1. D
2. A
3. C
4. E 5
5. B
6. D
7. D
8. B
9. C
10. A
11. C
12. D
13. A
14. B
15. B
16. B
17. D
18. A
19. E 25
20. D
21. B
22. C
23. A
24. E –10
25. C
26. C
27. B
28. D
29. C
30. A

1. $\dfrac{-3x-9}{(x^2+2)(x^2-1)}=\dfrac{Ax+B}{x^2+2}+\dfrac{C}{x-1}+\dfrac{D}{x+1}\rightarrow -3x-9=(Ax+B)(x-1)(x+1)+C(x^2+2)(x+1)+D(x^2+2)(x-1).$

   If $x=1$: $-12=6C\rightarrow C=-2$ If $x=-1$: $-6=-6D\rightarrow D=1$

   $-3x-9=Ax^3+Bx^2-Ax-B+Cx^3+Cx^2+2Cx+2C+Dx^3-Dx^2+2Dx-2D$. Equating corresponding parts:

   $\begin{cases}A+C+D=0\\ B+C-D=0\\ -A+2C+2D=-3\\ -B+2C-2D=-9\end{cases}$ Using the values of C and D from above: $\begin{matrix}A-2+1=0\rightarrow A=1\\ B-2-1=0\rightarrow B=3\end{matrix}$. $1+3+(-2)+1=3$. D.

2. $\displaystyle\sum_{n=1}^{20}\frac{1}{(3n-1)(3n+2)}=\sum_{n=1}^{20}\left(\frac{A}{3n-1}+\frac{B}{3n+2}\right).$

   $1=(3n+2)A+B(3n-1)$ If $n=-\dfrac{2}{3}$: $1=-3B\rightarrow B=-\dfrac{1}{3}$ If $n=\dfrac{1}{3}$: $1=3A\rightarrow A=\dfrac{1}{3}$

   $\displaystyle\sum_{n=1}^{20}\left(\frac{1}{3(3n-1)}-\frac{1}{3(3n+2)}\right)=\frac{1}{6}-\frac{1}{15}+\frac{1}{15}-\frac{1}{24}+\frac{1}{24}-\frac{1}{33}+\ldots-\frac{1}{186}=\frac{5}{31}.$ A.

3. When $n=1$, there are 4 teams and ${}_4C_2=6$ matches $\rightarrow a+b+c=6$

   When $n=2$, there are 5 teams and ${}_5C_2=10$ matches $\rightarrow 4a+2b+c=10$

   When $n=3$, there are 6 teams and ${}_6C_2=15$ matches $\rightarrow 9a+3b+c=15$

   Adding the first and third equations we get $10a+4b+2c=21$, so $5a+2b+c=\dfrac{21}{2}$. C.

4. $x=\begin{vmatrix}-1&1&-1\\1&1&2\\2&1&1\end{vmatrix}=(-1)(1-2)-1(1-4)+(-1)(1-2)=1+3+1=5.$ E.

5. Begin by dividing both equations by 50, then squaring both sides and adding the equations together:

   $(2\cos x)^2+(2\sin x)^2=25-40\cos y+16\cos^2 y+16\sin^2 y\rightarrow 4=25-40\cos y+16\rightarrow 40\cos y=37.$

   Since y is in Quadrant I, we have a $37-\sqrt{231}-40$ right triangle so $\tan y=\dfrac{\sqrt{231}}{37}$. B.

6. $\begin{cases}r=1-\sin\theta\\ r=\cos 2\theta\end{cases}\rightarrow 1-\sin\theta=1-2\sin^2\theta\rightarrow\theta=0,\pi,\dfrac{\pi}{6},\dfrac{5\pi}{6}$. The solutions are $(1,0),(1,\pi),\left(\dfrac{1}{2},\dfrac{\pi}{6}\right),\left(\dfrac{1}{2},\dfrac{5\pi}{6}\right)$.

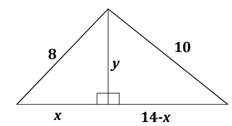
   The origin is also an intersection. We have four solutions and five intersections, so the sum is 9. D.

7. $y=\dfrac{20}{x}\rightarrow x^2+\dfrac{400}{x^2}=41\rightarrow x^4-41x^2+400=0\rightarrow(x^2-25)(x^2-16)=0\rightarrow(5,4),(-5,-4),(4,5),(-4,-5).$

   This is a rectangle with dimensions $\sqrt{2}$ and $\sqrt{162}$. The area will be $\sqrt{324}=18$. D.

8. When dividing by a quadratic, the remainder, if any, will be linear or constant. To take either into account, let the remainder be $ax+b$. $[Q(x)][(x-2)(x-1)]+ax+b=x^{100}-2x^{99}+4$. If $x=2$, $2a+b=4$. If $x=1$, $a+b=3$. Solving this system of equations we get $a=1$ and $b=2$, so the remainder is $x+2$. B.

