1. C	6. A	11. C	16. D	21. C	26. B
2. A	7. B	12. B	17. B	22. B	27. C
3. E	8. A	13. C	18. D	23. A	28. A
4. D	9. E	14. C	19. A	24. C	29. C
5. C	10. D	15. A	20. D	25. E	30. C

1. The results of the first leg are irrelevant, but do determine what order the remaining nine events must have. Thus, there are 3! ways for each leg to end, and nine legs which we need to count. $\frac{1}{6^9}$, C.

2. Using Cramer's Rule, x = $\frac{\begin{vmatrix} 1 & -3 & 1 \\ 3 & 7 & 10 \\ -1 & 2 & -2 \end{vmatrix}}{\begin{vmatrix} 1 & -3 & 1 \\ 2 & 7 & 10 \\ 3 & 2 & -2 \end{vmatrix}} = \frac{-9}{-153} = \frac{1}{17}$, A.

3. Simply multiply the top and bottom by 1/3 so that the constant added to the cosine is 1, this yields e = 2/3, E.

4. Using sum-to-product, sin(x)+sin(3x) = 2sin(2x)cos(x). The sin(2x) will cancel with the bottom to leave 2cos(x), D.

5. Notice that the x^4 term is missing. This allows us to factor the equation into $(x^5+1)(x^3+x^2+x+1)=0$. The only real solution here is from the x^5+1 term, which is x = -1. This is the only solution, C.

6. Using the triple scalar product formula from point A, AB = <-3, 2, -7>, AC = <1, -1, -5>, AD = <-4, -6, -7>.

AB x AC = <-17, -22, 1>. Then we take the dot product with AD to get 68 + 132 + (-7) = 193. The area of the parallelepiped is the absolute value, C.

7. There are $\binom{10}{5}$ ways to get from (0,0) to (5,5). Of these, $\binom{4}{2}\binom{6}{3}$ go through (2,2) and $\binom{8}{4}\binom{2}{1}$ go through (4, 4) and $\binom{4}{2}\binom{4}{2}\binom{2}{1}$ go through both. Thus our answer is $\binom{10}{5}-\left(\binom{4}{2}\binom{6}{3}+\binom{8}{4}\binom{2}{1}\right)+\binom{4}{2}\binom{4}{2}\binom{2}{1}$ = 64, B.

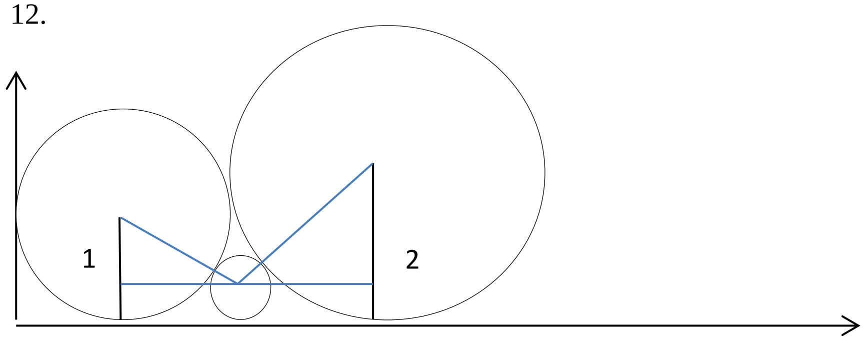
8. First we need to take the cross product of the two given vectors to get the vector orthogonal to the plane. <1, 5, 6> x <2, 3, 4> = <2, 8, -7>. Now we just plug in the given point to finish the equation. 2(7) + 8(6) + (-7)(9) = -1. A.

9. This cannot be done. If we consider that each domino must cover one white square and one black square and note that when we remove opposite corners of a chessboard that the two squares removed are the same color, there are 30 white (or black) squares and 32 black (or white) squares. Thus, 0, E.

10. Note that *x* is just defined as sin(2tπ). At t = $\frac{1}{3}$ we have $(\frac{\sqrt{3}}{2}, \frac{1}{2})$. At t = 1 we have (0, 1). Using distance formula, we get the distance between the two points as 1, D.

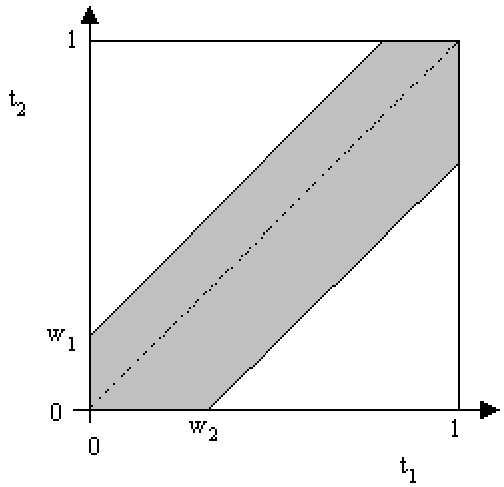
11. Note that each subset of the set {1, 2, 3, 4…39, 40} must be either 0, 1, 2, or 3 in mod 4. By symmetry, each of the four possible residues must appear an equal number of times. Thus, $\frac{1}{4}$, C.

