For this test, E) NOTA means None Of The Above. All inverse trigonometric functions are restricted to their tradition domains. The imaginary constant i is defined to be $i=\sqrt{-1}$, and $\text{cis}(\theta)=\cos(\theta)+i\sin(\theta)$. For this test, an infinite sum of 1's with alternating signs is taken to be undefined. Good Luck!

1) Determine the number of times on the interval $[0,2017\pi)$ that $\cos(3\theta)=0$.

A) 6048   B) 6051   C) 2017   D) 12102   E) NOTA

2) Determine the amplitude of the function $y=9\cos(x)+44\sin(x)$.

A) 9   B) 44   C) $\sqrt{2017}$ D) 53   E) NOTA

3) The expression $-\sin\theta\tan\theta$ is a simplified version of which of the following expressions.

A) $\dfrac{\sin\theta-\tan\theta}{\csc\theta-\cot\theta}$ B) $\dfrac{\cos\theta-\sin\theta}{\cos(\sin(\theta))}$

C) $\dfrac{\sin\theta+\cos\theta}{\tan\theta}$ D) $\dfrac{\tan(\theta-\pi)}{\sin\theta-\cos\theta}$ E) NOTA

For questions 4-6 use the following information: Victoria is standing a few feet away from a lamppost. For this problem, she is 6' tall and looks up at the top of the lamppost at an angle of α with the horizontal. She determines that $\tan\alpha=\dfrac{7}{9}$ and then walks toward the lamppost a few steps. When she stops, she is 7 feet away from the post and determines while looking up at the top of the post at angle β with the horizontal that $\tan\beta=\dfrac{9}{7}$.

4) How many feet did Victoria walk forward?

A) 7   B) 9   C) 9/7   D) 7/9   E) NOTA

5) How many feet taller is the lamppost than Victoria?

A) 7   B) 9   C) 9/7   D) 7/9   E) NOTA

6) If Victoria moves back to the position where she initially measured $\tan\alpha$, what is the tangent of the angle of inclination from the ground where she stands to the top of the flagpole?

A) 32/27   B) 35/27   C) 8/7   D) 5/4   E) NOTA

7) If $\theta > 0$ and $\cos\theta = \frac{1}{3}$, find $\cos 3\theta$.

A) 1/27 B) 1/3 C) –1 D) –23/27 E) NOTA

8) Jennifer wishes to model the vertical motion of a bucket attached to the outer edge of a vertical waterwheel. The wheel has diameter 2017 feet and is centered 1000 feet above the surface of the river it draws power from. If at time $t = 0$ seconds the bucket is at the apex of its path and the wheel makes 1 full counterclockwise rotation in 10 minutes, which of the following equations describes the vertical motion of the bucket, where *t* and *y* are in seconds and feet relative to the surface of the water, respectively?

A) $y = \frac{2017}{2}\cos\left(\frac{\pi}{300}t\right) + 1000$ B) $y = \frac{2017}{2}\sin\left(\frac{\pi}{600}t\right) - 1000$

C) $y = \frac{2017}{2}\cos\left(\frac{\pi}{150}t\right) - 1000$ D) $y = \frac{2017}{2}\sin\left(\frac{\pi}{300}t\right) + 1000$ E) NOTA

9) List all values of *x* is the function $f(x) = \sum_{k=0}^{\infty} \sin^k(x)$ undefined?

A) $\frac{\pi}{2}$ B) $\frac{3\pi}{2}$ C) $\frac{\pi}{2} + n\pi,\ n \in \mathbb{Z}$ D) $\frac{\pi + n}{2},\ n \in \mathbb{Z}$ E) NOTA

10) Simplify $\frac{\csc t - \sec t}{\cos^2 t - \sin^2 t}$.