9. Sketch the triangle and drop an altitude to the third side. Then set up a system of equations using the Pythagorean theorem.



$$\begin{cases} x^2+y^2=64 \\ y^2+(14-x)^2=100 \end{cases} \to \begin{cases} x^2+y^2=64 \\ x^2-28x+y^2=-96 \end{cases} \to 28x=160 \to x=\frac{40}{7}, y=\frac{16\sqrt{6}}{7}.\ \text{C.}$$

10. Multiply the first equation by xy to get $x^2-6y^2=5$ and substitute into the second equation.

$$\left(2y+1\right)^2-6y^2=5 \to y^2-2y+2=0 \to y=\frac{2\pm\sqrt{4-4(2)}}{2}=1\pm i \to x=1+2(1\pm i)=3+2i, 3-2i.$$

$$|a|+|b|+|c|+|d|=3+2+1+1=7.\ \text{A.}$$

11. $\begin{cases} D+Q=100 \\ 10D+25Q=1405 \end{cases} \to \begin{cases} D+Q=100 \\ 2D+5Q=281 \end{cases} \to 3Q=81 \to Q=27.$ C.

12. $\begin{cases} xy-5x-8y=0 \\ \log(x-11)+\log(y-5)=1 \end{cases} \to \begin{cases} xy-5x-8y=0 \\ (x-11)(y-5)=10 \end{cases} \to \begin{cases} xy-5x-8y=0 \\ xy-5x-11y=-45 \end{cases} \to$

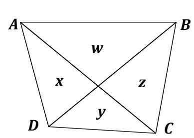
$3y=45 \to y=15, x=12 \to xy=180.$ D.

13. $m=\frac{4+3}{1-2}=-7 \to y-4=-7(x-1) \to y=4-7(11-1)=-66.$ A.

14. Dividing the second equation by the first equation, we have

$$\frac{x}{\sqrt{x}+\sqrt{y}} \circ \frac{\sqrt{x}+\sqrt{y}}{y}=9 \to \frac{x}{y}=9 \to x=9y.\ \text{By substitution,}$$

$$\frac{9y}{\sqrt{9y}+\sqrt{y}}=18 \to 9y=54\sqrt{y}+18\sqrt{y} \to y=8\sqrt{y} \to \sqrt{y}=8.\ \text{So, } y=64, x=576 \to \frac{\sqrt{576}}{\sqrt[3]{64}}=\frac{24}{4}=6.\ \text{B.}$$

15. 

$\begin{cases} w+x=1 \\ w+z=6 \\ x+y=2 \end{cases} \to$ Adding all equations together, we get $2x+2w+y+z=9$.

Multiplying the first equation by 2, we get $2w+2x=2$. Substituting this into the combined equation, we see the area will be 7. B.

16. $\begin{cases} (P+W)(6)=1750 \\ (P-W)(3)=540 \end{cases} \to \begin{cases} 3P+3W=875 \\ 3P-3W=540 \end{cases} \to 6P=1415 \to P=\frac{1415}{6}, W=\frac{335}{6} \to |P-W|=180.$ B.

17. Let x be the side length of the larger square and y be the length of the other. Then perimeter $P=3x+(x-y)+3y=256 \to 4x+2y=256 \to 2x+y=128$. The sum of the areas will be $x^2+y^2=3284$.

$\begin{cases} 2x+y=128 \\ x^2+y^2=3284 \end{cases} \to x^2+(128-2x)^2=3284 \to 5x^2-512x+13100=0 \to (x-50)(5x-262)=0.$ This gives

$(x, y) = (50, 28)$ or $\left(\frac{262}{5}, \frac{116}{5}\right)$. The maximum area of the foyer is $28^2 = 784$. D.

18. $\begin{cases} a+b+c=1 \\ a-b+c=5 \\ \frac{1}{4}a+\frac{1}{2}b+c=\frac{1}{2} \end{cases} \rightarrow \begin{cases} a+b+c=1 \\ a-b+c=5 \\ a+2b+4c=2 \end{cases} \rightarrow \begin{cases} 2a+2c=6 \\ 3a+6c=12 \end{cases} \rightarrow a=2, b=-2, c=1 \rightarrow \|\mathbf{m}\| = \sqrt{4+4+1} = 3$. A.

19. Matrix X must be a 2-by-1 matrix. Let $X = \begin{bmatrix} x \\ y \end{bmatrix}$. Then $\begin{bmatrix} 5 & -3 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix} \rightarrow \begin{cases} 5x-3y=5 \\ 4x-2y=10 \end{cases} \rightarrow$
$\begin{cases} -10x+6y=-10 \\ 12x-6y=30 \end{cases} \rightarrow x=10, y=15 \rightarrow x+y=25$. E.

20. $\begin{cases} a_2 = a_1 + d = 6 \\ a_{49} = a_1 + 49d = 342 \end{cases} \rightarrow 48d = 336 \rightarrow d = 7, a_1 = -1. a_{101} = -1+100(7) = 699$. D.

