Answers:

1. **A**	7. **B**	13. **C**	19. **C**	25. **D**
2. **C**	8. **B**	14. **C**	20. **B**	26. **E**
3. **B**	9. **D**	15. **A**	21. **C**	27. **A**
4. **B**	10. **A**	16. **E**	22. **A**	28. **C**
5. **E**	11. **D**	17. **C**	23. **A**	29. **D**
6. **B**	12. **A**	18. **D**	24. **B**	30. **D**

Solutions:

1. Draw a right triangle with legs 200m and h, the height of the rocket. We have $\tan\theta = \frac{h}{200}$ where θ is the angle of elevation. Differentiate to obtain $200\sec^2\theta\frac{d\theta}{dt} = \frac{dh}{dt}$. Plug in the given values to obtain $400\frac{m}{s} = \frac{dh}{dt}$. **A**

2. $a = \frac{dv}{dt} = \frac{2}{t} \rightarrow v_f = \int_1^3 \frac{2}{t}dt + v_0 = 2\ln 3 - 2\ln 1 + v_0$. We know $v_0 = 0$, giving us $v_f = 2\ln 3$. **C**

3. $v(t) = 2\ln t$. Using trapezoidal rule to approximate the area from $t = 1$ to $t = 3$, we have $d = \frac{1}{2}(2\ln 1 + 4\ln 2 + 2\ln 3) = 2\ln 2 + \ln 3 = \ln 12$. **B**

4. Use the washer method. The radius of each disk is $x = \sqrt{2y}$, so the area of each disk is $\pi(2y)$. Integrate over all y values of interest to find volume: $\pi\int_0^2 2y dy = 4\pi$. **B**

5. The rocket hits terminal velocity when the net forces acting on it equal zero. In other words, this is when $f_{drag} = f_{gravity} = weight$. Plugging in, $\frac{1}{2}pC(2^2\pi)v^2 = 100N \rightarrow v = \frac{5\sqrt{2}}{\sqrt{\pi}}$. **E**

6. We are given the payload must be $3000kg$. Using the density, we find the volume must be $2.5m^3$. The volume of the prism is given by $2.5 = \frac{s^2\sqrt{3}}{4}h \rightarrow h = \frac{10}{s^2\sqrt{3}}$. Surface area is $\frac{s^2\sqrt{3}}{2} + 3sh = \frac{s^2\sqrt{3}}{2} + \frac{10\sqrt{3}}{s}$. Take the derivative to and set equal to zero: $0 = s - \frac{10}{s^2} \rightarrow s = 10^{\frac{1}{3}}$. The function is concave up, so this must be a minimum. Plug back in to get $SA = \sqrt{3} * 10^{\frac{2}{3}} * \left(\frac{1}{2} + 1\right) = \sqrt{3} * 10^{\frac{2}{3}} * \frac{3}{2}$. Approximate $\sqrt{3} \approx 2$, $10^{\frac{1}{3}} \approx 2$. So we have $SA \approx 12$. **B**

7. Vertical velocity component is $h'(t) = -3t^2 + 200t + 3 \rightarrow h'(1) = 200$. Horizontal component is $x'(t) = 100e^t \rightarrow x'(1) = 100e$. Magnitude is given by $\sqrt{200^2 + (100e)^2} \approx \sqrt{200^2 + 100^2(2.7)^2} = 100\sqrt{4+7} = 100\sqrt{11} \approx 330$. **B**

8. We want to find when the vertical velocity of the rocket is zero. We differentiate $h(t)$ to find the velocity. $h'(t) = -3t^2 + 200t + 3 = 0$. Using the quadratic formula, we have positive root $\frac{200+\sqrt{200^2+36}}{6} \approx \frac{200}{3} \approx 67$. **B**

9. $\frac{dv}{dt} = -\frac{u}{M}\frac{dM}{dt} \rightarrow$ integrate both sides $\rightarrow \Delta v = -u(\ln m_f - \ln m_0) = u \ln\left(\frac{m_0}{m_f}\right)$. We know $m_0 = 135k$, and $m_f = 135k - 85k = 50k$, so $\Delta v = u \ln 2.7 \approx u$. **D**

10. Draw a right triangle on the bottom half of the sphere using the radius $R = 3$, the height h, giving the radius of the cross section $x^2 = R^2 - (R-h)^2$. Integrate this radius from 0 to h to get the volume of interest (disk method): $V = \pi \int_0^h (6h - h^2)dh = \frac{1}{4}\left(\frac{4}{3}\pi 3^3\right) \rightarrow 9h^2 - h^3 = 27 \rightarrow h^3 - 9h^2 + 27 = f(h)$. Using Newton's method, we have $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2 - \frac{-1}{-24} = 47/24$. **A**

11. From the previous problem, we have $V = \pi \int_0^h (6h - h^2)dh = \pi\left(3h^2 - \frac{h^3}{3}\right)$. Differentiating, we have $dV = \pi(6h - h^2)dh$. We know that when the height is half full, $h = 3$, and $dV = r$, so $dh = \frac{r}{9\pi}$. **D**

