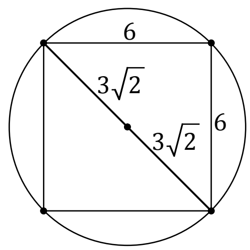
1. Since $\angle DEG$ is complementary to $\angle GEF$, $m\angle DEG + m\angle GEF = 90°$. Since $\angle DEG \cong \angle GEF$, $m\angle DEG = m\angle GEF$. Substitute the given angle measures into these equations. $(x+3y)+(2x+y)=90$ and $x+3y=2x+y$ and solve for x and y. x = 18 and y = 9.

   $x+y=27$.

2. $\dfrac{n(n+1)}{2}+1=\dfrac{5(5+1)}{2}+1=16$

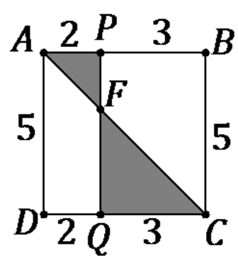
   16

3. $A=\pi r^2=\pi\left(3\sqrt{2}\right)^2=18\pi$



   18π

4. $\triangle APF \sim \triangle CQF$ $\dfrac{AP}{CQ}=\dfrac{2}{3}$. Since the ratio of the sides is $\dfrac{2}{3}$, the ratio of the areas is $\dfrac{2^2}{3^2}=\dfrac{4}{9}$.



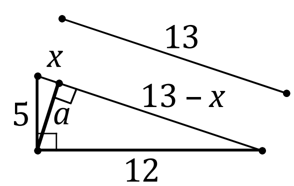
   $\dfrac{4}{9}$

5. $\dfrac{13}{5}=\dfrac{5}{x} \Rightarrow 13x=25 \Rightarrow x=\dfrac{25}{13}$

   $13-x=13-\dfrac{25}{13}=\dfrac{144}{13}$

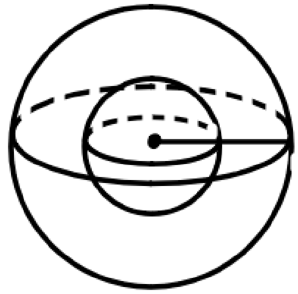
   $\dfrac{\frac{25}{13}}{a}=\dfrac{a}{\frac{144}{13}} \Rightarrow a^2=\dfrac{25\times144}{169} \Rightarrow a=\dfrac{60}{13}$

   $5+12+\dfrac{60}{13}=\dfrac{281}{13}$



6. The radius of the larger of two concentric spheres is r. The radius of the smaller of two concentric spheres is $\dfrac{r}{2}$. The probability the randomly selected point is nearer to the center

of the sphere is $\dfrac{\dfrac{4\pi\left(\frac{r}{2}\right)^3}{3}}{\dfrac{4\pi r^3}{3}} = \dfrac{1}{8}$.



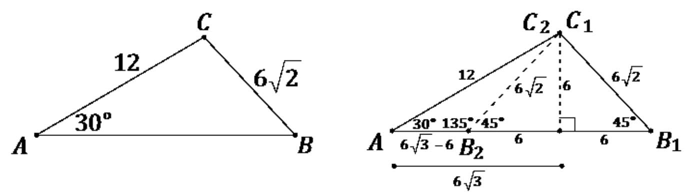
The probability that the randomly selected point is nearer to the surface of the sphere than to the center of the sphere is $1 - \dfrac{1}{8} = \dfrac{7}{8}$.

$\dfrac{7}{8}$

7. There are two solutions.

$m\angle B_1 = 45°, m\angle C_1 = 105°, AB_1 = 6 + 6\sqrt{3}$

$m\angle B_2 = 135°, m\angle C_2 = 15°, AB_2 = 6\sqrt{3} - 6$



$6\sqrt{3} - 6$

8. *x* is the radius of the cone.

$16 = \dfrac{2x \times 4x}{2}$

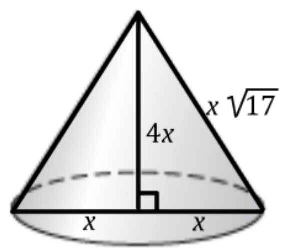
$x^2 = 4$

$x = 2$

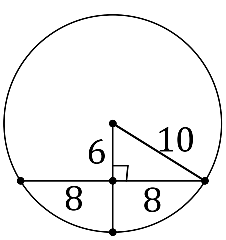
$SA = p \times 2^2 + p \times 2 \times 2\sqrt{17}$

$SA = 4p + 4\sqrt{17}p$

$n = 4 + 4\sqrt{17}$

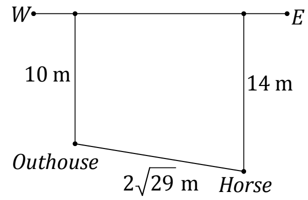


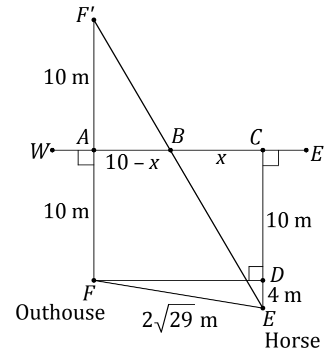
9.



