1. There are two areas to solve for: the triangle and the quarter circle. The area of the triangle can be found by: (0.5)bh. (0.5)(1)(1) = 0.5 (Each of the lengths for the base and height can be found on the graph.) The area of the

quarter circle is $(1/4)(pi)(r^2)=(1/4)(pi)(1^2)=pi/4$. The sum is $\frac{2+\pi}{4}$

- 2. A relative minimum can be found when the derivative goes from a negative to a positive value. That occurs at the value of x = 4.
- 3. Average rate of change is just the slope between the two coordinate pairs: (2,7) and (5,2.5). m = **-1.5** $\frac{1}{b-a}\int_{a}^{b}f(x)dx$
- 4. Average value of a function:

$$\frac{1}{2}\int_{0}^{2} f(x)dx = 0.4; \int_{0}^{2} f(x)dx$$

= 0.8; $-\int_{0}^{2} f(x)dx = -0.8 = \int_{2}^{0} f(x)dx$
= -**0.8**

5.
$$x \ln x - x \Big|_{1}^{e^{3}} = e^{3} (\ln e^{3}) - e^{3} - (1 \ln(1) - 1) = 2e^{3} + 1$$

6.
$$\frac{3x-4}{(x-3)(x+2)} = \frac{1}{x-3} + \frac{2}{x+2}$$
; $\int \frac{dx}{x-3} + 2\int \frac{dx}{x+2} = \ln|(x-3)(x+2)^2| + c$;
 $f(x) = (x-3)(x+2)^2$

- 7. A relative maximum comes when the derivative changes from an increasing (positive slope) to decreasing (negative slope). This is at x=10.
- 8. Points of inflection are where concavity changes signs, this is at **b** and g.
- 9. By direct substitution, I get 0/0 which is one of the indeterminate forms. So I use L'Hospital's: $\lim_{\theta \to 0} \frac{2\theta + 2}{2\cos(2\theta)} = \frac{2}{2} =$ 1

10.By definition:
$$\tan^{-1}(x)|_{1}^{\sqrt{3}} = \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}.$$



11.Area of Region A:
$$\int_{0}^{1} x^{\frac{1}{3}} - x dx = \frac{3}{4}x^{\frac{4}{3}} - (\frac{1}{2})x^{2}|_{0}^{1} = \frac{1}{4}$$

Area of Region B: $\int_{1}^{2} -x^{\frac{1}{3}} + x dx = \frac{3}{-4}x^{\frac{4}{3}} + (\frac{1}{2})x^{2}|_{1}^{2} = \frac{9}{4} - \frac{3}{4}(2\sqrt[3]{2})$
Sum $= \frac{5-3\sqrt[3]{2}}{2}$
12. $V = h(14 - 2h)(10 - 2h) = 4h^{3} - 48h^{2} + 140h$
 $\frac{dV}{dt} = 12h^{2} - 96h + 140$; $12h^{2} - 96h + 140 = 0 = 3h^{2} - 24h + 35$; Using
Quadratic formula: $h = \frac{24\pm 2\sqrt{39}}{6}$ By checking to see which would give the max
volume, I find that h must equal $h = \frac{12-\sqrt{39}}{3}$

$$13.y = 6(x^2 + 3)^{-1}; \frac{dy}{dx} = -6(x^2 + 3)^{-2}(2x)$$

To find the minimum slope, we must differentiate the slope and set equal to zero . Using product rule: $\frac{d^2y}{dx^2} = -12(x^2 + 3)^{-2} + (-2)(x^2 + 3)^{-3}(2x)(-12x) = 0; \frac{-12(x^2+3)+48x^2}{(x^2+3)^3} = 0; -12(x^2+3) + 48x^2 = 0; 48x^2 - 12x^2 - 36 = 0; x = \pm 1; \frac{dy}{dx}(1) = -\frac{12}{16}; \frac{dy}{dx}(-1) = \frac{12}{16}$ so the minimum is x=1.

14. To be continuous:
$$4a+b=-16+a$$
; $b = -16-3a$

To be differentiable: 2ax=-8; 2(2)a = -8; a=-2. By Substitution: b = -10.

15.
$$\int_0^{\frac{\pi}{4}} \sec^2 x \, dx + \int_0^{\frac{\pi}{4}} \sec^2 x \, tanx \, dx$$

The first integral can be found easily, but the second you must use a u – substitution. $tanx + \frac{1}{2}tan^2 x \Big|_{0}^{\frac{\pi}{4}} = \frac{3}{2}$

16. Tangent is the ratio of sine to cosine, so this expression can simplify to:

$$\int \frac{\pi}{0} \sin(5x) \, dx = -\frac{1}{5} \cos(5x) \Big|_{0}^{\frac{\pi}{20}} = \frac{2 - \sqrt{2}}{10}$$

17.By doing a u-substitution with $u = \sin(x)$, and $du = -\cos(x)dx$, we can rewrite the integral: $\int_{0}^{\frac{\pi}{2}} 2^{u} du = \frac{1}{\ln 2} 2^{sinx} \Big|_{0}^{\frac{\pi}{2}} = \frac{1}{\ln 2}$. The reciprocal is *ln2*. 18. The slope of the two lines forming the absolute value graph will be 1 or -1. At x = 4, the line is growing at a slope of **1**.

19.By using disk method: $2\pi \int_0^1 (\sqrt{1-x^2})^2 dx$; $2\pi \int_0^1 1 - x^2 dx$; $2\pi [x - \frac{1}{3}x^3] \Big|_0^1 \Big| = \frac{4}{3}\pi$

20. This integral requires a u sub and partial fractions. $u = e^x$

$$\int \frac{u}{(u+2)(u+1)} \left(\frac{du}{u}\right) = \int \frac{du}{(u+2)(u+1)} = \int \frac{-1}{u+2} du + \int \frac{1}{u+1} du = \ln \left|\frac{e^{x}+1}{e^{x}+2}\right| \Big|_{0}^{1} = \ln \left(\frac{e+1}{e+2}\right) - \ln \frac{2}{3}. A = \frac{3e+3}{2e+4}.$$
21. By Using L'Hospital's:
$$\lim_{x \to 0} \frac{\left(\frac{1}{2}\right)(25-x^{2})^{-\frac{1}{2}}(-2x)}{1} = \mathbf{0}$$
22. This is a slope field for a circle, which has eccentricity of $\mathbf{0}$.
23.
$$f(x) = 2\ln(x-1) - \ln(x+2) \to f'(x) = \frac{2}{x-1} - \frac{1}{x+2} \to f'^{(7)} = \frac{2}{6} - \frac{1}{9} = \frac{2}{3}$$

$$\frac{2}{9}$$

24. Rolle's Theorem



THIS IS HOW I ROLLE

25.

If f is differentiable at a point x_0 , then f must also be continuous at x_0 . In particular, any differentiable function must be continuous at every point in its domain. The converse does not hold: a continuous function need not be differentiable.

That makes the x-values where it is not differentiable at: 1, 2, and 4. Sum = 7