Answers:

1	A
2	C
3	C
4	D
5	A
6	C
7	B
8	A
9	A
10	A
11	C
12	D
13	E
14	B
15	C
16	C
17	D
18	C
19	B
20	C
21	E
22	A
23	C
24	C
25	E
26	B
27	E
28	D
29	D
30	B

Solutions:

1 A Need to check each answer to $1234° - k360°$ and $1234° - 4(360°) = -206°$.

2 C An even function is one such that $f(-x) = f(x)$. As long as one of the functions in the composition is even (namely cosine), the composition is even so II, III and IV are even.

3 C $$\sin(105°) + \sin(15°) = \sin(60° + 45°) + \sin(60° - 45°) = 2\sin(60°)\sin(45°)$$
$$= 2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{6}}{2}$$

4 D $2r = 2\sin\theta\cos\theta = \sin(2\theta)$. Since 2 is even, it will have twice as many or 4 petals.

5 A Call the two quantities A, B.
$$\tan(A + B) = \frac{\tan(A) + \tan(B)}{1 - \tan(A)\tan(B)} = \frac{\frac{2}{3} + (-5)}{1 - \left(\frac{2}{3}\right)(-5)} = \frac{-\frac{13}{3}}{\frac{13}{3}} = -1.$$
So, A+B is an angle whose tangent is -1. $A + B = -\frac{\pi}{4}$.

6 C By symmetry, the line from P to O, and the lines to the tangent point form a 30-60-90 triangle. The short side is the radius = 2, the hypotenuse is then 4.

7 B There are π radians in 180°, so
$$\frac{8\pi}{15}\left(\frac{180°}{\pi}\right) = 96°$$

8 A
$$i - 1 = \sqrt{2}\,cis\left(\frac{3\pi}{4}\right)$$
$$1 - i = \sqrt{2}\,cis\left(-\frac{\pi}{4}\right)$$
$$(i-1)^5(1-i)^3 = \sqrt{2}^8 cis\left(5\left(\frac{3\pi}{4}\right) - 3\left(\frac{\pi}{4}\right)\right) = 16\,cis(3\pi) = -16$$

9 A The cosine of the angle between the vectors is: $\frac{<2,3> \cdot <3,2>}{\|<2,3>\|\|<3,2>\|} = \frac{12}{13}$. The 3rd side of a right triangle with an angle of cosine 12/13 is 5. The tangent is $\frac{5}{12}$.

10 A arcsin has a domain of $[-1, 1]$. The range of secant is $(-\infty, -1] \cup [1, \infty)$. We need $\sec(x) = \pm 1$, so $\cos(x) = \pm 1$; $x \in \{k\pi | k\ is\ an\ integer\}$

11 C Obviously x=0 is a solution and since both sides are odd functions, we only need to consider one side. The first crossing will be at little less than π and the second and third crossing a little less and a little more than $\frac{5\pi}{2}$. On the next period, x/10 will be larger than 1. In total, there are 3 positive, 1 zero and 3 negative solutions = 7.

12 D The building, shadow, and the light beam forms a right triangle. So,

$$\frac{height}{shadow} = \tan 50°. \qquad height = 70\tan 50°$$

13 E Just a change of units.

$$\frac{2400\ rev}{min}\ \frac{2\pi\ rad}{1\ rev}\ \frac{1\ min}{60\ sec} = 80\pi\ rad/sec$$

14 B Since the triangle is isosceles, the obtuse angle must be between the two sides of 10. Since the cosine of an obtuse angle is negative, we must have $c^2 > a^2 + b^2 = 200$. c can be 15, 16, 17, 18 or 19 which sum to 85.

15 C

$$\cos^2\left(\frac{\pi}{12}\right) + \sin\left(2\left(\frac{\pi}{12}\right)\right) = \left(\frac{1+\cos\left(\frac{\pi}{6}\right)}{2}\right) + \sin\left(\frac{\pi}{6}\right) = \frac{\left(1+\frac{\sqrt{3}}{2}\right)}{2} + \frac{1}{2} = \frac{2+\sqrt{3}}{4} + \frac{1}{2}$$

$$= \frac{4+\sqrt{3}}{4}$$

16 C A vector orthogonal (perpendicular, normal) to the given plane is $< 3, -2, 1 >$. Just have to take dot product with the other normal vectors seeing which gives 0.

$$< 3,-2,1 > \cdot < 1,4,5 > = 0.$$

17 D $\sin\left(\frac{x}{3}\right)$ has period 6π and $\cos\left(\frac{x}{2}\right)$ has period 4π. The $LCM(6\pi, 4\pi) = 12\pi$.

18 C Geometric definition of parabola. Let the point be (a,b) and as an example line y=k. Equating squared distances.

$$(x-a)^2 + (y-b)^2 = (y-k)^2$$
$$(x-a)^2 = (2b-2k)y + k^2 - b^2$$

A parabola.

