

1. A

For example, take any vector lying along the real axis. Multiplying by i gives a purely imaginary number, lying along the imaginary axis.

2. C

This is the set of all 2D points of equal distance from the origin.

3. A

Using the quadratic equation, we have $\frac{-4 \pm \sqrt{16 - 4 \cdot 8 \cdot 5}}{2 \cdot 8} = -\frac{1}{4} \pm \frac{3}{4}i$

4. B

$$(1 - i)^2 = -2i, \text{ so } (1 - i)^{2016} = (-2i)^{1008} * (1 - i) = 2^{1008} * (1 - i)$$

5. C

Using the Pythagorean theorem for distance, $\sqrt{3^2 + 4^2} = 5$

6. E

By itself, k behaves just like the normal imaginary unit i . So $k^{2017} = k^{2016} * k = 1 * k = k$

7. B

We had given that $i*j*k = -1$, so $(i*j)*k = -1$. We know that $k*k = -1$, so $(i*j)$ must be k .

8. C

We know that $j*i = -i*j$, so $j*i*k = -i*j*k = 1$. Then $j*(i*k) = 1$, so $i*k = -j$.

9. B

$$i^3 j k i k^2 j = -i j k i (-1) j = i j k i j = -k * k * -k = k^3 = -k$$

10. D

FOILING gives $3i + 3ij + 2j + 2j^2 = 3i + 3k + 2j - 2$

11. E

All of those numbers are complex, they just aren't all imaginary.

12. A

$$\left(\frac{\sqrt{6}-\sqrt{2}}{4} + \frac{\sqrt{6}+\sqrt{2}}{4}i\right)^{2017} = (\text{cis } 75^\circ)^{2017} = \text{cis } 75^\circ = \frac{\sqrt{6}-\sqrt{2}}{4} + \frac{\sqrt{6}+\sqrt{2}}{4}i$$

13. B

If a polar conic has an eccentricity greater than one, it is a hyperbola.

14. B

All of the solutions are in the complex plane, so there are 6 complex solutions (and they are all distinct).

15. C

There are 254 factors in the expansion of the product, so we can pull out $i^{254} = -1$.

Then, due to the fact that $\log_a b = \frac{\log b}{\log a}$, all but the first and last values for n will cancel,

and we have $\prod_{n=2}^{255} \log_n(n+1) = \log_2 256 = 8$. So our final answer is -8.

16. C

We can factor the equation into $(z^2 + 1) * (z - 1) * (z + 2)$. Thus two of the solutions are real.

17. C

This is a spiral of Archimedes.

18. D

This is an 8-petaled rose, so there are eight lines of symmetry (four centered on petals, four between petals).

19. D

There are six solutions to this equation, forming a hexagon in the complex plane, with the length from the center to the vertex being 1. Then the total area is $6 * \frac{1^2\sqrt{3}}{4} = \frac{3\sqrt{3}}{2}$

20. E

$$\frac{(2\text{cis}\theta)^4 \text{cis}(4\theta)}{\text{cis}(8\theta)} = \frac{16(\text{cis}\theta)^4 * (\text{cis}\theta)^4}{(\text{cis}\theta)^8} = 16$$

21. D

By Euler's theorem, $e^{i\theta} = \text{cis}\theta \rightarrow e^{\frac{i\pi}{4}} = \cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}(1 + i)$

22. A

Let $x=1/a$ and $y=1/b$. Then we have $xi + 3y = 5$ and $2x - 4iy = 8$. Multiplying the first equation by $2i$ and adding to the second equation gives $6iy = 8 + 10i \rightarrow y = 5 - 4i$. Then

$$b = \frac{1}{5-4i}$$

23. B

The sum of an infinite geometric series is given by $\frac{a}{1-r} = \frac{2i}{1-\frac{i}{4}} = \frac{8i}{4-i} * \frac{4+i}{4+i} = \frac{32i-8}{17}$

24. C

For conics of the form $r = a + b\cos\theta$, if $a/b=1$, the conic is a cardioid.

25. B

The point is $4\sqrt{5}$ units from the origin, so it will take 4 sec.

26. D

$$\frac{(2 + 3i) \left(4 \operatorname{cis} \left(\frac{\pi}{2} \right) \right)}{e^{i\pi}} = \frac{(2 + 3i)(4)(i)}{-1} = -8i + 12$$

27. A

$$\cos^2(z) - \sin^2(z) + \sin(2z) = -1 \cos^2(z) - \sin^2(z) + 2\sin(z)\cos(z) = (\cos(z) + i\sin(z))^2 = \operatorname{cis}^2(z). \text{ Then } \operatorname{cis}(z) = e^{\frac{i\pi}{2}}, \text{ so } z = \frac{\pi}{2}$$

28. A

The sequence is $i + -1 + -i + 1 + i + \dots + i$. Each set of four terms will cancel, leaving only the very last term, so the entire sum is i .

29. A

Similar to the previous problem, each set of four terms will multiply out to -1 . There are $2016/4=504$ sets of four terms, so the first 2016 terms multiply out to 1 . This leaves the last term, i .

30. E

$$\begin{aligned} e^{i\pi/3} \cdot e^{i\pi/4} &= \operatorname{cis} \left(\frac{\pi}{3} \right) * \operatorname{cis} \left(\frac{\pi}{4} \right) = \left(\frac{1}{2} + \frac{\sqrt{3}i}{2} \right) \left(\frac{\sqrt{2}}{2} + \frac{i\sqrt{2}}{2} \right) \\ &= \frac{1}{4} (\sqrt{2} + i\sqrt{2} + i\sqrt{6} - \sqrt{6}) \end{aligned}$$