

1. A	6. C	11. C	16. A	21. D	26. D
2. B	7. D	12. D	17. E	22. A	27. A
3. B	8. D	13. B	18. B	23. B	28. A
4. A	9. D	14. D	19. D	24. B	29. E
5. C	10. A	15. E	20. A	25. C	30. A

1. Note that  $2047 = 1111111111_2$ .  $2017 = 2047 - 30$ .  $30 = 11110_2$ . So  $2017 = 1111111111_2 - 11110_2 = 11111100001_2$ . A.

2. The formula for an arithmetic-geometric series, where the  $n$ th term of the series,  $x_n = a_n g_n$  with  $a_n$  as the arithmetic series with common difference  $d$  and  $g_n$  as the geometric series with common ratio  $r$ . is  $\frac{d g_2}{(1-r)^2} + \frac{x_1}{1-r}$ . Here we have  $2n$  as our arithmetic series and  $\frac{1}{2^n}$  as our geometric series.  $d = 2$ ,  $r = \frac{1}{2}$ ,  $x_1 = 0$ ,  $g_2 = \frac{1}{4}$ . Thus our answer is  $\frac{2(\frac{1}{2})}{(1-\frac{1}{2})^2} + \frac{0}{1-\frac{1}{2}} = 4$ , B.

3. Write the equation as  $y = \frac{x+2}{x-2}$ . Replace  $x$  with  $y$  and  $y$  with  $x$ . This gives us  $x = \frac{y+2}{y-2}$ . Then  $xy - 2x = y + 2$ . And  $y(x - 1) = 2 + 2x$ . Thus  $g(x) = \frac{2+2x}{x-1}$ , B.

4. Plugging in 0 for  $x$ , we get  $y = c$ . Using Vieta's formulas, we can find that  $rs$  is also equal to  $c$ . Thus,  $rs = y$ , A.

5. The boring numbers are: 2, 6, 10, 14, 22, 26, 30, 34, 38, 42, 46, 58, 62, 66, 70, 74, 78, 82, 86, 94. Summing these gives us 940, C.

6. We can clearly see that the digit 0 cannot be used here since it would have to be the leftmost digit, making the number not technically 5 digits. Thus, there are 9 digits to choose from. From these 9 digits, we can choose 5 to be in our number. Once we have chosen those 5 digits, there is only one way to arrange them such that they are in strictly increasing order. Thus, our answer is  $\binom{9}{5} = 126$ . C

7.  $f(0) = 4(0) - 4f(-1) + 8f(-2) = 0 - 4(0) + 8(4) = 32$ .

$$f(1) = 4(1) - 4f(0) + 8f(-1) = 4 - 4(32) + 8(0) = -124.$$

$$f(2) = 4(2) - 4f(1) + 8f(0) = 8 - 4(-124) + 8(32) = 760, D.$$

8. The roots of unity are defined as the solutions to  $x^n = 1$ . Our problem asks for the sum of the roots using  $n = 2017$ . As is clear from the equation and Vieta's formulas,  $-\frac{b}{a} = 0$ , D.

9. The area of one triangle defined by the center of the circle and two consecutive vertices of the dodecagon is equal to  $1/12$  of the total area. The angle at the center of the circle is equal to  $360/12 = 30$  degrees. Each leg of the isosceles triangle is 4. Using the formula  $A = \frac{1}{2}ab\sin(C)$ , we can calculate the area of one triangle to be  $\frac{1}{2}(4)(4)(\sin(30)) = 4$ . Thus the total area of the dodecagon is 48. The area of the circle is  $16\pi$ . The ratio is  $\frac{3}{\pi}$ , D.

10. In order to find this distance, we can find the distance from the line to the center of the circle, and then subtract the radius of the circle. The distance from the line to the center of the circle,  $(2, 0)$  is defined by  $L = \frac{3(2)+(-1)(0)+15}{\sqrt{3^2+(-1)^2}} = \frac{21}{\sqrt{10}}$ . By completing the square, we can find the equation of the circle to be  $(x - 2)^2 + y^2 = 40$ , so our radius is  $2\sqrt{10}$ . Thus, the distance between the points is  $\frac{21\sqrt{10}}{10} - 2\sqrt{10} = \frac{\sqrt{10}}{10}$ , A.

