1. Answer: E

Solution: $\frac{x^3y^{-\frac{1}{2}}z^{\frac{15}{16}}}{x^{-3}(y^3)^{\frac{1}{2}}\left(z^{\frac{1}{2}}\right)^{-\frac{1}{4}}} = \frac{x^6y^{-\frac{1}{2}}z^{\frac{15}{16}}}{y^{\frac{3}{2}}z^{-\frac{1}{8}}} = \frac{x^6z^{\left(\frac{15}{16}+\frac{2}{16}\right)}}{y^{\left(\frac{3}{2}+\frac{1}{2}\right)}} = \frac{x^6z^{\frac{17}{16}}}{y^2}$

2. Answer: C

Solution: Simplified, the equations look like this: $y = e^x + e^{-x} - 2$ and $y = \frac{(e^x+2)}{2e^{-x}+1}$

Setting these equations equal to one another yields that $\frac{(e^x+2)}{2e^{-x}+1} = e^x + e^{-x} - 2$

Simplifying yields that $e^x + 2 = 2 + 2e^{-2x} - 4e^{-x} + e^x + e^{-x} - 2$

Thus, 0 = $2e^{-2x} - 3e^{-x} - 2$

0 = $(2e^{-x} + 1)(e^{-x} - 2) \rightarrow x = -\ln 2, \;\; y = \frac{1}{2}$

3. Answer: A

Solution: $4^{3\cot x} = 4^{4\cos x}$

$3\cot x = 4\cos x \rightarrow \frac{3\cos x}{\sin x} = 4\cos x \rightarrow \sin x = \frac{3}{4}$. The answer has sine value 1.

4. Answer: B

Solution: There are 4 terms in the expansion containing z^{-3}. Those terms contain the variables (other than z^{-3}) x^3, x^2y, xy^2, and y^3, respectively. In fact, these terms can actually be treated as a binomial themselves such that the equation to get the z^{-3} terms follows $\binom{6}{3}(3x-4y)^3z^{-3}$. Thus, the sum is $\binom{6}{3} * (-1)^3(-1)^3 = 20$.

5. Answer: A

Solution: $\frac{2}{\log_2 36}+\frac{1}{\log_3 36}+\frac{1}{\log_4 36}+\frac{2}{\log_9 36}+\frac{1}{\log_{12} 36}=\frac{\log 2^2}{\log 36}+\frac{\log 3}{\log 36}+\frac{\log 4}{\log 36}+$

$\frac{\log 9^2}{\log 36}+\frac{\log 12}{\log 36}=\frac{\log(4*3*4*81*12)}{\log 36}=\frac{\log 46656}{\log 36}=3$

6. Answer: D

Solution: $29*10^x+100*10^{-x}-10^{2x}=104 \rightarrow 10^{3x}-29*10^{2x}+104*$

$10^x-100=0=(10^x-2)^2(10^x-25)$

$x=\log 2,\ \log 25\,; \log 2+\log 25=\log 50$

7. Answer: A

Solution: There are three powers of 2 in the list, so for the base 2 log, the probability that Ul wins is $\frac{3}{10}$. There are two powers of 3, so the probability that Ul wins is $\frac{2}{10}$ for the base 3 log. For all other logs, the probability Ul wins is $\frac{1}{10}$. Thus, the overall probability Ul wins is $\frac{13}{100}$.

8. Answer: B

Solution: $\left(\frac{1}{\log_7 4}+\frac{1}{\log_{14} 2}+\frac{1}{\log_{17} 4}\right)=\frac{\log 7}{\log 4}+\frac{\log 14}{\log 2}+\frac{\log 17}{\log 4}=\frac{\log 7}{2\log 2}+\frac{\log 14}{\log 2}+$

$\frac{\log 17}{2\log 2}=\frac{\log 7+2\log 14+\log 17}{2\log 2}=\log_4 23324 \rightarrow 4^{\log_4 23324}=23324$

9. Answer: B

Solution: $\lim_{n\to\infty}\left(1+\frac{3}{n}\right)^n = e^3 = 20.086 \approx 20$

10. Answer: A

Solution: $V = \frac{1}{3}\pi r^2 h \rightarrow V = \frac{1}{27}\pi h^3 \rightarrow t \cdot log27 = \frac{1}{27}\pi 3^3 = \pi$

$$t = \frac{\pi}{log27} = \frac{\pi}{3log3}$$

11. Answer: D

Solution: $3^2 - 2^3 = 1 \rightarrow 1^{17} - 17^1 = -16$

12. Answer: D

Solution: $7^{6x+4} = 6^{18x+2} \rightarrow (6x+4)\log 7 = (18x+2)\log 6$

$6x\log 7 - 18x\log 6 = 2\log 6 - 4\log 7$

$$x = \frac{2\log 6 - 4\log 7}{6(\log 7 - 3\log 6)} = \frac{\log 6 - 2\log 7}{3(\log 7 - 3\log 6)}$$

