Alpha Trigonometry Nationals 2015

Answers:

1. C
2. A
3. B
4. D
5. B
6. B
7. C
8. D
9. A
10. A
11. C
12. A
13. B
14. B
15. D
16. B
17. C
18. D
19. A
20. E (1)
21. D
22. B
23. C
24. C
25. A
26. E (no solution)
27. B
28. C
29. D
30. C

Solutions

1.	C	$y=\frac{-b}{2a}=\frac{-(-12)}{2(-2)}=-3$
2.	A	$x=r\cos\theta, x=-2\cos\left(\frac{5\pi}{4}\right)=-2\left(-\frac{\sqrt{2}}{2}\right)=\sqrt{2}$ $y=r\sin\theta, x=-2\sin\left(\frac{5\pi}{4}\right)=-2\left(-\frac{\sqrt{2}}{2}\right)=\sqrt{2}$
3.	B	$f(x)$ cannot have negative inside the root and cannot include undefined values so $\left[\frac{\pi}{2},\frac{3\pi}{2}\right]$ must be excluded.
4.	D	$1-\csc x+\sin x-\csc x\sin x=1-\frac{1}{\sin x}+\sin x-1=\frac{-1+\sin^2 x}{\sin x}=$ $\frac{-\cos^2 x}{\sin x}=-\cot x\csc x$
5.	B	$\lim_{x\to 0}\frac{\cos 4x+4}{\cos x}=\frac{\cos 0+4}{\cos 0}=\frac{1+4}{1}=5$
6.	B	$\sin^2\left(\frac{3\pi}{4}\right)+\sin^2\left(\frac{5\pi}{4}\right)+\sin^2\left(\frac{7\pi}{4}\right)+\ldots=$ $\left(\frac{\sqrt{2}}{2}\right)^2+\left(-\frac{\sqrt{2}}{2}\right)^2+\left(-\frac{\sqrt{2}}{2}\right)^2+\ldots=$ $\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\ldots=\infty$
7.	C	Period $=\frac{2\pi}{4098\pi}=\frac{1}{2049}$
8.	D	$\cos\theta=\frac{A\cdot B}{\lvert A\rvert\cdot\lvert B\rvert}$ $0=\langle x,3,-2\rangle\cdot\left\langle -4,\frac{5}{2},12\right\rangle$ $-4x+\frac{15}{2}-24=0$
9.	A	$r=\frac{14}{3-1\cos\theta}$ $r=\frac{\frac{14}{3}}{1-\frac{1}{3}\cos\theta}$ $e=\frac{1}{3}<1$ Thus it must be an ellipse.

10.	A	$\left(\frac{3\sqrt{2}}{2}-\frac{3\sqrt{6}}{2}i\right)^4=\left[3\sqrt{2}\left(\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)\right]^4=\left[3\sqrt{2}cis\frac{5\pi}{3}\right]^4=$ $\left(3\sqrt{2}\right)^4 cis\frac{4*5\pi}{3}=324cis\frac{20\pi}{3}=-162+162\sqrt{3}i$
11.	C	$3\sin^2 x=3-3\cos^2 x$ $3-3\cos^2 x-5\cos x=1$ $3\cos^2 x+5\cos x-2=0$ $(3\cos x-1)(\cos+2)=0$ $\cos x=\frac{1}{3},-2$ But -2 is not a solution since cosine cannot be less than -1.
12.	A	$2\cos 120°\cdot\tan 315°+\sin 90°\cdot\csc^2(-1485°)=$ $2\left(-\frac{1}{2}\right)\left(-1\right)+\left(1\right)\left(2\right)=3$
13.	B	$opposite=4$ $adjacent=1$ $hypotenuse=\sqrt{17}$ $1+\left(\frac{1}{\sqrt{17}}\right)^2+\left(\frac{4}{1}\right)^2-\left(\frac{4}{\sqrt{17}}\right)^2=$ $1+\frac{1}{17}+16-\frac{16}{17}=$ $\frac{274}{17}$
14.	B	$Tan^{-1}\left(\frac{4}{3}\right)$ $opposite=4$ $adjacent=3$ $hypotenuse=5$ $\sec\theta=\frac{hyp}{adj}=\frac{5}{3}$
15.	D	$\left(8x+26\right)+\left(10x-8\right)=90$ $x=4$ $\sqrt{x}=2$
16.	B	Solve for *t*, plug in and simplify: $\frac{x^2}{9}+\frac{y^2}{49}=1$ Thus a=3 and the length must be 2a = 6.