12.



The distance between the center of the circle of radius 1 and the smaller circle is 1+x, where x is the radius of the smaller circle. Similarly for the circle of radius 2, we have 2+x. The length of the line segment connecting the two draw radii of the circles can be computed as $\sqrt{(2+1)^2-(2-1)^2} = 2\sqrt{2}$. Now we set up a systems of equations using 2 unknowns.

13. Note that $\Phi^2 = \Phi + 1$. This gives us that $\Phi^4 = (\Phi+1)^2 = \Phi^2 + 2\Phi + 1 = 3\Phi + 2$, C.



14.

Using geometric probability, Anthony stays for 2 hours, which is half of 4 so w_1 will be 0.5, and Daniel stays for half an hour, which is one-eighth of 4, so w_2 will be .125. Calculating the area of the two outer triangles, we get $.5^2/2 + .875^2/2 = 65/128$. So our answer is 1-65/128 = 63/128, C.

15. Notice that this is just a model for the infinite sum of $\frac{n}{2^n}$. Call this sum S. Then 2S = $\sum \frac{n}{2^{n-1}}$. So when we take 2S-S=S, we get S = $\sum \frac{1}{2^{n-1}}$ as n goes from 0 to infinity. This sum is 4, A.

16. In order to find the eigenvalues, we subtract x from 1 and 4 in the matrix and then take the determinant and set it equal to 0. $(1-x)(4-x)-(2)(2)=0$. Simplifying, $5x-x^2=0$. Thus the eigenvalues are 0,5 D.

17. Notice that there is a remainder of 2 for both 5 and 13, so combining these two mods we get 2 (mod 65). X = 4 (mod 7) and x = 5 (mod 11). Rewriting x, let x = 5 + 11k. This is 4 (mod 7). So 5 + 11k = 4 (mod 7). Note that 11 is just 4 (mod 7) so, 5+4k = 4 (mod 7). Solving this, we get k = 5. Plugging this back in, we have x = 60 (mod 77). Now we just combine 2 (mod 65) and 60 (mod 77). Let x = 60 + 77m. This is equal to 2 (mod 65) so 60 + 77m = 2 (mod 65). Note that 77 = 12 (mod 65). Now we have 60 + 12m = 2 (mod 65). Solving this, we get m = 6. Plugging back in, we now have x = 60 + 77(6) = 522. So x is 522 (mod 65 x 77), which, using the fact that 7x11x13 = 1001, is 5005. So we want the second smallest, which is 522 + 5005 = 5527, B.

18. We can split the top into $(3x-8)^2+1$. Let y = 3x-8. We now want the minimum of $\frac{y^2+1}{y}$. Splitting this up, we get $y+\frac{1}{y}$. From our initial assumption, y >0, so we can use AM-GM here to see that $y+\frac{1}{y}$ has a minimum at $y=\frac{1}{y}$, so y = 1. Finally we solve $3x-8=1$ to get x = 3, D.

19. Using sum to product on sin(50) – sin(70), we get 2cos(60)sin(-10), which simplifies to sin(-10). Sin(-10) = -sin(10), so this equation boils down to 0, A.

20. Plotting this on a Cartesian plane with +x East and +y North, we find that Nikolai is at the point (-3, 0) and Diego is at the point (1,1). Using $A=\frac{1}{2}absin(C)=\frac{1}{2}hb$, we find that $A=\frac{3}{2}, b=\sqrt{(-3-1)^2+(0-1)^2}=\sqrt{17}$. Plugging this in, we get $b=\frac{3\sqrt{17}}{17}$. So a+b+c = 3+17+17=37, D.

21. To get our $x^5+\frac{1}{x^5}$, we first take $x+\frac{1}{x}$ to the fifth power. Expanding, this gives $x^5+5x^3+10x+\frac{10}{x}+\frac{5}{x^3}+\frac{1}{x^5}=y^5$. We see the $10(x+\frac{1}{x})$, which is just 10y. To get the $x^3+\frac{1}{x^3}$ term, we cube $x+\frac{1}{x}$ and get $x^3+3x+\frac{3}{x}+\frac{1}{x^3}=y^3$. So $x^3+\frac{1}{x^3}=y^3-3y$. Substituting this back in, we get $y^5-5(y^3-3y)-10y=y^5-5y^3+5y$, C.