6

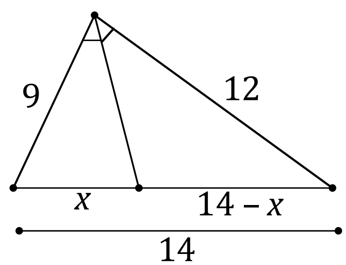
10. To minimize the distance, reflect point F (or point E) over the river. Using the Pythagorean Theorem, $FD = 10$. Since $ACDF$ is a rectangle, $AC = 10$. $\Delta ABF' \sim \Delta CBE$ $\frac{10}{14} = \frac{10 - x}{x}$ $x = \frac{35}{6}$ Using Pythagorean Theorem $F'B = \frac{65}{6}$ and $FB = \frac{65}{6}$ and $EB = \frac{91}{6}$
Cowboy Bob would walk 26 meters.





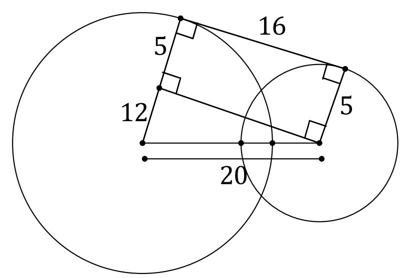
26 meters

11.
$$\frac{9}{x} = \frac{12}{14 - x}$$



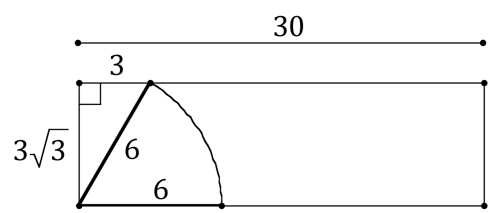
6 and 8

12. The common external tangent measures 16.



16

13. The triangle formed is a 30°-60°-90° triangle. The angle across from 3 is a 30° angle. The angle in the sector is 60°.
$$\frac{3 \cdot 3\sqrt{3}}{2} + \frac{1}{6} \cdot \pi (6)^2 = \frac{9\sqrt{3}}{2} + 6\pi$$



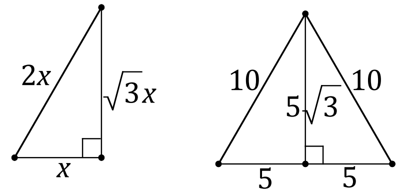
$$\frac{9\sqrt{3} + 12\pi}{2}$$

14. Put both equations in the form $Ax + By + C = 0$. $4x + 3y - 15 = 0$ and $4x + 3y - 20 = 0$ The distance between two parallel lines can be found using the formula $\frac{|C_2 - C_1|}{\sqrt{A^2 + B^2}}$. $\frac{|-15 - (-20)|}{\sqrt{4^2 + 3^2}} = 1$

1

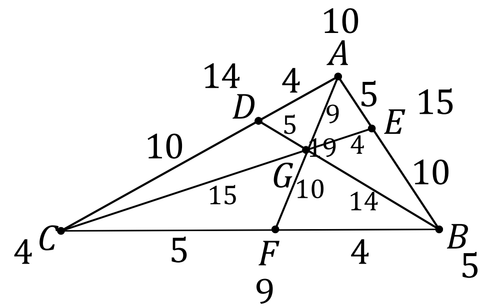
15. $x+\sqrt{3}x+2x=15+5\sqrt{3}$, so $x=5$. Each edge of the regular octahedron measures 10. The formula for the volume of a regular octahedron is