A) $\frac{4(\cos t - \sin t)}{\sin 4t}$ B) $\frac{\cos t - \sin t}{\sin t}$ C) $\frac{4(\sin t - \cos t)}{\sin 4t}$ D) $\frac{\cos 2t}{\sin 4t}$ E) NOTA

11) Given: $0 < x < \pi$ and $\frac{\pi}{2} < y < \frac{3\pi}{2}$. If $\cos x = -\frac{5}{13}$ and $\sin y = -\frac{8}{17}$, find $\tan(x - y)$.

A) 220/21 B) –220/21 C) 20/3 D) –20/3 E) NOTA

12) What is the straight line distance between the points $\left(2\sqrt{2}, \frac{\pi}{4}, \frac{\pi}{4}\right)$ and $\left(\sqrt{3}, \frac{7\pi}{4}, \pi - \tan^{-1}\left(\sqrt{2}\right)\right)$? Coordinates are given in spherical coordinates.

A) 3 B) $\sqrt{11}$ C) $\sqrt{13}$ D) 4 E) NOTA

13) $\left[(2\operatorname{cis}3^\circ)(5\operatorname{cis}11^\circ)\right]^3 =$

A) $10\operatorname{cis}(14^\circ)$ B) $1000\operatorname{cis}(2744^\circ)$ C) $1000\operatorname{cis}(42^\circ)$ D) $\sqrt[3]{10}\operatorname{cis}(\sqrt[3]{14}^\circ)$ E) NOTA

14) Evaluate $\cos\left(2\cot^{-1}\left(\frac{15}{11}\right)\right)$.

A) $\frac{15}{\sqrt{346}}$ B) $\frac{225}{346}$ C) $\frac{11}{15}$ D) $\frac{52}{173}$ E) NOTA

15) Evaluate $\cos\left(\sin^{-1}\left(-\frac{3}{4}\right)\right)$

A) $\frac{4}{5}$ B) $\frac{\sqrt{7}}{4}$ C) $-\frac{4}{5}$ D) $-\frac{\sqrt{7}}{4}$ E) NOTA

16) Express the following as a complex number in cis(x) form:

$$e^{i\frac{\pi}{2}}e^{i\frac{\pi}{3}}e^{i\frac{\pi}{4}}e^{i\frac{\pi}{5}}e^{i\frac{\pi}{6}}$$

A) $\operatorname{cis}\left(\frac{\pi}{20}\right)$ B) $\operatorname{cis}\left(\frac{\pi^5}{720}\right)$ C) $\operatorname{cis}\left(\frac{60\pi}{87}\right)$ D) $\operatorname{cis}\left(\frac{87\pi}{60}\right)$ E) NOTA

17) Compute the tangent of the smaller angle between the planes $x - y + 2z = 7$ and $-4x + 7y - z = 12$.

A) $\frac{-\sqrt{227}}{13}$ B) $\frac{\sqrt{227}}{13}$ C) $\frac{-13\sqrt{11}}{6}$ D) $\frac{13\sqrt{11}}{6}$ E) NOTA

18) Let us say that for a given angle $0 < \theta < \pi/2$, call *A* the point where the terminal side of θ in standard position intersects the unit circle centered at the Origin *O*. The line segment that is tangent to the unit circle at A meets the *x*-axis at *E*. Which trigonometric function can be geometrically represented as the length of segment *OE*?

A) $\tan\theta$ B) $\sin\theta$ C) $\csc\theta$ D) $\sec\theta$ E) NOTA

19) A lesser known trig function is called the chord of theta, shortened to $\text{crd}(\theta)$. It is the length of the chord from the point A (as defined in problem 18) to either $(1,0)$ or $(-1,0)$, whichever is closer. It is always positive. Which of the following is $\text{crd}(\theta)$ equal to?

A) $r\theta$ B) $\frac{\sin\theta}{2}$ C) $2\cos\frac{\theta}{2}$ D) $2\sin\frac{\theta}{2}$ E) NOTA

For Questions 20 and 21: *ABCDEFGH* is given to be a regular octagon with side length 2.

