}$ B. $\frac{15}{91}$ C. $\frac{1}{9}$ D. $\frac{16}{93}$ E. NOTA
- 3. In an international cricket tournament, there are teams from at least four different countries in a group, and each country in a group must play every country in the group exactly once. The number of matches *M* that will be played when there are *n* teams in a group can be found by $M = an^2 + bn + c$. Find the value of 5a + 2b + c. (*n*=1 represents the four-team minimum, *n*=2 represents five teams.)
 - A. $\frac{23}{6}$ B. $\frac{19}{2}$ C. $\frac{21}{2}$ D. $\frac{71}{8}$ E. NOTA
- 4. Anastasia was at the board, attempting to solve a system of equations (in (*x*, *y*, *z*)) by Cramer's rule when she was called to the office to check out. Greta was then called upon to finish the problem.

If Anastasia had correctly written $y = \begin{vmatrix} 2 & -1 & -1 \\ -2 & 1 & 2 \\ 1 & 2 & 1 \end{vmatrix}$, what will be the value of the determinant in $\begin{vmatrix} 2 & 1 & -1 \\ -2 & 1 & 2 \\ 1 & 1 & 1 \end{vmatrix}$, what will be the value of the determinant in the numerator when solving for x? A. -5 B. -1 C. 1 D. -3 E. NOTA

5. Find the value of $\tan y$, where $0 < y < \frac{\pi}{2}$, given $\begin{cases} 100\cos x + 200\cos y = 250\\ 100\sin x + 200\sin y = 0 \end{cases}$.

A.
$$\frac{\sqrt{2962}}{37}$$
 B. $\frac{\sqrt{231}}{37}$ C. $\frac{40\sqrt{231}}{231}$ D. $\frac{37\sqrt{231}}{231}$ E. NOTA

6. The system of equations $\begin{cases} r = 1 - \sin q \\ r = \cos 2q \end{cases}$, where r > 0 and $0 \le q < 2p$, has *m* solutions and the graphs overlap *n* times. Find m + n. Do not consider a point as a solution unless the representation solves both equations. For example, $\left(5, \frac{p}{4}\right)$ is not the same as $\left(-5, \frac{3p}{4}\right)$ unless both points solve both equations.

A. 5 B. 6	C. 8	D. 9	E. NOTA
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7. The points of intersection of the graphs of xy = 20 and $x^2 + y^2 = 41$ are joined to form a convex quadrilateral. Find the area of the quadrilateral. A. 40 B. 10 C. 24 D. 18 E. NOTA 8. Find the remainder for when $(x^{100} - 2x^{99} + 4)$ is divided by $(x^2 - 3x + 2)$. B. x + 2C. x + 4D. 4 A. x - 2 E. NOTA 9. The sides of a triangle are of length 8, 10, and 14. Find the altitude onto the side of length 14. B. $8\sqrt{2}$ C. $\frac{16\sqrt{6}}{7}$ D. $\frac{12\sqrt{3}}{5}$ A. √33 E. NOTA 10. The two solutions to $\begin{cases} \frac{x}{y} - \frac{6y}{x} = \frac{5}{xy} \\ x - 2y = 1 \end{cases}$ can be written in the form (a + bi, c + di), (a - bi, c - di), where $i = \sqrt{-1}$. Find the value of |a| + |b| + |c| + |d|. B. $2\sqrt{2} + 2\sqrt{13}$ C. 11 A. 7 D. 9 E. NOTA 11. Emma has a total of 100 coins, consisting only of dimes and quarters. If the total value of the coins is \$14.05, how many quarters does she have? A. 73 B. 40 C. 27 D. 56 E. NOTA 12. If the solution to the system of equations $\begin{cases} xy - 5x - 8y = 0\\ \log(x - 11) + \log(y - 5) = 1 \end{cases}$ is the ordered pair (x, y), find the value of the product *xy*. C. 120 D. 180 A. 48 B. 240 E. NOTA 13. If the graph of ax + by = 8 passes through (2, -3) and (1, 4), it also passes through (11, k). Find k. C. $\frac{18}{7}$ D. $\frac{38}{7}$ B. 74 E. NOTA A. -66 14. The solution to $\begin{cases} \frac{x}{\sqrt{x} + \sqrt{y}} = 18\\ \frac{y}{\sqrt{x} + \sqrt{y}} = 2 \end{cases}$ is the ordered pair (x, y). Find the value of $\frac{\sqrt{x}}{\sqrt[3]{y}}$. C. 4 D. 8 B. 6 A. 12 E. NOTA 15. In convex quadrilateral ABCD, the area of triangle DAB is 1, that of triangle ABC is 6, and that of triangle *CDA* is 2. Find the area of triangle *BCD*. A. 5 B. 7 C. 4 D. 3 E. NOTA

Alpha Systems of Equations

16. A plane flew 1750 km in 6 hours with a tail wind of constant velocity. It then flew back 540 km in 3 hours with a head wind of the same velocity as the tail wind. Find the positive difference between the speed of the plane and the speed of the wind. All answers are in km/hr.

