1. 4 students compete in a decathlon. One of the students, Ankit, can't seem to get out of fourth place, and can always see the other three students ahead of him. Given that the placement of the other three students is completely random for each event, what is the probability that Ankit sees the other three in the same order for every event in the decathlon?

A. $\frac{1}{3^9}$ B. $\frac{1}{3^{10}}$ C. $\frac{1}{6^9}$ D. $\frac{1}{6^{10}}$ E. NOTA

2. Solve the following systems of equations for *x*:

$$x - 3y + z = 1$$
$$2x + 7y + 10z = 3$$
$$3x + 2y - 2z = -1$$

A. $\frac{1}{17}$ B. $-\frac{1}{17}$ C. $\frac{3}{17}$ D. $-\frac{3}{17}$ E. NOTA

3. Find the eccentricity of the conic section defined by the equation $r = \frac{5}{3+2\cos(\Theta)}$.

A. 5 B. 3 C. 2 D. 1 E. NOTA

4. Simplify: $\frac{\sin(x)+\sin(3x)}{\sin(2x)}$.

A. $\sin(x)$ B. $\cos(x)$ C. $2\sin(x)$ D. $2\cos(x)$ E. NOTA

5. How many distinct real solutions are there to the equation:

$x^8 + x^7 + x^6 + x^5 + x^3 + x^2 + x + 1 = 0$?

A. 8 B. 4 C. 1 D. 0 E. NOTA

6. Compute the volume of the parallelepiped defined by the vertices A(11, 7, 11), B(8, 9, 4), C(12, 6, 6), and D(7, 1, 4).

A. 193 B. $\frac{193}{6}$ C. 71 D. $\frac{71}{6}$ E. NOTA

7. How many ways are there to get from the origin to (5, 5) moving only vertically up or horizontally to the right if one of your coordinates of your path must always be an integer and if you can go through neither of the points (2, 2) nor (4, 4)?

A. 252 B. 64 C. 72 D. 63 E. NOTA

8. Find the equation of the plane containing both vectors <1, 5, 6> and <2, 3, 4> and going through the point (7, 6, 9).

A. $2x + 8y - 7z = -1$ B. 2x+8y-7z=1 C. 2x-8y+7z=29 D. 2x-8y+7z=-29 E. NOTA

9. In how many ways can you tile an 8x8 chessboard with 1 pair of opposite corners removed using dominoes, given that the dominoes cannot overlap? Each domino covers two squares, either vertically or horizontally but not diagonally.

A. 31! B. 2^{31} C. 31 D. $\frac{62!}{31!}$ E. NOTA

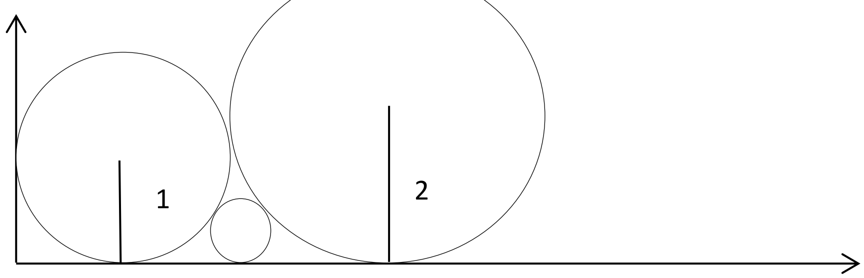
10. A point in space moves governed by the parametric equations $x = 2\cos(\mathrm{t}\pi)\sin(\mathrm{t}\pi)$, $\mathrm{y} = sin\frac{t\pi}{2}$. What is the distance between the point's locations at t = 1/3 and t = 1?

A. $\frac{\sqrt{6}-\sqrt{2}}{2}$ B. $\frac{\sqrt{3}}{2}$ C. $\frac{1}{2}$ D. 1 E. NOTA

11. What is the probability that the sum of the elements of an arbitrary subset of {1,2,3…,39,40} is congruent to 1 mod 4?

