For all questions, E "NOTA" means none of the above answers is correct.

1. Find the equation of the directrix closest to the pole of the following conic section:

$$r = \frac{12}{6 - 3\cos\theta}$$

a) $x = 4$ b) $x = -4$ c) $y = 4$ d) $y = -4$ e) NOTA

2. Find the shortest distance from point P (1, 2, -1) to the line joining P_1 (0, 0, 0) and P_2 (-1, 0, 2).

a) $\frac{1}{\sqrt{5}}$ b) $\sqrt{5}$ c) $\sqrt{\frac{21}{5}}$ d) $\sqrt{17}$ e) NOTA

3. Four symmetrically placed congruent circles with radius 1 intersect at a single point. Find the area of the shaded pairwise intersection.
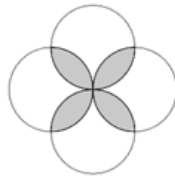
a) $2\pi - 4$ b) $2\pi - 2$ c) π d) 2π e) NOTA

4. A square is inscribed in a semi-circle and its associated diameter. The semi-circle has a radius of 10. What is the area of the region inside the semi-circular region but outside the square?

a) $100\pi - 25$ b) $100\pi - 50$ c) $50\pi - 25$ d) $50\pi - 50$ e) NOTA

5. Consider the following diagram where circle C is inscribed in an equilateral triangle and an altitude is drawn from a vertex to the corresponding base. Find the sine of the angle ABC.
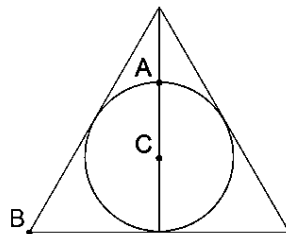
a) $\frac{\sqrt{21}}{14}$ b) $\frac{\sqrt{7}}{10}$ c) $\frac{1}{2}$ d) $\frac{2}{\sqrt{7}}$ e) NOTA

6. The lines $y = \frac{5}{12}x$ and $y = \frac{4}{3}x$ are drawn in the coordinate plane. Find the slope of the line that bisects the smaller angle formed by these lines.

   a) $-\frac{9}{7}$ b) $\frac{4}{11}$ c) $\frac{7}{9}$ d) $\frac{11}{4}$ e) NOTA

7. The line $y = \frac{3x+7}{4}$ intersects the circle $x^2 + y^2 = 25$ at (a, b) and (c, d). Find the sum of the absolute values of only the integers in the set (a, b, c, d).

   a) 7   b) 9   c) 11   d) 22   e) NOTA

8. A circle is tangent to the y-axis at the point (0, 2) and passes through the point (8, 0). Find the radius of the circle.

   a) 4   b) $\frac{17}{4}$ c) $2\sqrt{17}$ d) Not Enough Info   e) NOTA

9. Square ABCD has sides of length 4, and M is the midpoint of CD. A circle with radius 2 and center M intersects a circle with radius 4 and center A at points P and D. What is the distance from P to AD?

   a) 3   b) $\frac{16}{5}$ c) $\frac{13}{4}$ d) $2\sqrt{3}$ e) NOTA

10. For some positive real number r, the line $x + y = r$ is tangent to the circle $x^2 + y^2 = r$. Find r.

   a) 2   b) $2\sqrt{2}$ c) 4   d) $4\sqrt{2}$ e) NOTA

For questions 11-12, we'll be checking out the matrix representation of conic sections.

The general conic section $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ can be expressed in matrix form as
$$Q = \begin{bmatrix} 2A & B & D \\ B & 2C & E \\ D & E & 2F \end{bmatrix}$$

11. A certain conic has $Q = \begin{bmatrix} 4 & 4 & 3 \\ 4 & 8 & 1 \\ 3 & 1 & 6 \end{bmatrix}$. Classify this conic

   a) Hyperbola   b) Parabola   c) Circle   d) Ellipse   e) NOTA

12. A conic with $|Q|=0$ is considered degenerate. Which of the following conics is degenerate?

   a) $3x^2 - 12x + 4y^2 - 8y + 16 = 4$ b) $4x^2 - 4xy + 7y^2 - 24 = 0$ c) $2xy = 1$
   d) $31x^2 + 12xy + y^2 - 50x - 125 = 0$ e) NOTA

For questions 13-15, let's examine the parameters of conic sections in more detail.

The **linear eccentricity** is the distance between the center and the focus (or one of the two foci).
The **latus rectum** is the chord parallel to the directrix and passing through the focus (or one of the two foci). The **focal parameter** is the distance from the focus (or one of the two foci) to the directrix.

