

1. Evaluate  $\begin{vmatrix} -13 & 6 & 14 & 2 \\ 10 & 0 & 4 & 18 \\ 7 & 3 & 2 & 1 \\ 5 & 0 & 2 & 9 \end{vmatrix}$ .
- A. 1      B. -1      C. 2      D. 0      E. NOTA
2. Find  $\|u\|$  where  $u$  is  $\langle -48, 55 \rangle$ .
- A. 73      B. 77      C. 63      D. 67      E. NOTA
3. A parallelepiped has three sides  $\overrightarrow{AB} = \langle 1, -7, 3 \rangle$ ,  $\overrightarrow{AC} = \langle 4, 2, 6 \rangle$ , and  $\overrightarrow{AD} = \langle 5, -3, 9 \rangle$ . Determine the volume of the parallelepiped.
- A. 144      B. 2      C. 120      D. 72      E. NOTA
4. For matrix  $m = \begin{bmatrix} 9 & 2 & 11 & 2 \\ 13 & 4 & 9 & 6 \\ 8 & -7 & 4 & 0 \\ 5 & 17 & -3 & 2 \end{bmatrix}$ , find the cofactor of  $m_{34}$ .
- A. -894      B. -633      C. -1029      D. -693      E. NOTA
5. The vector  $v = \langle x + 1, x, 2 \rangle$ . Find the value of  $x$  that minimizes the absolute value of  $v$ .
- A. -1      B. 0      C.  $-\frac{1}{2}$       D. -2      E. NOTA
6. Find the equation of the plane containing the vectors  $u = \langle 1, -4, 5 \rangle$  and  $v = \langle 6, 0, 3 \rangle$  and the point  $P = (7, 10, 2)$ .
- A.  $-12x + 27y + 24z = 236$       B.  $-9x + 14y - 7z = 63$   
 C.  $19x - 13y + 6z = 15$       D.  $-12x + 13y - 7z = 32$       E. NOTA

7. Let  $A$  and  $B$  both be square matrices of the same dimensions. Then, the operation  $\odot$  is defined

such that  $A \odot B = AB + (AA)B + A(BB)$ . Determine  $\begin{bmatrix} 7 & 3 \\ 2 & 0 \end{bmatrix} \odot \begin{bmatrix} 1 & 4 \\ -3 & 1 \end{bmatrix}$ .

- A.  $\begin{bmatrix} -110 & 23 \\ -22 & 16 \end{bmatrix}$  B.  $\begin{bmatrix} -100 & 233 \\ -12 & 42 \end{bmatrix}$  C.  $\begin{bmatrix} -6 & 93 \\ 6 & -18 \end{bmatrix}$  D.  $\begin{bmatrix} -105 & 295 \\ -24 & 86 \end{bmatrix}$  E. NOTA

8. Let  $\theta$  represent the acute angle between the vectors  $u = \langle 1, 2 \rangle$  and  $v = \langle t+2, t+1 \rangle$ . Find  $\cos\theta$  at  $t = 2$ .

- A.  $\frac{2}{5}$  B.  $\frac{2\sqrt{5}}{5}$  C.  $\frac{2}{25}$  D.  $\frac{2}{125}$  E. NOTA

9. Let  $m = \begin{bmatrix} x^2 & 0 & x^2 + 8x - 6 \\ 0 & x^2 + 2x + 1 & 0 \\ 1 & 0 & x \end{bmatrix}$ . Find  $\lim_{x \rightarrow 3} \frac{|m|}{x-3}$

- A. 208 B. 117 C. 99 D. 63 E. NOTA

10. Let  $f(x) = x^3 + 3x^2 + 3x - 7$ . Each point on the graph of  $y = f(x)$  is transformed by

$\begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$ , resulting in  $y_1 = f^T(x_1)$  ( $\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$ ). Find the value of  $f^T(3)$ .

- A. 1 B. 2 C. 3 D. 4 E. NOTA

11. For what real value of  $t$  are the vectors  $\langle 7, 9, 3 \rangle$  and  $\langle 6, -4, t^2 + 2\sqrt{2}t \rangle$  orthogonal?

- A.  $-4\sqrt{2}$  B.  $-2$  C.  $-2\sqrt{2}$  D.  $-\sqrt{2}$  E. NOTA

12. Let  $m = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ . Find  $(m^{23})_{14}$ .

- A. 14 B. 23 C. 28 D. 46 E. NOTA

13. Which of the following is an eigenvalue for the matrix  $\begin{bmatrix} 1 & -8 \\ -11 & 4 \end{bmatrix}$ ?
- A. 14      B. 5      C. -12      D. -42      E. NOTA
14. Let  $A$  represent the projection of  $\langle 3, -2, 5 \rangle$  onto  $\langle 7, 0, 1 \rangle$ , and let  $B$  represent the projection of  $\langle 1, 9, 4 \rangle$  onto  $\langle -6, -2, 7 \rangle$ . Find  $25A \cdot 89B$
- A. -2584      B. 100      C. -1820      D. -1456      E. NOTA
15. If  $m$  is a  $5 \times 5$  matrix such that  $|m| = 7$ . Find  $|3m|$ .
- A. 21      B. 234375      C. 50421      D. 1701      E. NOTA
16. For matrices  $A$  and  $B$ ,  $|A| = 4$ ,  $\text{adj}(A) = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \\ 0 & 1 & 10 \end{bmatrix}$ , and  $B = \begin{bmatrix} -1 & -2 & -3 \\ -4 & -5 & -7 \\ -10 & -6 & -9 \end{bmatrix}$ . Find  $(A^{-1}B)_{23} * (A^{-1}B)_{31}$ .
- A.  $\frac{4095}{16}$       B.  $\frac{1763}{8}$       C. 780      D.  $\frac{2665}{8}$       E. NOTA
17. Mike buys 17 Harambe dolls and 23 Jimi Hendrix CDs for \$206. John buys 19 Harambe dolls and 13 Jimi Hendrix CDs for \$154. Find the price of 21 Harambe dolls and 43 Jimi Hendrix CDs.
- A. \$302      B. \$342      C. \$338      D. \$320      E. NOTA