19 B Using a rotation matrix:

$$\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}4\\2\end{bmatrix} = \begin{bmatrix}1/2 & -\sqrt{3}/2\\ \sqrt{3}/2 & 1/2\end{bmatrix}\begin{bmatrix}4\\2\end{bmatrix} = \begin{bmatrix}2-\sqrt{3}\\ -2\sqrt{3}-1\end{bmatrix}$$

Now switch the sign of y.

$$(2-\sqrt{3}, -2\sqrt{3}-1)$$

20 C

$$2^{\cos\theta} < 1 \text{ and } 5^{\sin\theta} > 1$$

means $\cos\theta < 0, \sin\theta > 0$. The angles must terminate in the second quadrant. Only $\frac{3\pi}{4}$ works.

21 E The angle between the points measures 45°, the Law of Cosines can find the squared distance.

$$d^2 = 3^2 + 5^2 - 2(3)(5)\cos(45°) = 34 - 15\sqrt{2}$$

22 A Divide the region into 2 pieces. The sector has area: $\pi r^2 \left(\frac{A}{2\pi}\right) = \frac{A}{2} r^2$. The triangle has base r, and height $r\sin A$, with area $\frac{1}{2}\sin A\, r^2$. The total area is

$$\left(\frac{\mathrm{A} + \sin A}{2}\right) r^2$$

23 C $r(2\sin\theta + 5\cos\theta) = 10$, use $y = r\sin\theta$ and $x = r\cos\theta$ and simplify.

$$2y + 5x = 10$$

A line so that the length of the graph is just the distance between (0,5) and (2,0) or $\sqrt{2^2 + 5^2} = \sqrt{29}$.

24 C $1^2 = (\sin^2 x + \cos^2 x)^2$

$$= \sin^4 + 2\sin^2 x\cos^2 + \cos^4 x$$

$$= \sin^4 x + \cos^4 x + \frac{\sin^2 2x}{2} = \sin^4 x + \cos^4 x + \frac{8}{49}.$$

yielding an answer of $\frac{41}{49}$.

25 E

$$\cos\left(\frac{\pi}{5}\right) + \cos\left(\frac{2\pi}{5}\right) + \cos\left(\frac{3\pi}{5}\right) + \cos\left(\frac{4\pi}{5}\right) + \cos(\pi) = \cos(\pi) = -1.$$

We have $\cos\left(\frac{\pi}{5}\right) = -\cos\left(\frac{4\pi}{5}\right)$, and $\cos\left(\frac{2\pi}{5}\right) = -\cos\left(\frac{3\pi}{5}\right)$.

26 B Let:

$$8x = 4(2)(\sin\frac{\pi}{16})(\cos\frac{\pi}{16})(\cos\frac{\pi}{8})(\cos\frac{\pi}{4})$$

$$= 2(2)\sin\left(\frac{\pi}{8}\right)\cos\left(\frac{\pi}{8}\right)\cos\left(\frac{\pi}{4}\right)$$

$$= 2\sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{4}\right)$$

$$= \sin\left(\frac{\pi}{2}\right) = 1.\ \ x = \frac{1}{8}.$$

27 E The area of the triangle is:

$$A = \frac{1}{2}(ab)\sin(C).$$

To maximize, let $\sin(C) = 1$; giving an area of $\frac{1}{2}(20)(25) = 250$.

28 D The amplitude is (400-100)/2=150, the coefficient of 'sin'. The vertical shift will be (400+100)/2=250. The coefficient of x is $\frac{2\pi}{period} = \frac{\pi}{6}$. Finally, since the peak is 4 months into the cycle rather than 12/4=3 months, the phase shift is 1 month.

$$s(x) = 150\sin\left(\frac{\pi}{6}(x-1)\right) + 250$$

29 D So, $\sin(\theta) = 0$ only when $\theta = k\pi$. The solutions for $\sin(x)$, $\sin(2x)$ and $\sin(3x)$ are subsets of the others. $\sin(4x)$ has 4 solutions: $\frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi$. $\sin(5x)$ adds an additional 4 solutions (1,2,3,4 over 5 π), $\sin(6x)$ provides 4 more: (1, 2, 4, 5 over 6 π) for a total of 12.

30 B Assign the variables: h – the height of the tree, x – distance from ground Mary to tree and y – distance from ground Nancy to tree. Now 3 equations.

$$\sin B = \frac{h}{x};\ \sin C = \frac{h}{y}$$

$$d^2 = x^2 + y^2 - 2xy\cos A$$

Solve the first 2 eqns for x and y and substitute into the third.

$$d^2 = \left(\frac{h}{\sin B}\right)^2 + \left(\frac{h}{\sin C}\right)^2 - 2\left(\frac{h}{\sin B}\right)\left(\frac{h}{\sin C}\right)\cos A$$

Finally, solve for h.

$$h = \frac{d\sin B\sin C}{\sqrt{\sin^2 B + \sin^2 C - 2\sin B\sin C\cos A}}$$