11. We can square both sides of the equation to get  $\sin^2(x) + 2 \sin(x) \cos(x) + \cos^2(x) = \frac{289}{225}$ .  $\sin^2(x) + \cos^2(x) = 1$ . Therefore,  $2 \sin(x) \cos(x) = \sin(2x) = \frac{64}{225}$ , C.

12. Completing the square, our equation becomes  $4(x - 4)^2 - 25(y - 4)^2 = 36$ . Converting into a hyperbola, we find that  $a = 3$  and  $b = \frac{6}{5}$ . The length of the latus rectum is  $\frac{2b^2}{a} = \frac{2(\frac{6}{5})^2}{3} = \frac{24}{25}$ , D.

13. The equation for the sum of the first  $n$  perfect squares is  $\frac{n(n+1)(2n+1)}{6}$ . So  $x = n(n+1)(2n+1)$ , where  $n = 2017$ . Since we only want the last two digits, we can take everything in mod 100.  $2017 \pmod{100} = 17$ .  $2017+1 \pmod{100} = 18$ .  $2(2017)+1 \pmod{100} = 35$ . Our product is now  $17 * 18 * 35 = 630 * 17 = 10710$ . The last two digits are 10, B.

14. The radius of the cone is 10 meters. Thus, the volume is  $\frac{1}{3}\pi 10^2(12) = 400\pi$ . The volume of the water that would flow into the cone is  $250\left(\frac{472}{60}\right) = 1966\frac{2}{3}$ . More water flows into the cone than the volume of the cone, thus the answer is D.

15. Note that  $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}^n = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2^n \end{pmatrix}$ . The magnitude of each matrix is then just  $2^n$ . Thus summing  $2^n$  from  $n=1$  to 5 is just  $2+4+8+16+32 = 62$ , E.

16. Notice that  $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots = 4\left(\frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{6^2} + \dots\right)$ . Then  $\frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{6^2} + \dots = \frac{\pi^2}{24}$ . If we subtract this from  $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots$ , we get that  $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} = \frac{\pi^2}{6} - \frac{\pi^2}{24} = \frac{\pi^2}{8}$ , A.

17. If we plot this onto a Cartesian plane, our math will get much simpler. Let the triangle have points at the origin  $(0,0)$ ,  $(0, 24)$ , and  $(70, 0)$ . We know that the circumcenter is at the midpoint of the hypotenuse which goes from  $(0, 24)$  to  $(70, 0)$ , so the orthocenter is located at  $(35, 12)$ . Using our formula for the inradius,  $r_s = A$ . Our area is just  $\frac{1}{2}(24)(70) = 840$ . The semi perimeter is  $(0.5)(24+70+74) = 84$ . (Our triangle is double a 12-35-37). Our inradius is 10, so our incenter is located at  $(10, 10)$ . Using the distance formula, our distance is  $\sqrt{(35 - 10)^2 + (12 - 10)^2} = \sqrt{629}$ , E.

18. Note that there are two distinct Pythagorean triples with 50 as the hypotenuse. These are  $(30, 40, 50)$  and  $(14, 48, 50)$ . There are multiple ways to solve for  $x_1, x_2, y_1,$  and  $y_2$ , but in the end, all four of them must be distinct. As a result, the only option is for each of  $x_1, x_2, y_1,$  and  $y_2$  to take on a value from  $\{30, 40, 14, 48\}$ . The sum of these values must be 132, B.

19. To solve this problem, we first find all of the intersection points that define our shape. These points are  $(0,0)$  from the first two equations,  $(0, 4)$  from the first and third equations,  $(4, 0)$  from the second

and fourth equations (not that (6, 0) is thrown out from the second and third equations since the fourth equation would fail in that case). From the third and fourth equations, we get  $(\frac{84}{17}, \frac{12}{17})$ . Checking each of these points for  $5x+3y$  yields: 0, 12, 20, and  $\frac{456}{17}$ . Of these,  $\frac{456}{17}$  is clearly the largest. D.

20. After  $n$  miles, Everett's odometer will read  $2^n \pmod{100}$ . After 10 miles, Everett's odometer will read  $2^{10} = 1024 \pmod{100} = 24$ .  $2^{20} = (2^{10})^2$ , so when we take  $2^{20} \pmod{100}$  we get  $(2^{10})^2 \pmod{100}$  which is  $24^2 \pmod{100} = 576 \pmod{100} = 76$ , A.