18. A  $1 \times 2$  matrix is multiplied by a  $2 \times 3$  matrix. The resultant matrix is multiplied by a  $3 \times 7$  matrix, and that resultant matrix is multiplied on the left by a  $5 \times 1$  matrix. That matrix is then multiplied by a  $7 \times 11$  matrix. If  $n$  is the number of rows of this final matrix and  $m$  is the number of columns of the final matrix, find  $m^n$ .

- A. 1331      B. 161051      C. 16807      D. 78125      E. NOTA

19. A vector  $\langle v^2, v, 2 \rangle$  has a magnitude of  $2\sqrt{6}$ . Find the product of the two possible real values of  $v$ .

- A. -4      B. -6      C. -5      D. 20      E. NOTA

20. The displacement vector of a particle at time  $t$  is  $\langle 5t^4 + 6, 15t^3 - 13t^2, 4t^5 \rangle$ . Which of the following vectors represents the jerk of the particle at time  $t = 2$ ?

- A.  $\langle 240, 90, 960 \rangle$     B.  $\langle 120, 90, 480 \rangle$     C.  $\langle 240, 30, 160 \rangle$     D.  $\langle 120, 60, 480 \rangle$     E. NOTA

Questions 21-25 pertain to the matrix  $m = \begin{bmatrix} 1 & 0 & 2 \\ 3 & 0 & -4 \\ -7 & -2 & 9 \end{bmatrix}$ .

21. Find the product of the determinant and the trace of the matrix.

- A. -60      B. -200      C. -36      D. -40      E. NOTA

22. Find the product of the roots to the characteristic polynomial of the matrix.

- A. 4      B. 20      C. 40      D. 16      E. NOTA

23. Find  $(m^3)_{33}$ .

- A. 595      B. -165      C. 729      D. 451      E. NOTA

24. Find  $\sum_{j=1}^3 \sum_{i=1}^3 m_{ij}$ .

- A. 1      B. -7      C. -17      D. 2      E. NOTA

25. The null space of  $3 \times 3$  matrix  $m$  is the set of all vectors  $v$  such that  $mv = 0_{3 \times 1}$ . Which of the following vectors is in the null space of  $m$ ?

- A.  $\langle 2, 4, -1 \rangle$     B.  $\langle -1, 3, -2 \rangle$     C.  $\langle 2, 3, -4 \rangle$     D.  $\langle 1, -1, 0 \rangle$     E. NOTA

26. An African swallow flies on a path given by  $\langle 1, -4, 5 \rangle + \langle 1, 0, 2 \rangle t$ . A European swallow flies on a path given by  $\langle 4, -6, 4 \rangle + \langle 3, -2, -1 \rangle s$ . Find the point at which their paths intersect.

- A.  $\langle 4, -6, 4 \rangle$     B.  $\langle 1, -4, 5 \rangle$     C.  $\langle 7, -8, 3 \rangle$     D.  $\langle 2, -4, 7 \rangle$     E. NOTA

27.  $\begin{vmatrix} 83 & 76 \\ 62 & 57 \end{vmatrix}^{-1} \begin{vmatrix} 97 & 67 \\ 42 & 29 \end{vmatrix} = ?$

- A. 1      B. 6859      C. 361      D. 19      E. NOTA

Questions 28-30 pertain to the vector  $v = \langle 64\sqrt{6} + 64\sqrt{2}, 64\sqrt{6} - 64\sqrt{2} \rangle$

28. Write the vector in the form  $\langle \text{magnitude}, \text{angle} \rangle$ .

- A.  $\langle 256, \frac{\pi}{12} \rangle$     B.  $\langle 256, -\frac{\pi}{12} \rangle$     C.  $\langle 64, \frac{\pi}{12} \rangle$     D.  $\langle 64, \frac{5\pi}{12} \rangle$     E. NOTA

29. Find the product of the  $i$  and  $j$  components of  $v$ .

- A. 32768    B. 8192    C. 4096    D. 16384    E. NOTA

30. Woo! You made it this far, so all you have to do is determine which of the following vectors is equivalent to  $v$ . The answer choices are written in the form  $\langle magnitude, angle \rangle$ .

- A.  $\langle -256, \frac{\pi}{12} \rangle$     B.  $\langle 256, \frac{977\pi}{12} \rangle$     C.  $\langle 256, \frac{625\pi}{12} \rangle$     D.  $\langle -256, \frac{547\pi}{12} \rangle$     E. NOTA