

For each question, "E) NOTA" indicates that none of the above answers is correct.

1. The first term of an arithmetic sequence is 2017. The 2017<sup>th</sup> term is 1. What is the 200<sup>th</sup> term of this sequence?

- A) 200                  B) 1817                  C) 1818                  D) 2116                  E) NOTA

2. How many terms are in the arithmetic sequence 40, 33, 26, ..., -3964 ?

- A) 573                  B) 574                  C) 575                  D) 576                  E) NOTA

3. What is the sum of the multiples of 11 between 200 and 2000?

- A) 354200              B) 177100              C) 356400              D) 178200              E) NOTA

4. A digital sum of an integer is given by the iteration of the sum of all its digits until a single-digit number is reached. Let  $F_n$  represent the  $n$ th Fibonacci number for  $n = 1, 2, 3, \dots$ . What is the digital sum of  $F_{20}$ ? For our purposes,  $F_1 = F_2 = 1$ .

- A) 1                      B) 6                      C) 5                      D) 2                      E) NOTA

5. Once simplified, what is the number of terms in the expansion of  $(2i + 3j + 4k + l)^{10}$  ?

- A) 286                  B) 45                      C) 40                      D) 78                      E) NOTA

6. What is the geometric mean of 98 and 2?

- A) 10                      B) 50                      C) 14                      D) 49                      E) NOTA

7. Evaluate  $\sum_{n=1}^{\infty} \left(\frac{3}{4}\right)^n$ .

- A) 0                      B) 3                      C) 1                      D) 2                      E) NOTA

8. Evaluate  $\sum_{n=1}^{\infty} \sin^{2n} x$  when  $x \neq \left(\frac{k\pi}{2}\right)$  for any integer  $k$ .

- A)  $\cos^2 x - 1$       B)  $\tan^2 x$               C)  $\sin^2 x - 1$       D)  $\cos^2 x$               E) NOTA

9. Let  $r(x)$  be a cubic polynomial with  $r(-1) = 2$ ,  $r(1) = 16$ ,  $r(3) = 131$ , and  $r(5) = 425$ .

What is the value of  $r(-3)$ ?

- A) 11                      B) 17                      C) 88                      D) 89                      E) NOTA

10. Let  $p_1 = 3$ , and let  $p_{k+1} = 4p_k + 5$  for all integers  $k \geq 1$ . What is the base-8 representation of  $p_6$ ?

- A) 1022221              B) 11251                      C) 11244                      D) 10252                      E) NOTA

11. What is the sum of the infinite series  $\frac{2}{6} + \frac{3}{36} + \frac{4}{216} + \dots$ , where numerators increase by 1 and denominators increase by a factor of 6 from term to term?

- A)  $\frac{3}{5}$                       B)  $\frac{2}{5}$                       C)  $\frac{5}{12}$                       D)  $\frac{11}{25}$                       E) NOTA

12. Erica and Tommy are playing a game in which each of them blindly picks a digit in the infinite decimal expansion of  $\frac{3}{7}$  (excluding the leading 0 before the decimal point). Erica wins the game if the product of their digits is even, while Tommy wins if the product of their digits is odd. What is the probability that Tommy wins?

- A)  $\frac{1}{4}$                       B)  $\frac{1}{2}$                       C)  $\frac{3}{4}$                       D)  $\frac{2}{3}$                       E) NOTA

13. The sides of a right triangle form a geometric progression. If  $\theta$  is the smallest angle in the triangle, what is  $\cot(\theta)$ ?

- A)  $\frac{\sqrt{5}-1}{2}$                       B)  $\frac{\sqrt{5}+1}{4}$                       C)  $\sqrt{\frac{\sqrt{5}+1}{2}}$                       D)  $\sqrt{\frac{1-\sqrt{5}}{2}}$                       E) NOTA

14. Evaluate  $\sum_{n=2}^{\infty} \frac{6}{n^2-1}$ .

- A) 3                      B)  $\frac{9}{2}$                       C) 4                      D)  $\frac{7}{2}$                       E) NOTA

15. Evaluate:  $\sqrt{30 + \sqrt{30 + \sqrt{30 + \dots}}}$

- A) 3                      B) 7                      C) -5                      D) 6                      E) NOTA

16. What is the sum of the 67 distinct least positive odd integers?

- A) 4489      B) 4556      C) 2278      D) 1089      E) NOTA

17. A super ball dropped from a height of 30 yards rebounded on each bounce  $\frac{3}{8}$  of the height from which it fell. How far (in yards) did the ball travel before coming to rest?

