MAΘ Nationals Buffalo
Alpha Trigonometry Solutions

1) B
2) C
3) A
4) E
5) B
6) B
7) D
8) A
9) C
10) A
11) A
12) E
13) C
14) D
15) B
16) D
17) B
18) D
19) D
20) C
21) E
22) A
23) B
24) A
25) A
26) D
27) B
28) C
29) D
30) E

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1) The base cosine function has two zeroes in the span of 2π so in the span of 2π, $\cos 3\theta$ will have 6 zeroes. That means there are $\frac{2017\pi}{2\pi}\times 6 = 6051$

2) Amplitude will be $A^2 = 9^2 + 44^2 \Rightarrow A = \sqrt{9^2+44^2} = \sqrt{2017}$

3) $\frac{\sin\theta - \tan\theta}{\csc\theta - \cot\theta} = \frac{\sin\theta\left(1-\frac{1}{\cos\theta}\right)}{\frac{1}{\sin\theta}(1-\cos\theta)} = \sin^2\theta\frac{\frac{\cos\theta-1}{\cos\theta}}{1-\cos\theta} = -\sin^2\theta\frac{1}{\cos\theta} = -\sin\theta\tan\theta$

4) Let A be the point at which Victoria started, B be the point 7 feet away from the post and the distant from A to B be x. We know the height above Victoria's head is $7\tan\beta = 9$, and we know that $(x+7)\tan\alpha = h = 9 \Rightarrow x = \frac{32}{7}$

5) $\frac{h}{7} = \tan\beta = \frac{9}{7}$

6) The flagpole is 15 feet, so $\tan\theta = \frac{15}{81/7} = \frac{35}{27}$.

7) Using angle addition twice (3x=x+2x, then again on the 2x) one gets to $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$. Plugging in 1/3, we get $4\cdot\frac{1}{27} - 3\frac{1}{3} = \frac{-23}{27}$

8) The amplitude is 2017/2, the modifier for t will be $\frac{2\pi}{600} = \frac{\pi}{300}$, and the vertical shift is +1000 (the waterwheel will extend a little into the water). If when t=0 the bucket is at the top, we can use cosine for the sinusoidal here as cos0=1. Putting this all together we have $y = \frac{2017}{2}\cos\left(\frac{\pi}{300}t\right) + 1000$

9) This is a geometric series and is only defined when $|\sin x| < 1$. Values where |sinx|=1 make it undefined. sinx=1 when $x = \frac{\pi}{2} + k\pi$ for integer *k.*

10)

$$\frac{\csc t - \sec t}{\cos^2 t - \sin^2 t} = \frac{\frac{1}{\sin t} - \frac{1}{\cos t}}{\cos 2t} = \frac{\cos t - \sin t}{\cos t \sin t \cos 2t} = \frac{\cos t - \sin t}{\sin 2t/_2 \cos 2t} = \frac{\cos t - \sin t}{\sin 4t/_4} = 4\frac{\cos t - \sin t}{\sin 4t}$$

11) $\tan(x-y)=\dfrac{\tan x-\tan y}{1+\tan x\tan y}$. We know that cosx=-5/13 which means tanx=-12/5 and since siny=-8/17, tany must be 8/15. Plugging in we get $\dfrac{\frac{-12}{5}-\frac{8}{15}}{1+\left(\frac{-12}{5}\right)\left(\frac{8}{15}\right)}=\dfrac{220}{21}$

12) The conversion from spherical is $\begin{array}{l} x=r\cos\theta\sin\varphi \\ y=r\sin\theta\sin\varphi \\ z=r\cos\varphi \end{array}$. Plugging in to the given points become $\left(\sqrt{2},\sqrt{2},2\right)$ and $(1,-1,-1)$. Their distance is

$$\sqrt{\left(\sqrt{2}-1\right)^2+\left(\sqrt{2}+1\right)^2+(2+1)^2}=\sqrt{15}$$

13) When multiplying cis functions, you multiply the moduli and add the arguments. This gives $\left[10\text{cis}\left(14^\circ\right)\right]^3=1000\text{cis}(42^\circ)$

14) Let $t=\cot^{-1}\left(\dfrac{15}{11}\right)$. This means that $\cos(t)=\cos\left(\cot^{-1}\left(\dfrac{15}{11}\right)\right)=\dfrac{15}{\sqrt{11^2+15^2}}=\dfrac{15}{\sqrt{346}}$.

Then $\cos 2t=2\cos^2 t-1=2\left(\dfrac{15}{\sqrt{346}}\right)^2-1=2\bullet\dfrac{225}{346}-1=\dfrac{52}{173}$

15) $\sin^{-1}\left(-\dfrac{3}{4}\right)$ is defined to be in Quadrant IV, so the cosine of this angle is positive.

Further, the leg along the x-axis would have length $\sqrt{7}$, so the cosine is $\dfrac{\sqrt{7}}{4}$.