21. Note that  $e^{i\theta} = \cos(\theta) + i\sin(\theta)$ . So  $e^{e^{i\theta}} = e^{\cos(\theta)+i\sin(\theta)} = e^{\cos(\theta)}e^{i\sin(\theta)}$ . The magnitude of  $e^{ix}$  is always 1. So when we take the magnitude of our new equation, we get  $|(e^{\cos(\theta)})(1)| = |e^{\cos(\theta)}|$ . This is maximized when  $\cos(\theta)$  is maximized, which occurs when  $\theta=0$ , D.

22. Subtracting  $x+4$  from both sides and multiplying through by  $-1$  (don't forget to flip the inequality) gives us  $\frac{x^3+3x^2+5x-9}{x^2-2} \geq 0$ . Factoring the top, we find that 1 is a root, but that the other roots are imaginary. So, our roots come at 1 and  $\pm\sqrt{2}$ . The roots at  $\pm\sqrt{2}$  are not included in the solution set since they give division by 0. Thus, our answer is  $(-\sqrt{2}, 1] \cup (\sqrt{2}, \infty)$ . A

23. We can convert these two lines into their equivalent vectors. The first equation gives us the vector  $\langle 4, 3 \rangle$ . The second equation gives us the vector  $\langle 12, 5 \rangle$ .  $\cos(\theta) = \frac{a \cdot b}{|a||b|} = \frac{4(12)+3(5)}{5 \cdot 13} = \frac{63}{65}$ , B.

24. Doing a quick partial fraction decomposition gives us that  $\frac{3n+4}{n^3+3n^2+2n} = \frac{2}{n} - \frac{1}{n+1} - \frac{1}{n+2}$ . When we sum this infinitely, everything with denominator greater than or equal to 3 cancels out. Thus, the series sums to  $\frac{2}{1} - \frac{1}{2} + \frac{2}{2} = \frac{5}{2}$ , B.

25. We can simplify this triangle into a similar one with  $\frac{1}{3}$  the scale for simpler numbers since scaling down the sides won't change the value of the angles. The law of cosines on this smaller 8, 13, 13 triangle gives  $64 = 169 + 169 - 338 \cos(y)$ . Solving for  $\cos(y) = \frac{137}{169}$ , C.

26. The chance of pulling each coin is  $\frac{1}{3}$ . The chance of pulling the first coin and getting heads is  $\frac{1}{3}(\frac{1}{2}) = \frac{1}{6}$ . The chance of pulling the second coin and getting heads is  $\frac{1}{3}(\frac{2}{3}) = \frac{2}{9}$ . The chance of pulling the third coin and getting heads is  $\frac{1}{3}(\frac{3}{4}) = \frac{1}{4}$ . The total probability of getting heads is  $\frac{1}{6} + \frac{2}{9} + \frac{1}{4} = \frac{23}{36}$ . So the probability of flipping tails is  $\frac{13}{36}$ , D.

$$27. \text{proj}_{\vec{v}} \vec{u} = \frac{\vec{u} \cdot \vec{v}}{|\vec{v}|^2} \vec{v} = \frac{\langle 2, 1, 4 \rangle \cdot \langle -3, -1, 2 \rangle}{\sqrt{(-3)^2 + (-1)^2 + 2^2}} \vec{v}, A$$

28. Using Lagrange's method of differences, since we know that it is a cubic polynomial, we only need 4 levels.

- |    |     |     |      |      |      |
|----|-----|-----|------|------|------|
| 1) | 120 | 129 | 70   | -51  | -228 |
| 2) | 9   | -59 | -121 | -177 |      |
| 3) |     | -68 | -62  | -56  |      |

4)                                  6                  6

Our answer is -228, A.

29. Note that  $\cosh^2(x) - \sinh^2(x) = 1$ . To get here, we take  $\frac{(x-5)^2}{16} - \frac{(y+2)^2}{36} = 1$ . However, notice that  $\cosh(x)$  is always positive, meaning that we have only the right half of a hyperbola. Not the entire hyperbola. E.

30. Elliot must go a total of  $(10-2)+(10-3) = 15$  in the y-direction. Elliot also goes 8 in the x-direction. The shortest path that exists is  $\sqrt{8^2 + 15^2} = 17$ . A