21. A homogeneous system of equations is one in which all the constant values are 0. B.

22. $\begin{cases} 4^{x+y} = 60 \\ 3^{x-y} = 5 \end{cases} \rightarrow \begin{cases} x+y = \log_4 60 \\ x-y = \log_3 5 \end{cases} \rightarrow 2x = \log_4 60 + \log_3 5 \rightarrow x = \frac{1}{2}\log_4 60 + \frac{1}{2}\log_3 5 \rightarrow x = \log_4 \sqrt{60} + \log_3 \sqrt{5} \rightarrow$
$x = \log_4 2\sqrt{15} + \log_3 \sqrt{5} \rightarrow \log_{2^2} 2\sqrt{15} + \log_3 \sqrt{15} \rightarrow \frac{1}{2}\log_2 2 + \frac{1}{2}\log_2 \sqrt{15} + \log_3 \sqrt{5} \rightarrow \frac{1}{2} + \log_2 \sqrt[4]{15} + \log_3 \sqrt{5}$. C.

23. $(6x+15y)(8x+7y) = 129 \rightarrow (2x+5y)(8x+7y) = 43 = 43 \cdot 1$. This leads to four possible systems.

$\begin{cases} 2x+5y=43 \\ 8x+7y=1 \end{cases} \rightarrow$ doesn't give integers $\qquad \begin{cases} 2x+5y=1 \\ 8x+7y=43 \end{cases} \rightarrow (8, -3)$

$\begin{cases} 2x+5y=-43 \\ 8x+7y=-1 \end{cases} \rightarrow$ doesn't give integers $\qquad \begin{cases} 2x+5y=-1 \\ 8x+7y=-43 \end{cases} \rightarrow (-8, 3) \rightarrow \qquad (8)(-8)(3)(-3) = 576$. A.

24. $888_9 = ABC_{18} \rightarrow 8 \cdot 9^2 + 8 \cdot 9^1 + 8 \cdot 9^0 = A \cdot 18^2 + B \cdot 18^1 + C \cdot 18^0$
$= 4A \cdot 9^2 + 2B \cdot 9^1 + C \cdot 9^0$

$4A = 8 \quad 2B = 8 \quad C = 8$
$A = 2 \quad B = 4 \quad C = 8 \quad \rightarrow \quad A - B - C = 2 - 4 - 8 = -10$. E.

25. Let S represent sop, J represent jun, and N represent sen.
$\begin{cases} 3S+5J+7N=120 \\ S+J+N=30 \end{cases} \rightarrow \begin{cases} 3S+5J+7N=120 \\ 3S+3J+3N=90 \end{cases} \rightarrow 2J+4N=30 \rightarrow J+2N=15$. The ordered triples (S, J, N) are as follows: $(22,1,7), (21,3,6), (20,5,5), (19,7,4), (18,9,3), (17,11,2), (16,13,1)$. There are 7 cases overall. C.

26. Let $f(x) = ax+b$. Then, $a(ax+b)+b = ax^2+ab+b = 3x-2$. Equating corresponding parts, we have $a^2 = 3$ and $ab+b = -2$. If $a = \sqrt{3}$, then $\sqrt{3}b+b = -2 \rightarrow b = \frac{-2}{\sqrt{3}+1} = 1-\sqrt{3}$. If $a = -\sqrt{3}$, then $-\sqrt{3}b+b = -2 \rightarrow$

$b=\dfrac{-2}{-\sqrt{3}+1}=1+\sqrt{3}$. $f(-1)=\sqrt{3}(-1)+1-\sqrt{3}=1-2\sqrt{3}$ or $-\sqrt{3}(-1)+1+\sqrt{3}=1+2\sqrt{3}$. The only positive value is $1+2\sqrt{3}$, so $A+B+C=6$. C.

27. Using row reduction we have $\begin{bmatrix} 1 & 1 & -1 & 2 \\ 1 & 2 & 1 & 3 \\ 1 & 1 & k^2-5 & k \end{bmatrix} \to \begin{bmatrix} 1 & 1 & -1 & 2 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & k^2-4 & k-2 \end{bmatrix}$. If $k^2-4=0$ and $k-2 \neq 0$, then there is no solution. $k=-2$ is the only value that does this. B.

28. $(x+y)^2=x^2+2xy+y^2$ $\qquad$ $x^3+y^3=(x+y)(x^2-xy+y^2)$

$(x+y)^2=10+2(3)=16$ $\qquad$ $=(x+y)(10-3)$

$\to x+y=4$ $\qquad$ $=(4)(7)$

$=28$ D.

29. $2x^2=x^2+x+6 \to x^2-x-6=0 \to (x-3)(x+2)=0 \to (3,18),(-2,8)$. $m=\dfrac{18-8}{3+2}=2$.

$y-8=2(x+2) \to 2x-y+12=0$. C.

30. Let the first equation represent the product of the roots of a polynomial, the second equation the sum of the product of the roots taken two at a time, and the third equation the sum of the roots. The polynomial could be $x^3-6x^2-x+30=0 \to (x+2)(x-3)(x-5)=0$. Then, $5=\log_2 a, 3=\log_3 b, -2=\log_5 c$.

$(a,b,c)=\left(32,27,\dfrac{1}{25}\right)=\left(2^5,3^3,5^{-2}\right)$. $2+5+3+3+5+(-2)=16$. A.