16) $e^{i\frac{\pi}{2}}e^{i\frac{\pi}{3}}e^{i\frac{\pi}{4}}e^{i\frac{\pi}{5}}e^{i\frac{\pi}{6}}=e^{i\frac{87\pi}{60}}=\text{cis}\left(\dfrac{87\pi}{60}\right)$

17) The normal vector to the plane $ax+by+cz=d$ is $\langle a,b,c\rangle$. Our two normal vectors for the problem are $u=\langle 1,-1,2\rangle$ and $v=\langle -4,7,-1\rangle$. Their cosine is $\cos\theta=\dfrac{u\bullet v}{|u||v|}=\dfrac{1(-4)+(-1)7+1(-2)}{\sqrt{66}\sqrt{6}}=\dfrac{-13}{6\sqrt{11}}$. Since the angle is acute, this means that the tangent must be $\left|\dfrac{-\sqrt{227}}{13}\right|=\dfrac{\sqrt{227}}{13}$

18) Similar triangles gives that $\frac{\sin\theta}{\cos\theta}=\frac{AE}{1}\Rightarrow AE=\tan\theta$. Solving for OE via Pythagorean Theorem, we get $OE^2=1^2+AE^2=1+\tan^2\theta\Rightarrow OE=\sec\theta$

19) Using law of cosines with both sides equal to 1 (unit circle), $(\text{crd}\,\theta)^2=1^2+1^2-2(1)(1)\cos\theta$. We now have $\text{crd}\,\theta=\sqrt{2-2\cos\theta}=2\sin\frac{\theta}{2}$

20) Length AD=DG. Inspecting your drawing of this Octagon, length AD is a side length plus two of the "legs" created by the diagonal side lengths if you consider them a hypotenuse. This is equal to $2+2\sqrt{2}$. Finding AG is easy with law of cosines $AG^2=2^2+2^2-2(2)(2)\cos 135^\circ\Rightarrow AG=2\sqrt{2+\sqrt{2}}$. Summing and dividing by 2 we get $2+2\sqrt{2}+\sqrt{2+\sqrt{2}}$

21) Use Law of Cosines
$AG^2=AD^2+DG^2-2(AG)(DG)\cos(\angle ADG)\Rightarrow 8+4\sqrt{2}=(24+16\sqrt{2})(1-\cos\angle ADG)$.
This means that $\cos\angle ADG=\frac{\sqrt{2}}{2}\Rightarrow m\angle ADG=\frac{\pi}{4}$.

22) f(x) simplifies to $f(x)=\sqrt{\frac{x^2-2}{x^2-1}}$ which has asymptotes at x=1, x=-1, and y=1

23) Call the point on the left side of the middle line A. The top triangle's angle at A represents $\tan^{-1}(3)$. The bottom angle at A represents $\tan^{-1}(2)$. All that's left is that middle angle. Notice that the segment from A going down and to the right is the same length as the one going from where that segment ends to the up and right. They also form a right angle (similar triangles!) so the triangle formed with those two segments as legs has angle at A equal to $\frac{\pi}{4}$, or $\tan^{-1}(1)$. The sum of these three angles at A is 180 degrees or π .

24) Set the two equations equal $\sin\pi t=\cos(\pi t-\pi)$. The RHS= $-\cos\pi$ t so the equation becomes $\sin\pi t+\cos\pi t=0\Rightarrow\sqrt{2}\cos(\pi t+\pi/4)=0$. t=3/4 would be the first time they meet but plugging back in you will notice the arguments are never the same. This means that they may be at the same y value but never the same x value. They will never meet.

25) Area of the triangle is $\frac{1}{2}ab\sin C=\frac{5\cdot 7}{2}\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right)=\frac{35\sqrt{2}}{8}\left(\sqrt{3}-1\right)$

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26) $\sin 2x = 2\sin x \cos x = 2 \bullet \frac{1}{4} \bullet \frac{\sqrt{15}}{4} = \frac{\sqrt{15}}{8}$. Draw a triangle for angle 2x and note that $\tan 2x = \frac{-\sqrt{15}}{7}$. $\sqrt{\tan^2 2x - \sin^2 2x} = \sqrt{\frac{15}{49} - \frac{15}{64}} = \sqrt{\frac{15^2}{56^2}} = \frac{15}{56}$

27) If n=20, there are 4 solutions in each quadrant and one on each directional axis. Since they are evenly spaced, adding one will shift all of the solutions clock-wise so that there will be 5 in the first quadrant so n=21.

28) $\left(\frac{x}{2}\right)^2 + 1 = \left(\frac{y}{3}\right)^2 \Rightarrow$ hyperbola

29) $\lim_{x \to 0} \frac{\sin(2x)}{\sin(x)} = \lim_{x \to 0} \frac{2\sin(x)\cos(x)}{\sin(x)} = 2\lim_{x \to 0} \cos x = 2$

30) Max of f(x) is 7 because 3(1)+4. Max of 4f(x+1)-3 is 4*7-3=25