A. 60 B. 180 C. 90 D. 120 E. NOTA

17. A carpenter is building a square room with a (smaller) square adjacent foyer according to the floor plan at the right. If the total area is to be 3284 ft² and the total perimeter is to be 256 ft, find the maximum area of the foyer in ft².



- A. $\frac{19321}{25}$ B. $\frac{68644}{25}$ C. $\frac{13456}{25}$ D. 784 E. NOTA
- 18. The parabola $y = ax^2 + bx + c$ passes through the points (1, 1), (-1, 5), and (0.5, 0.5). Find the magnitude of vector **m** if **m** = ai + bj + ck.

A. 3 B.
$$\frac{\sqrt{34}}{2}$$
 C. $\frac{\sqrt{94}}{2}$ D. 7 E. NOTA

19. Find the sum of the elements in matrix X so that $\begin{bmatrix} 5 & -3 \\ 4 & -2 \end{bmatrix} X = \begin{bmatrix} 5 \\ 10 \end{bmatrix}$ A. 10B. 15C. 30D. -15E. NOTA

20. An arithmetic sequence has second term 6 and 50^{th} term 342. Find the 101^{st} term in this sequence.

- A. 702 B. 723.75 C. 717.5 D. 699 E. NOTA
- 21. Which of the following is a homogeneous system of equations?

A.
$$\begin{cases} 3x + 4y = 8\\ 6x + 8y = 16 \end{cases}$$
B.
$$\begin{cases} 3x + 4y = 0\\ 7x - 6y = 0 \end{cases}$$
$$\begin{cases} 3x + 4y = 12\\ -9x - 12y = -48 \end{cases}$$
D.
$$\begin{cases} 3x + 4y = 11\\ 5x - y = 9 \end{cases}$$
E. NOTA

22. Solve for x: $\begin{cases} 4^{x+y} = 60 \\ 3^{x-y} = 5 \end{cases}$ A. B. C. D. E. $243 + \log_{\frac{4}{3}} 2\sqrt{3} \qquad \log_2 2\sqrt[4]{15} + \log_3 \sqrt{5} \qquad \frac{1}{2} + \log_2 \sqrt[4]{15} + \log_3 \sqrt{5} \qquad \log_2 \sqrt{15} + \log_3 \sqrt{5} \qquad \text{NOTA} \end{cases}$

23. For all $x, y \in \mathbb{Z}$, find the product of all integer values of x and y for which (x, y) is a solution to (6x + 15y)(8x + 7y) = 129. A. 576 B. 441 C. 784 D. 676 E. NOTA

24. When written in base 18, $888_9 = ABC_{18}$. Find the value of A - B - C.

A. 2 B. 4 C. -8 D. -3 E. NOTA

- 25. Each sop grader has 3 legs, each jun grader has 5 legs, and each sen grader has 7 legs. In a room are 30 graders, each of whom is a sop, jun, or sen. If there is a total of 120 legs, how many possible combinations of graders are there, if there is at least one of each type of grader in the room?
 - A. 5 B. 6 C. 7 D. 8 E. NOTA

26. A linear function y = f(x) satisfies f(f(x)) = 3x - 2. The positive possible value of f(-1) can be written in the form A+B√C, where A, B, and C are integers, and C is not divisible by the square of any integer greater than 1. Find A+B+C.
A. 2
B. 4
C. 6
D. 8
E. NOTA

27. Find the value of k so that the system $\begin{cases} x_1 + x_2 - x_3 = 2\\ x_1 + 2x_2 + x_3 = 3\\ x_1 + x_2 + (k^2 - 5)x_3 = k \end{cases}$ has no solution.

A. 2 B. -2 C. $\frac{1+\sqrt{5}}{2}$ D. no such *k* E. NOTA

28. Find the value of
$$x^3 + y^3$$
 such that
$$\begin{cases} xy = 3 \\ x^2 + y^2 = 10. \\ x + y > 0 \end{cases}$$
A. 64 B. 16 C. 52 D. 28 E. NOTA

29. The graphs of the parabolas $y = 2x^2$ and $y = x^2 + x + 6$ intersection in two points. Find the equation of the line that passes through these two points.

A. y - 2x + 18 = 0 B. 2x - y - 18 = 0 C. 2x - y + 12 = 0 D. 2x - y + 4 = 0 E. NOTA

30. The ordered triple that satisfies the system of equations shown below is the ordered triple (a, b, c), where a > b > c. (a, b, c) can be written in the form (d^e, f^g, h^k) , where d, e, f, g, h, and k are all integers, and d, e, and f are positive integers that are as small as possible. Find the value of d + e + f + g + h + k.

A. 16

$$\begin{cases} (\log_2 a)(\log_3 b)(\log_5 c) = -30\\ (\log_2 a)(\log_3 b) + (\log_2 a)(\log_5 c) + (\log_3 b)(\log_5 c) = -1\\ (\log_2 a) + (\log_3 b) + (\log_5 c) = 6 \end{cases}$$

A. 16
B. -8
C. 12
D. -4
E. NOTA