17.	C	$(2, 780°) = (1, \sqrt{3})$ $\left(3, -\frac{26\pi}{3}\right) = \left(-\frac{3}{2}, -\frac{3\sqrt{3}}{2}\right)$ $D = 5$
18.	D	$\frac{2}{3}\sec\left(x - \frac{9\pi}{4}\right) = \frac{2}{3}\sec\left(x - \frac{\pi}{4}\right)$ Neither the original nor answer choice D can have $-\frac{\pi}{4}$ as a value.
19.	A	The trace is the sum of the main diagonals which are 1 and $-\sqrt{2}$.
20.	E	$\cos^4 x + \sin^4 x + 4\sin^2 x\cos^2 x - 2\sin^2 x\cos^2 x =$ $\cos^4 x + \sin^4 x + 2\sin^2 x\cos^2 x =$ $\left(\cos^2 x + \sin^2 x\right)^2 =$ 1
21.	D	$\cos\left(Arc\cos\frac{4}{5}\right)\cos\left(Arc\sin\frac{15}{17}\right) + \sin\left(Arc\cos\frac{4}{5}\right)\sin\left(Arc\sin\frac{15}{17}\right) =$ $\left(\frac{4}{5}\right)\left(\frac{8}{17}\right) + \left(\frac{3}{5}\right)\left(\frac{15}{17}\right) =$ $\frac{77}{85}$
22.	B	The triangle is two 5-12-13 right triangles so the vertex angle of the isosceles triangle is double the acute angle of the 5-12-13 right triangle.
23.	C	$\cos(75°) = \frac{\sqrt{6} - \sqrt{2}}{4}$ $\sin(18°) = \frac{\sqrt{5} - 1}{4}$ $\left(\frac{\sqrt{6} - \sqrt{2}}{4}\right)\left(\frac{\sqrt{5} - 1}{4}\right) =$ $\frac{\sqrt{30} - \sqrt{10} - \sqrt{6} + \sqrt{2}}{16}$
24.	C	$f(x) = 6 - \sin^2 x - 4\sin x$ $\sin x = t$ $6 - t^2 - 4t = y$ $-(t+2)^2 + 10 = y$ $0 \le t \le 1$ $\max = 6$ $\min = 1$

25.	A	$2e^{i\frac{\pi}{3}}+4e^{i\frac{\pi}{3}}-3e^{i\frac{4\pi}{3}}=$ $6e^{i\frac{\pi}{3}}-3e^{i\frac{4\pi}{3}}=$ $6\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)-3\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)=$ $3+3\sqrt{3}i+\frac{3}{2}+\frac{3\sqrt{3}}{2}i=$ $\frac{9}{2}+\frac{9\sqrt{3}}{2}i=$ $9e^{i\frac{7\pi}{3}}$
26.	E	$\sec x+\csc 2x=\frac{1}{\cos x}+\frac{1}{2\sin x\cos x}$ $\frac{2\sin x+1}{2\sin x\cos x}=\frac{1}{2\sin x\cos x}$ $\sin x=0,\pi$ But both solutions make the equation undefined so there are not solutions.
27.	B	The two circles intersect at the point (1, 1) and (0, 0). The area of one of the sectors from 0 to $\frac{\pi}{2}$ is $\frac{\pi}{4}$. The area of the right triangle inside the sector is $\frac{1}{2}$. Taking the triangle out of the sector leaves have of the bounded region. Double it to get the area of the region.
28.	C	Solving using any systems of equations methods will give you $\sin x=\frac{2}{5}$ and $\cos y=\frac{3}{5}$. From this you can solve for $\cos x=\frac{\sqrt{21}}{5}$ and $\sin y=\frac{4}{5}$.
29.	D	$(1+\sec\theta+\tan\theta)(1-\csc\theta+\cot\theta)=$ $1-\csc\theta+\cot\theta+\sec\theta-\sec\theta\csc\theta+\csc\theta+\tan\theta-\sec\theta+1=$ $2+\cot\theta-\sec\theta\csc\theta+\tan\theta=$ $2+\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta\cos\theta}+\frac{\sin\theta}{\cos\theta}=$ $2+\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}-\frac{1}{\sin\theta\cos\theta}=$ 2
30.	C	$\sin(\sin^{-1}x+2\cos^{-1}x-\cos^{-1}(x))=$ $\sin(\sin^{-1}x+\cos^{-1}x)=$ $\sin(\sin^{-1}x)\cos(\cos^{-1}x)+\cos(\sin^{-1}x)\sin(\cos^{-1}x)=$ $x^2+\left(\sqrt{1-x^2}\right)\left(\sqrt{1-x^2}\right)=$ 1