22. Using PIE, there are 180+120+80-(40+30+30)+x=300 students. Solving for x, we get 20, B.

23. Notice that from $e^{i\Theta}=\cos(\Theta)+i*\sin(\Theta)$ that we can derive $\cos(\Theta)=\frac{e^{i\Theta}+e^{-i\Theta}}{2}$. Now we can just substitute in -*i***ln*(2+$\sqrt{3}$) for Θ. This gives us just ln's in the exponent of e, which cancel. The result is $2+\sqrt{3}+\frac{1}{2+\sqrt{3}}=4$. And then we divide out the 2 from the original denominator to get our answer of 2, A.

24. We can combine the logs and bring over the 1 as a 10 to get $x^2-3x-10=0$. Solutions here include x=-2 and x=5. X=-2 gives and undefined log(x), so the only solution is 5. C.

25. The eccentricity of the ellipse is $\frac{c}{a}$. Note that $c=\sqrt{a^2-b^2}=\sqrt{25-9}=4$. So the eccentricity of our hyperbola will be $\frac{5}{4}$. A quick check shows that the x^2 term in the hyperbola will be positive (the major

axis is the x for the ellipse) and have a denominator of 25 so that the vertices are the same. Using $e = \frac{c}{a}$ with $e = \frac{5}{4}, a = \sqrt{25} = 5$ we can find $c = \frac{25}{4}$, and since $c = \sqrt{a^2 + b^2}$, we can find $b = \frac{15}{4}$, so our equation is $\frac{x^2}{25} - \frac{y^2}{\left(\frac{15}{4}\right)^2} = 1$, which is not given. E.

26. Notice that this is a recursive sequence with $a_1 = 1, a_2 = 2, a_3 = 4, a_n = a_{n-1} + a_{n-2} + a_{n-3}$. There is 1 way to get a 1x1x1 block. You can either make 2 1x1x1blocks or a single 2x1x1 block to get to 2x1x1. For 3x1x1, you can add a 2x1x1 to the 1x1x1 or a 1x1x1 to the two ways to get to 2x1x1 or you could just make a 3x1x1 block. For each successive block, you can add 3x1x1 to each method of getting to a block 3x1x1 less, 2x1x1 to each method of getting to a block 2x1x1 less, and 1x1x1 to each method of getting to a block 1x1x1 less.

27. Here we use $x' = xcos\theta + ysin\theta$ and $y' = -xcos\theta + ysin\theta$. Where $\cot(2\theta) = \frac{A-C}{B} = \frac{5-5}{2} = 0$. So θ = 45°. Substituting in for x' and y' we get $5(xcos(45) + ysin(45))^2 - 2\big(xcos(45) + ysin(45)\big)\big(-xcos(45) + ysin(45)\big) + 5\big(-xcos(45) + ysin(45)\big)^2$ + (things without an x^2) = 0. Expanding, we get a coefficient of 6 on the x^2 term, C.

28. To calculate the denominator, we use stars and bars on each candy separately. There are $\binom{11}{4} = 330$ ways to distribute both the Snickers and the Hersheys, and $\binom{5}{1} = 5$ ways to give out the KitKat. Multiplying these out gives a denominator of 544500. If one child has 5 Snickers and a KitKat, there are 2 Snickers left and all the Hershey's. There are 5 ways to choose a child, $\binom{6}{2}$ ways to give out the remaining Snickers and still $\binom{11}{4}$ ways to give out the Hershey's. Multiplying all of this out gives us $\frac{1}{22}, A$.

29. Hailang loses his roll and Alec wins on his first try with probability $\frac{3}{4} \times \frac{1}{6} = \frac{1}{8}$.

Hailang loses twice and Alec loses his first then wins his second with probability $\frac{3}{4} \times \frac{5}{6} \times \frac{3}{4} \times \frac{1}{4} = \frac{15}{128}$.

Alec wins his third try with probability $\frac{3}{4} \times \frac{5}{6} \times \frac{3}{4} \times \frac{3}{4} \times \frac{5}{6} \times \frac{1}{4} = \frac{75}{1024}$

At this point we repeat and have an infinite series with common ratio $\frac{3}{4} \times \frac{5}{6} \times \frac{3}{4} \times \frac{3}{4} \times \frac{5}{6} \times \frac{3}{4} = \frac{225}{1024}$. Which is the probability that everyone fails every time. Thus $p = \frac{\frac{1}{8}+\frac{15}{128}+\frac{75}{1024}}{1-\frac{225}{1024}} = \frac{323}{799}$.

30. Using symmetry, we can see that x is just measuring half the length of the diagonal of the sheet of paper, which is $\sqrt{8^2 + 15^2} = 17$. Thus, 8.5, C.