Alpha Trigonometry Nationals 2015
Note: For all questions, answer "(E) NOTA" means none of the above answers is correct.

1. Find the axis of symmetry of the following curve: $x = -2\sin\left(\frac{\pi}{2}\right)y^2 + 12\cos(\pi)y + 3$

(A) $x = -3$ (B) $x = 3$ (C) $y = -3$ (D) $y = 6$ (E) NOTA

2. Change the following polar coordinates into Cartesian form: $\left(-2, \frac{5\pi}{4}\right)$

(A) $(\sqrt{2}, \sqrt{2})$ (B) $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ (C) $(1,1)$ (D) $(-1,-1)$ (E) NOTA

3. What is the domain of $f(x) = \sqrt{\sec x}$ on the interval $0 \le x < 2\pi$?

(A) $[0, 2\pi)$ (B) $[0, \frac{\pi}{2}) \cup (\frac{3\pi}{2}, 2\pi)$ (C) $[0, \frac{\pi}{2}) \cup \left(\frac{\pi}{2}, \frac{3\pi}{2}\right) \cup \left(\frac{3\pi}{2}, 2\pi\right)$ (D) $[0, \pi]$ (E) NOTA

4. Simplify: $(1 - \csc x)(1 + \sin x)$

(A) $\sin x$ (B) $\cos x$ (C) $\cot x \csc x$ (D) $-\cot x \cos x$ (E) NOTA

5. Evaluate: $\lim_{x \to 0} \frac{\cos 4x + 4}{\cos x}$

(A) 4 (B) 5 (C) 0 (D) undefined (E) NOTA

6. Evaluate the following sum: $\sum_{n=1}^{\infty} \left[\sin\left(\frac{\pi(2n+1)}{4}\right)\right]^2$

(A) 1 (B) the series diverges (C) π (D) π^2 (E) NOTA

7. What is the period of of the function $f(x) = 7^5 \csc(4098\pi x)$

(A) $\frac{\pi}{2049}$ (B) $\frac{1}{4098}$ (C) $\frac{1}{2049}$ (D) $\frac{7^5}{2\pi}$ (E) NOTA

8. If the cosine of the acute angle formed by the vectors $\langle x, 3, -2 \rangle$ and $\left\langle -4, \frac{5}{2}, 12 \right\rangle$ is equal to 0, then what is x?

(A) $\frac{33}{2}$ (B) 3 (C) 0 (D) $-\frac{33}{8}$ (E) NOTA

9. Give the best name for the graph represented by the equation: $3r - r\cos\theta = 14$?

(A) ellipse (B) parabola (C) hyperbola (D) cardiod (E) NOTA

10. Evaluate: $\left(\frac{3\sqrt{2}}{2} - \frac{3\sqrt{6}}{2}i\right)^4$

(A) $-162 + 162\sqrt{3}i$ (B) $162 - 162\sqrt{3}i$ (C) $105 - 105\sqrt{3}i$ (D) $-105 + 105\sqrt{3}i$ (E) NOTA

11. What is $\cos x$ if $3\sin^2 x - 5\cos x = 1$?

(A) -2 (B) -2 and $\frac{1}{3}$ (C) $\frac{1}{3}$ (D) no solution (E) NOTA

12. What is the value of the expression $2\cos 1200° \times \tan 1035° + \sin 1170° \times \csc^2(-1485°)$?

(A) 3 (B) $\sqrt{3}$ (C) $-\frac{1}{2}$ (D) 0 (E) NOTA

13. Given $\cot x = \frac{1}{4}$, find $1 + \cos^2 x + \tan^2 x - \sin^2 x$.

(A) 18 (B) $\frac{274}{17}$ (C) 16 (D) $\frac{304}{17}$ (E) NOTA

14. Evaluate: $\sec\left(Tan^{-1}\left(\frac{2x}{3}\right)\right)$ when x = 2.

(A) $\frac{3\sqrt{7}}{7}$ (B) $\frac{5}{3}$ (C) $\frac{25}{3}$ (D) $\frac{5}{4}$ (E) NOTA

15. Find $\sqrt{x}$, if x is the smallest degree measure solution to the equation $\cot(8x + 26°) - \tan(10x - 8°) = 0$ on the interval $0° < x < 45°$.