13. Answer: A

Solution: $\log_3 3^{243} = 243 \rightarrow \log_3 243 = 5 \rightarrow \log_5 5 = 1$

14. Answer: C

Solution: $\log_7 1024 * \log_3 14641 * \log_{13} 16807 * \log_2 2197 * \log_{11} 59049 =$

$\log_7 16807 * \log_3 59049 * \log_{13} 2197 * \log_2 1024 * \log_{11} 14641 = 5 * 10 * 3 *$

$10 * 4 = 6000$

15. Answer: B

Solution: $\frac{df(x)}{dx} = e^{sinx} * \cos x$. At $x = 0, e^{sin0} * \cos 0 = 1 * 1 = 1$

16. Answer: B

Solution: $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{ne^{2n}} (1 - e^2)^n = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} (e^{-2} - 1)^n$, so this summation represents $\ln e^{-2} = -2$

17. Answer: D

Solution: $x = \sqrt{16256 - x} \rightarrow x^2 = 16256 - x \rightarrow x^2 + x - 16256 = 0$

$(x + 128)(x - 127) = 0 \rightarrow$ Since the answer must be positive, $x = 127$

18. Answer: A

Solution: The number of digits in the expansion is equal to the log base-10 of this number rounded up. $2017 \log 6 = 2017(\log 2 + \log 3) \approx 2017(\,0.77815) - the\ correct\ answer\ is\ actually\ achieved\ even\ with\ 0.778 = 1569.52855.$ This number rounds up to 1570.

19. Answer: B

Solution: This is $\sin x \cos x$, which, when written in Euler notation, is $\frac{e^{ix} - e^{-ix}}{2i} * \frac{e^{ix} + e^{-ix}}{2} = \frac{e^{2ix} - e^{-2ix}}{4i}$

20. Answer: D

Solution: $-64\sqrt{3} - 64i = 128cis\left(\frac{7\pi}{6}\right)$. $\left(128cis\left(\frac{7\pi}{6}\right)\right)^{\frac{1}{7}} = 2cis\left(\frac{\pi}{6}\right) = \sqrt{3} + i$

21. Answer: C

Solution: A simple graphical analysis of $y = e^x - x - x^e$ has two solutions, one near $x = 1.5$ and the other near $x = 3.5$. As x approaches both positive and negative infinity, the graph clearly increases without bound. Similarly, a graphical analysis of $y = x^{2e} - e^{2x} + 2x + 2$ will yield three solutions. $2^3 = 8$

22. Answer: E

Solution: $\log_2(3x-4) = \log_8(x-2) \rightarrow (3x-4)^3 = x-2$. Expanding and simplifying yields $27x^3 - 108x^2 + 143x - 62 = 0$. The only real solution to this is 1, but that doesn't work in the equation.

23. Answer: B

Solution: e to 9 decimal places (the billionths place) is 2.718281828, so the answer is 8.

24. Answer: B

Solution: The cubes' diagonals are $\sqrt{3} * \log_{11} 17$, $\sqrt{3} * \log_{289} 169$, and $\sqrt{3} * \log_{13} 121$. Multiplying these values together yields $3\sqrt{3} * \log_{11} 17 * \log_{289} 169 * \log_{13} 121 = 3\sqrt{3} * \log_{11} 121 * \log_{13} 169 * \log_{289} 17 = 6\sqrt{3}$

25. Answer: A

Solution: $\lim_{x \to 0} e^x = e^0 = 1$

26. Answer: D

Solution: The only real solutions are 2 and 4 (this can be seen graphically), so the sum of the solutions is 6.

27. Answer: C

Solution: Analysis of the bottom equation yields that there are three reasonable combinations for x and y: $(12^1, 18^3), (12^2, 18^2),$ and $(12^3, 18^1)$. Plugging these values into the top equation yields that $(12^2, 18^2)$ is the correct solution. $144 + 324 = 468 = 2^2 * 3^2 * 13$

28. Answer: B

Solution: The hypotenuse of the triangle is e^x, and the side opposite the angle is $2\ln x$. Thus, the side adjacent to the angle is $\sqrt{e^{2x} - 4(\ln x)^2}$. The tangent of the angle is opposite over adjacent, which is $\frac{2\ln x}{\sqrt{e^{2x}-4(\ln x)^2}}$.

29. Answer: E

Solution: $f^{-1}(x) = 10^x$, so the domain is all reals.

30. Answer: D

Solution: $x = e$. $(e^{e^{-1}})^e = e$.