A. $\frac{1}{40}$ B. $\frac{1}{8}$ C. $\frac{1}{4}$ D. $\frac{389}{1560}$ E. NOTA

12. In the first quadrant, we draw a circle of radius 1 tangent to both the x- and y- axes. Then we draw a circle of radius 2 next to it that is tangent to the x-axis and the first circle. Finally, we draw a circle tangent to the first two circles and the x-axis. What is the radius of this circle?



A. $\sqrt{5}-2$ B. 6-4$\sqrt{2}$ C. 4-2$\sqrt{2}$ D. $2\sqrt{2}-2$ E. NOTA

13. Compute the golden ratio, Φ, taken to the fourth power in terms of Φ. Φ^4 =?

A. 2 Φ + 1 B. Φ + 1 C. 3 Φ + 2 D. 5 Φ + 3 E. NOTA

14. Anthony and Daniel are trying to meet up to get lunch at Chipotle. They decide to meet sometime between 10am and 2pm, but both of them have forgotten exactly when they said to meet. Daniel is very busy and will show up some time between 10am and 2pm but only stay for 30 minutes. On the other hand, Anthony has nothing to do with himself and will show up between 10am and 2pm and stay for 2 hours. What is the probability that the two are both at Chipotle at the same time?

A. $\frac{61}{128}$ B. $\frac{31}{64}$ C. $\frac{63}{128}$ D. $\frac{1}{2}$ E. NOTA

15. Assume that the probability of correctly answering question *n* on an infinite exam is $\frac{1}{2^{n-1}}$. Question *n* is also worth *n* points. How many points do you expect to get on the exam?

A. 4 B. 8 C. 16 D. ∞ E. NOTA

16. Find the eigenvalues of the matrix $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$.

A. 0, 4 B. 1, 4 C. 2, 2 D. 0, 5 E. NOTA

17. Drew likes when people stand in straight lines, so he always tries to arrange all of his friends into columns. When he tries with five lines of equal length, he finds two people left over. When he tries with seven, he finds four people left over. When he tries with eleven, he finds five people left over. When he tries with thirteen, he finds two people left over. What is the second smallest number of friends Drew could have?

A. 522 B. 5527 C. 1237 D. 6242 E. NOTA

18. Find the value of x that minimizes $\frac{9x^2-48x+65}{3x-8}$ given that x > $\frac{8}{3}$.

A. $\frac{1}{2}$ B. 1 C. 2 D. 3 E. NOTA

19. Compute $\sin 50^\circ + \sin 10^\circ - \sin 70^\circ$.

A. 0 B. 1 C. $\frac{1}{2}$ D. $-\frac{\sqrt{6}-\sqrt{2}}{4}$ E. NOTA

20. Nikolai and Diego just got out of class and head home. The two start at the same point, the origin, and Nikolai heads due west at 3 mph while Diego heads northeast at 2 mph. Nikolai walks for an hour while Diego walks for $\frac{\sqrt{2}}{2}$ hours. The altitude from the origin of the triangle made by their starting point and ending points can be written as $\frac{a\sqrt{b}}{c}$ where all three letters are positive integers, no perfect square besides 1 divides b and a and c are relatively prime. Compute a+b+c.

A. 71 B. 61 C. 54 D. 37 E. NOTA

21. $x + \frac{1}{x} = y$. Compute $x^5 + \frac{1}{x^5}$ in terms of y.

A. y^5 B. y^5-5y^3-10y C. $y^5 - 5y^3$+5y D. $y^5 - 5y^3 - 5y$ E. NOTA

22. In a school of 300 students, everyone is taking at least one of the following languages: French, Spanish, or German. There are 180 people taking Spanish, 120 people taking French, and 80 people taking German. Of these, there are 40 taking Spanish and French, 30 taking Spanish and German, and 30 taking German and French. How many students are taking all three?

A. 10 B. 20 C. 30 D. 40 E. NOTA

23. Euler's constant, *e*, can take non-real values in its exponent. Similarly, sin and cos can take complex inputs. What is cos(-*i***ln*(2+$\sqrt{3}$))?