13. Find the length of the latus rectum of the $\frac{x^2}{9} - \frac{y^2}{16} = 1$.

   a) $\frac{18}{4}$ b) $\frac{32}{3}$ c) 12   d) 16   e) NOTA

14. If the major axis of an ellipse is equal to 4 and the eccentricity is .5, find the linear eccentricity.

   a) .5   b) 1   c) 2   d) 4   e) NOTA

15. If the focal parameter of a parabola is equal to 3, what is the length of the latus rectum?

   a) $\frac{1}{2}$ b) $\frac{3}{2}$ c) 3   d) 6   e) NOTA

16. Find the sine of the dihedral angle between two faces of a regular tetrahedron.

   a) $\frac{\sqrt{3}}{2}$ b) $\frac{2\sqrt{3}}{3}$ c) $\frac{2\sqrt{2}}{3}$ d) $\frac{2\sqrt{6}}{3}$ e) NOTA

17. Find the volume of the following quadric surface $\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{1} = 1$

   a) 4π b) 6π c) 8π d) 36π e) NOTA

18. In cylindrical coordinates, the graph of $r = z$ is intersected by the graph, in rectangular coordinates, of $x = 5$. What is the shape of this intersection?

   a) Hyperbola   b) Circle   c) Ellipse   d) Point   e) NOTA

19. Eliminate the parameter from the parametric equations $x = 4e^{\frac{t}{4}}, y = 3e^t$

   a) $y = 3\ln x$ b) $y = \frac{3x^4}{64}$ c) $y = \ln\frac{x}{4}$ d) $y = 3x$ e) NOTA

20. A laser is shot from vertex A of square ABCD of side length 1, towards point P on BC so that BP=$\frac{3}{4}$. The laser reflects off the sides of the square, until it hits another vertex, at which point it stops. What is the length of the path the laser takes?
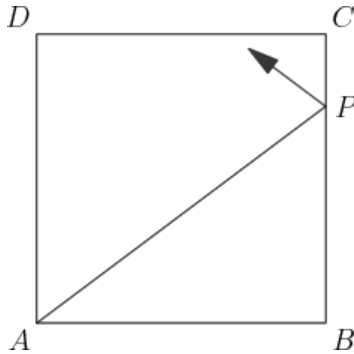
   a) $\frac{5}{3}$ b) $\frac{10}{4}$ c) 5   d) ∞ e) NOTA

21. Find the number of petals traced out by the polar curve $r = \sin(\theta)\cos(\theta)\cos(2\theta)\cos(4\theta)$

   a) 8   b) 16   c) 32   d) 64   e) NOTA

22. The eight roots of $x^8 - 1 = 0$ are graphed in the complex plane. Find the area of the octagon created by connecting all eight points in clockwise order about the origin.

   a) $2 - \sqrt{2}$ b) $\sqrt{2}$ c) $2\sqrt{2}$ d) $2 + 2\sqrt{2}$ e) NOTA

23. Points A and B are in the first quadrant, and O is the origin. If the slope of OA is 1 and the slope of OB is 7, and OA and OB have the same length, then compute the slope of AB.

   a) $-\frac{2}{3}$ b) $-\frac{3}{4}$ c) $-\frac{6}{7}$ d) $-\frac{1}{6}$ e) NOTA

24. A plane passes through the center of a cube and is perpendicular to one of the cube's diagonals. How many edges of the cube does the plane intersect?

   a) 4   b) 5   c) 6   d) 8   e) NOTA

25. The polar curve $r = \cos(\theta) + \frac{1}{4}$ is plotted in the Cartesian coordinate system. Find the maximum distance of any point on this curve to the origin.

   a) $\frac{1}{4}$ b) 1   c) $\frac{5}{4}$ d) $\frac{9}{4}$ e) NOTA

26. The point $(3, 4)$ is rotated 120° clockwise about the origin. Find the distance between the original point and the new rotated point.

   a) $5\sqrt{3}$ b) 10   c) $5\sqrt{2 + \sqrt{3}}$ d) 25   e) NOTA

27. Find the volume of the figure that results from the rotation of the circle $x^2 + y^2 = 25$ about the line $y = -6$.

   a) 150π b) 300π c) 600π d) 800π e) NOTA

28. For $\theta \epsilon [0, 2\pi]$, how many times do the graphs $r^2 = 25 \cos(2\theta)$ and $r = 4$ intersect?

   a) 1   b) 2   c) 3   d) 4   e) NOTA

29. Find the equation of the plane determined by the three points $(2, -1, -1)$, $(1, 4, 0)$, and $(-3, 4, 1)$.

   a) $5x - 3y + 20z + 7 = 0$ b) $-x + 5y + z - 19 = 0$ c) $5x + 5y + 20z - 25 = 0$

   d) $3x - 3y + 12z + 9 = 0$ e) NOTA

30. Nate likes squares. A square is inscribed in an ellipse such that two sides of the square respectively pass through the two foci of the ellipse. The square has a side length of 4. The square of the length of the minor axis of the ellipse can be written in the form $a + b\sqrt{c}$ where a, b, and c are positive integers and c is not divisible by the square of any prime. Find the sum $a + b + c$.

   a) 9   b) 13   c) 18   d) 21   e) NOTA