(A) 3 (B) $3\sqrt{5}$ (C) 5 (D) 2 (E) NOTA

16. What is the length of the minor axis of the ellipse given by the parametric equations

$x = 3\cos t$ and $y = 7\sin t$

(A) 3 (B) 6 (C) 7 (D) 14 (E) NOTA

17. What is the distance between the following points given in polar form:

$(2, 780°)$ and $\left(3, -\frac{26\pi}{3}\right)$

(A) 1 (B) $\sqrt{17}$ (C) 5 (D) $\sqrt{5}$ (E) NOTA

18. Which of the following shares all asymptotes with the function $-\frac{3}{2}\csc\left(x + \frac{\pi}{4}\right)$?

(A) $\frac{3}{2}\csc\left(x - \frac{9\pi}{4}\right)$ (B) $-\frac{3}{2}\csc\left(x - \frac{\pi}{4}\right)$ (C) $\sec\left(2x - \frac{\pi}{4}\right)$ (D) $\frac{2}{3}\sec\left(x - \frac{9\pi}{4}\right)$ (E) NOTA

19. What is the trace of the matrix $\begin{bmatrix} \sec\left(\frac{5\pi}{4}\right) & \sin^2(14°) \\ \frac{1}{\sin^2(28°)} & \cot^2\left(\frac{17\pi}{4}\right) \end{bmatrix}$?

(A) $1 - \sqrt{2}$ (B) 1 (C) 0 (D) $-\sqrt{2}$ (E) NOTA

20. If $\sec x = \frac{5}{4}$, evaluate $\cos^4 x + \sin^4 x + 4\sin^2 x\cos^2 x - 2\sin^2 x\cos^2 x$.

(A) $\frac{3}{5}$ (B) $\frac{4}{5}$ (C) -1 (D) 0 (E) NOTA

21. Simplify $\cos\left(Arc\cos\frac{4}{5} - Arc\sin\frac{15}{17}\right)$.

(A) $-\frac{13}{85}$ (B) $\frac{84}{85}$ (C) $\frac{44}{85}$ (D) $\frac{77}{85}$ (E) NOTA

22. In an isosceles triangle, two sides have length 26 and the length of the third side is 20. What is the measure of the angle that is not congruent to either of the other two angles?

(A) $\sin^{-1}\frac{5}{13}$ (B) $2\sin^{-1}\frac{5}{13}$ (C) $\tan^{-1}\frac{12}{5}$ (D) $2\tan^{-1}\frac{12}{5}$ (E) NOTA

23. $(\cos 75°)(\sin 18°) =$

(A) $\frac{\sqrt{30}+\sqrt{10}+\sqrt{6}+\sqrt{2}}{16}$ (B) $\frac{\sqrt{2}\left(\sqrt{15}-1\right)}{16}$ (C) $\frac{\sqrt{2}\left(\sqrt{15}-\sqrt{5}-\sqrt{3}+1\right)}{16}$ (D) $\frac{5+4\sqrt{5}}{16}$ (E) NOTA

24. What is the positive difference between the maximum and minimum values of the function $f(x)=6-\sin^2 x-4\sin x$ on the interval $\left[0,\frac{5\pi}{6}\right]$?

(A) 3 (B) 4 (C) 5 (D) 6 (E) NOTA

25. Evaluate $2e^{i\frac{\pi}{3}}+4e^{i\frac{\pi}{3}}-3e^{i\frac{4\pi}{3}}$.

(A) $9e^{i\frac{7\pi}{3}}$ (B) $3e^{i\frac{7\pi}{3}}$ (C) $3e^{i2\pi}$ (D) 0 (E) NOTA

26. Find the sum of the solutions to the equation $\sec x+\csc 2x=\frac{\sec x}{2\sin x}$ on the interval $[0,2\pi)$.

(A) π (B) 2π (C) 3π (D) $\frac{3\pi}{2}$ (E) NOTA

27. What is the area of the region bounded by $r=2\sin\theta$ and $r=2\cos\theta$?

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{2}-1$ (C) $\frac{\pi-2}{4}$ (D) 1 (E) NOTA

28. From the system of equations $\begin{cases} 8\sin x+18\cos y=14 \\ 6\sin x+16\cos y=12 \end{cases}$, if x and y have terminal side in Quadrant I, then what is the product of $\sin y$ and $\cos x$?

(A) $\frac{12}{25}$ (B) $\frac{2\sqrt{21}}{25}$ (C) $\frac{4\sqrt{21}}{25}$ (D) $\frac{6}{25}$ (E) NOTA

29. Evaluate $(1+\sec\theta+\tan\theta)(1-\csc\theta+\cot\theta)$

(A) -2 (B) 1 (C) 0 (D) 2 (E) NOTA

30. What is the value of the expression $\sin(\sin^{-1}x+2\cos^{-1}x-\cos^{-1}(x))$?

(A) -1 (B) 0 (C) 1 (D) $\frac{\sqrt{2}}{2}$ (E) NOTA