A. 2 B. 1 C. 0 D. $\sqrt{2}$ E. NOTA

24. Find the sum of the solutions to log(x)+log(x-3)=1.

A. 2 B. 3 C. 5 D. 7 E. NOTA

25. Find the equation of a hyperbola whose vertices are the same as those of the endpoints of the major axis of the ellipse defined by $\frac{x^2}{25} + \frac{y^2}{9} = 1$ given that the eccentricity of the hyperbola is equal to the reciprocal of the eccentricity of the ellipse.

A. $\frac{x^2}{25} - \frac{y^2}{9} = 1$ B. $\frac{y^2}{25} - \frac{x^2}{9} = 1$ C. $\frac{x^2}{25} - \frac{y^2}{16} = 1$ D. $\frac{y^2}{25} - \frac{x^2}{9} = 1$ E. NOTA

26. You have a giant 12 foot x 1 foot x 1 foot block of cheese that you want to sell. Unfortunately, no one wants 12 cubic feet of cheese. So you decide to break it up into smaller pieces. The pieces that sell are 1 foot x 1 foot x 1 foot, 2 feet x 1 foot x 1 foot, and 3 feet x 1 foot x 1 foot. How many different ways can you cut your cheese to sell?

A. 455 B. 836 C. 610 D. 927 E. NOTA

27. Rotate the graph of the equation $5x^2-2xy+5y^2-4x-6y+7=0$ the least possible positive angle to remove the *xy* term. What is the coefficient of x^2 in the new equation (written in general form)?

A. 4 B. 5 C. 6 D. 7 E. NOTA

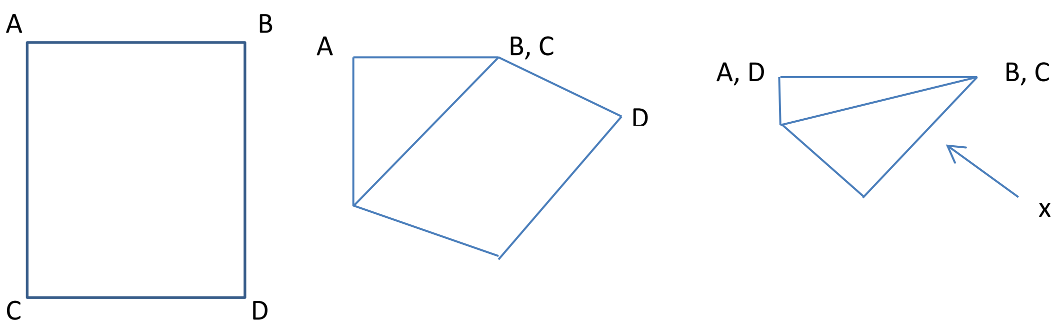
28. Elliot is giving away old Halloween candy to little kids. There are 5 kids at his door and he has 7 Snickers, 7 Hershey's, and one KitKat. The kids are running around outside his door, so he throws the candy to them at random. Given that no pieces of candy are left behind, what is the probability that one kid gets exactly 5 Snickers and the KitKat?

A. $\frac{1}{22}$ B. $\frac{4}{165}$ C. $\frac{1}{33}$ D. $\frac{2}{33}$ E. NOTA

29. Hailang and Alec are playing a game with a tetrahedral die, a cubic die, and an octahedral die. Each die is labelled with one number on each face from 1 to *n* where *n* is the number of faces on the die, and all dice are fair. A player wins by rolling a multiple of 4. The game proceeds with the first player rolling the tetrahedral die. The second player then rolls the cubic die. The first player rolls the octahedral die, the second player rolls the tetrahedral die, and so on. What is the probability that Alec wins if Hailang goes first?

A. $\frac{1}{8}$ B. $\frac{1}{3}$ C. $\frac{323}{799}$ D. $\frac{225}{323}$ E. NOTA

30. An 8x15 piece of paper is folded twice by taking each pair of opposite corners and laying them atop each other as shown in the diagram. What is the length of x?



A. 7.5 B. 8 C. 8.5 D. 15 E. NOTA