20) Find the semi-perimeter of triangle *ADG*.

A) $2+2\sqrt{2}$ B) $2\sqrt{2+\sqrt{2}}$

C) $2+2\sqrt{2}+\sqrt{2+\sqrt{2}}$ D) $\sqrt{2+\sqrt{2+\sqrt{2}}}$ E) NOTA

21) Find the measure of angle *ADG*.

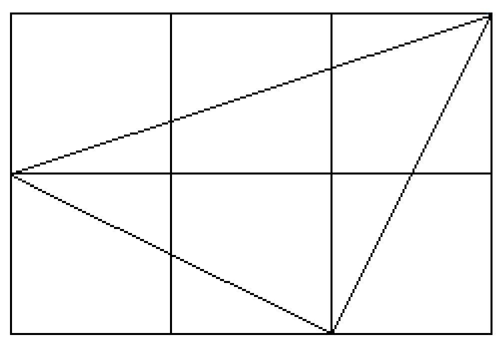
A) $\frac{2\pi}{3}$ B) $\frac{\pi}{2}$ C) $\frac{\pi}{3}$ D) $\frac{\pi}{6}$ E) NOTA

22) Find all asymptotes of the function $f(x)=\cos\left(\csc^{-1}\left(\tan\left(\sec^{-1}(x)\right)\right)\right)$.

A) $x=\pm1,\ y=1$ B) $x=\pm1,\ y=-1$

C) $x=1,\ y=\pm1$ D) $x=-1,\ y=\pm1$ E) NOTA

23) Compute $\tan^{-1}(1)+\tan^{-1}(2)+\tan^{-1}(3)$. The diagram to the right may be of some use to you.



A) $\frac{\pi}{2}$ B) π C) $\frac{\pi}{4}$ D) $\frac{\pi}{6}$ E) NOTA

24) Brandon and Andrew are insects that are flying along the paths $y=\sin x,\ x=\pi t$ and $y=\cos x,\ x=\pi t-\pi$, respectively, starting at *t*=0. Will they ever collide, and if so what is the first positive *t* value at which they collide?

A) No B) Yes, *t*=1/4 C) Yes, *t*=1/2 D) Yes, *t*=3/4 E) NOTA

25) What is the area of the triangle which has legs of length 5 and 7 which are separated by an angle of 15 degrees?

A) $\frac{35\sqrt{2}}{8}\left(\sqrt{3}-1\right)$ B) $\frac{35\sqrt{2}}{4}\left(\sqrt{3}-1\right)$

C) $\frac{35\sqrt{3}}{8}\left(\sqrt{2}-1\right)$ D) $\frac{35\sqrt{3}}{4}\left(\sqrt{2}-1\right)$ E) NOTA

26) Given that $\sec x = 4$, compute $\sqrt{\tan^2 2x - \sin^2 2x}$.

A) $\frac{\sqrt{15}}{7}$ B) $\frac{\sqrt{7}}{15}$ C) $\frac{\sqrt{15}}{56}$ D) $\frac{15}{56}$ E) NOTA

27) Find the smallest n such that $x^n - 1 = 0$ has at least 5 solutions in the first quadrant (but not on an axis) of the Argand plane.

A) n = 20 B) n =21 C) n = 25 D) n = 24 E) NOTA

28) What shape is the graph of $\begin{matrix} x = 2\tan t \\ y = 3\sec t \end{matrix}$?

A) Circle B) Parabola C) Hyperbola D) Ellipse E) NOTA

29) Evaluate $\lim_{x\to 0}\frac{\sin(2x)}{\sin(x)}$

A) Does Not Exist B) 0 C) 1 D) 2 E) NOTA

30) Given that $f(x) = 3\sin(2x) + 4$, find the maximum value of $4f(x+1) - 3$

A) 3 B) 4 C) 14 D) 21 E) NOTA