$$V=\frac{s^3\sqrt{2}}{12}=\frac{10^3\sqrt{2}}{12}=\frac{500\sqrt{2}}{3}$$



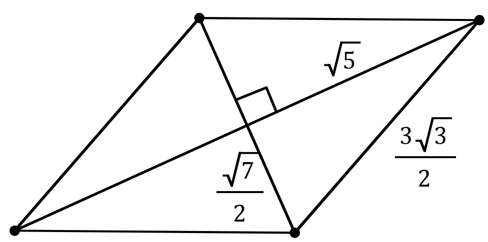
$\frac{500\sqrt{2}}{3}$

16. Using Ceva's Theorem $\frac{CF}{FB}=\frac{BE}{EA}=\frac{AD}{DC}=1$



$\frac{AG}{AF}=\frac{9}{19}$

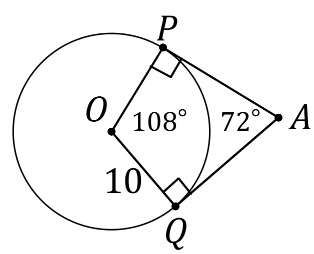
17. The diagonals of a rhombus are the perpendicular bisectors of each other. The sides of the rhombus measure $\frac{3\sqrt{3}}{2}$. The perimeter is $6\sqrt{3}$.



$6\sqrt{3}$

18. Tangents to a circle are perpendicular to the radii at the point of tangency. Radii in the same circle are congruent and tangents from the same point to the same circle are congruent. $OPAQ$ is a kite. ÐA is supplementary to ÐO. $m\overset{\frown}{PQ}=m\angle O$

Area of the sector $=\frac{m\text{ arc }\cdot\pi r^2}{360°}=\frac{108°\pi 10^2}{360°}=30\pi$



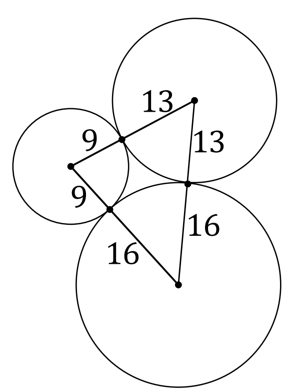
30p

19. s = semiperimeter of the triangle

$$A=\sqrt{s(s-a)(s-b)(s-c)}$$

$$A=\sqrt{38(38-22)(38-25)(38-29)}$$

$$A=12\sqrt{494}$$



$12\sqrt{494}$

20. The length of the midsegment of a trapezoid equals the average of the bases.

$$\frac{\left(x^2 - x\right)+\left(x^2 - 3\right)}{2} = 2x^2 + 2x - 3$$

Substituting $\frac{1}{2}$ for x in $\overline{RS}$ gives a negative answer.

$$x = \frac{1}{2}, -3$$

$$2(-3)^2 + 2(-3) - 3 = 9$$

9

21.

$$\frac{\text{height of smaller pyramid}}{\text{height of larger pyramid}} = \frac{5}{8}$$

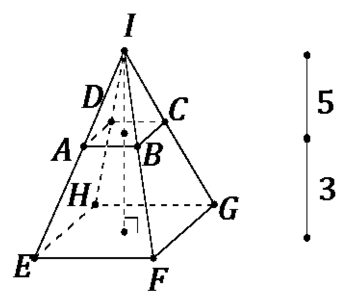
ratio of areas: $\frac{25}{64}$

$\frac{25}{64} = \frac{A}{64}$ Area of $ABCD$ = 25

Volume of frustum = Volume of larger pyramid – Volume smaller pyramid

$$\frac{64 \times 8}{3} - \frac{25 \times 5}{3} = 129$$

129



22.

$$\begin{array}{ccccc} \hat{i} & \hat{j} & \hat{k} & \hat{i} & \hat{j} \\ 1 & 2 & 5 & 1 & 2 \\ 4 & -2 & -3 & 4 & -2 \end{array}$$

$$-6\hat{i} + 20\hat{j} - 2\hat{k} - 8\hat{k} + 10\hat{i} + 3\hat{j} = 4\hat{i} + 23\hat{j} - 10\hat{k}$$

$4\hat{i} + 23\hat{j} - 10\hat{k}$

23. The circumcenter is equidistant from the vertices. The triange is a right triangle with the right angle at (3, 3). The midpoint of the hypotenuse is equidistant from the vertices. $\left(\frac{-3+6}{2}, \frac{5+12}{2}\right) = \left(\frac{3}{2}, \frac{17}{2}\right)$

$\left(\frac{3}{2}, \frac{17}{2}\right)$ or $\left(1\frac{1}{2}, 8\frac{1}{2}\right)$ or $(1.5, 8.5)$

24. $b^2 = a^2 + b^2 - 2ab\cos B$

$b^2 = 7^2 + 5^2 - 2 \times 7 \times 5 \cos 45^\circ$

$b^2 = 74 - 35\sqrt{2}$

25. Each interior angle is supplementary to each exterior angle. $180 - 144 = 36$

$$\frac{360}{n} = 36$$

$$n = 10$$

decagon