- A) 60      B) 66      C) 80      D) 90      E) NOTA

18. What is the matrix equivalent of  $\begin{bmatrix} \frac{1}{3} & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{5} \end{bmatrix} + \begin{bmatrix} \frac{1}{6} & \frac{1}{8} \\ \frac{1}{12} & \frac{1}{10} \end{bmatrix} + \begin{bmatrix} \frac{1}{12} & \frac{1}{32} \\ \frac{1}{36} & \frac{1}{20} \end{bmatrix} + \dots + \begin{bmatrix} \frac{1}{3 \cdot 2^{n-1}} & \frac{1}{2 \cdot 4^{n-1}} \\ \frac{1}{4 \cdot 3^{n-1}} & \frac{1}{5 \cdot 2^{n-1}} \end{bmatrix} + \dots?$

- A)  $\begin{bmatrix} \frac{2}{3} & \frac{2}{3} \\ \frac{3}{8} & \frac{2}{5} \end{bmatrix}$       B)  $\begin{bmatrix} \frac{1}{2} & \frac{3}{4} \\ \frac{3}{4} & \frac{2}{5} \end{bmatrix}$       C)  $\begin{bmatrix} \frac{1}{2} & \frac{2}{3} \\ \frac{3}{4} & \frac{3}{5} \end{bmatrix}$       D)  $\begin{bmatrix} \frac{1}{2} & \frac{3}{4} \\ \frac{5}{8} & \frac{3}{5} \end{bmatrix}$       E) NOTA

19. Find  $y > 6$ , given that  $\sum_{j=y}^{17} (j - 6)^2 = 506$ .

- A) 7      B) 8      C) 9      D) 10      E) NOTA

20. A cube is inscribed in a sphere of radius 3. A second sphere is inscribed inside the cube and a second cube is then inscribed in this second sphere. Given that this pattern continues indefinitely, what is the surface area of the 10<sup>th</sup> sphere?

- A)  $\frac{4\pi}{81}$       B)  $\frac{4\pi}{243}$       C)  $\frac{4\pi}{729}$       D)  $\frac{4\pi}{2187}$       E) NOTA

21. Timmy created an infinite amount of circles all with a special property. Starting with the first circle of radius 1, each successive circle that Timmy created has a radius numerically equal to the area enclosed by the previous circle. What is the enclosed area of the 11<sup>th</sup> circle that Timmy created?

- A)  $\pi^{2047}$       B)  $\pi^{1023}$       C)  $\pi^{4095}$       D)  $\pi^{8191}$       E) NOTA

22. What is the sum of the 999 distinct least positive integers?

- A) 500000      B) 499500      C) 999000      D) 1000000      E) NOTA

23. Which of the following ensures that  $\sum_{n=2}^{\infty} xy^n$  converges?

- I.  $|y| < 1$       II.  $|x| < 1$       III.  $x = 0$

- A) II only      B) I only      C) I or III      D) III only      E) NOTA

24. To settle a bet and be claimed champion, Alvin, Dan, and Nicholas roll, in alphabetical order, a fair six-sided die until one of them rolls a 6. What is the probability that Nicholas will be the champion?

- A)  $\frac{30}{91}$       B)  $\frac{25}{91}$       C)  $\frac{25}{216}$       D)  $\frac{1}{3}$       E) NOTA

25. The formula  $l(n) = \frac{n}{2}(3n - 1)$  is the generating function for which set of numbers?

- A) Catalan      B) triangular      C) Fibonacci      D) pentagonal      E) NOTA

26. What is the constant term in the expansion of  $\left(z^2 - \frac{4}{z}\right)^6$ ?

- A) 256      B) 3840      C) 1536      D) 384      E) NOTA

27. What are the last two digits in the sum when the expression  $\sum_{n=0}^{100} n!$  is evaluated?

- A) 13      B) 00      C) 34      D) 14      E) NOTA

28. The first three terms of a harmonic sequence are 72, 80, and 90. What is the seventh term in this sequence?

- A) 102      B) 150      C) 180      D) 360      E) NOTA

29. An arithmetic sequence and its indices have been translated into Roman numerals as follows:  $XCIX, CVI, CXIII, \dots$

What is the  $C'$ 'th term in this sequence (in Roman numerals)?

- A)  $DCCXCII$       B)  $DCCXCIX$       C)  $DCCC$       D)  $DCCCIII$       E) NOTA

30. Express the sum of the base 8 geometric series  $.6_8 - .06_8 + .006_8 - .0006_8 + \dots$  as a fraction in base 8.

- A)  $\frac{2_8}{4_8}$       B)  $\frac{6_8}{9_8}$       C)  $\frac{6_8}{11_8}$       D)  $\frac{3_8}{4_8}$       E) NOTA