12. The waveform is some sinusoid: $v(t) = A\sin(t)$. Calculate the RMS value of this function by finding the average value over a period: $avg = \frac{1}{2pi}\int_0^{2pi} A^2 \sin^2 t\, dt = \frac{A^2}{2}$. Take the square root of this: $v_{RMS} = \frac{A}{\sqrt{2}} = 120 \rightarrow A = 120\sqrt{2}$. **A**

13. $P = i_{bat}v_{bat} = \frac{V}{r+R}(iR) = \frac{V^2R}{(r+R)^2}$. Take the derivative of this with respect to R and set equal to zero: $\frac{(r+R)^2 - 2R(r+R)}{(r+R)^4} = 0 \rightarrow r = R$ **C**

14. The mean (or expected value) is $\int_{t=0}^{t=\infty} t\lambda e^{-\lambda t} dt = \frac{1}{\lambda} = 20 \rightarrow \lambda = \frac{1}{20}$ **C**

15. $P(T > 20) = \int_{20}^{\infty} \lambda e^{-\lambda t} dt = e^{-1}$ **A**

16. These curves are both halves of an ellipse. The top is $\frac{y^2}{9} + x^2 = 1$ and the bottom is $\frac{y^2}{4} + x^2 = 1$. We take half the area of the top and half the area of the bottom: $\frac{1}{2}(3\pi + 2\pi) = \frac{5\pi}{2}$. **E**

17. $A = \int \frac{1}{2} r^2 d\theta = \int_0^{2\pi} \frac{1}{2}(2 + 2\cos\theta)^2 d\theta = 6\pi$. **C**

18. We want to find the x intercept of the line through $(-1, 2)$ and tangent to the curve (hill) $2x - x^2 \rightarrow m = 2 - 2x$. This is the slope between our transmitter and some point on the curve, so $m = 2 - 2x = \frac{(2x-x^2)-2}{x-(-1)} \rightarrow x = -1 \pm \sqrt{5}$. Only the positive root can possibly lie on the curve in the area we care about, so $m = 2 - 2(-1+\sqrt{5}) = 4 - 2\sqrt{5}$. We now solve for the x coordinate of $(x_0, 0)$ on the line: $4 - 2\sqrt{5} = \frac{0-2}{x_0+1} \rightarrow \mathrm{x}_0 = 1 + \sqrt{5}$. **D**

19. Differentiate using quotient rule and set the derivative equal to zero: $\frac{(t+2)^2 - 2t(t+2)}{(t+2)^4}$. We are interested in the zero at $t = 2$. Plugging this into the original function, we get $1/8$. **C**

20. The station will maximize its listeners when $\frac{dL}{dt} = 0 = kL(780 - 12L) \rightarrow L = 65$ **B**

21. The solution to the differential equation is of the form $P(t) = \frac{65}{1+Ce^{kt}}$. Using $P(0) = 1$ and $P(1) = 1$, we find that $C = 64$ and $k = \ln\left(\frac{11}{128}\right)$. We now plug in $t = 2$ and get $\frac{2*65*128}{2*128+121} \approx \frac{2*65}{3} = 43.333$. Notice that since we approximated 121 as 128, our approximation is a bit lower than the true value, as we increased the denominator while approximating. **C**

22. $\int f_h^2(t)dt = \int f_e^2(t) - 2jf_e(t)f_o(t) - f_o^2(t)dt = \int f_e^2(t) - f_o^2(t)dt$ since the middle term is an odd function integrated from some –a to a, which is always 0. **A**

23. One statement is true. **A**
   I. False. For example, consider aperiodic signals $f_1(t) = t + \sin t$, $f_2(t) = -x + \sin t$. Their sum $f_1 + f_2 = 2\sin t$ is periodic
   II. False. Consider two signals f_1, f_2 with periods 1 and $\sqrt{2}$ respectively. These two signals will never “match up,” as $n(1) \neq m(\sqrt{2})$ for any integer n, m. Thus, their sum is not periodic
   III. True. This signal has period 1
   IV. False. The period of the cosine is 1, while the period of the sine is irrational. Thus, these signals never “match up,” similar to II.

24. $\lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |\sin(t)|^2 dt = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} \sin^2(t)\, dt = \frac{1}{2\pi}\int_0^{2\pi} \sin^2(t)\, dt = \frac{1}{2}$ since the average value of the signal is the same as the average value over its period. $0 < \frac{1}{2} < \infty$, so this is a power signal. Intuitively, you only need to see the signal has a non-zero and non-infinite average value. **B**

25. There are an infinite number of pulses which do not get any smaller, so $E_x = \infty$. However, since the interval between the pulses increases but the pulses’ magnitude does not, the average power over any interval approaches zero $P_x = 0$. Thus, the signal is neither an energy nor a power signal. **D**

26. $\sin(t^2) \approx t^2 - \frac{t^6}{6}$ by the Taylor series for $\sin(t)$. Plugging in 2, we get $4 - \frac{64}{6} = -\frac{20}{3}$. **E**

27. $E_x = \lim_{T\to\infty}\int_{-T}^{T}|x(t)|^2 dt = \int_0^{\infty} e^{-2at}dt = \left[-\frac{1}{2a}e^{-2at}\right]_0^{\infty} = \frac{1}{2a}$. **A**

28. $X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}dt = \int_0^{\infty} e^{-(a+j\omega)t}dt = \left[-\frac{1}{a+j\omega}e^{-(a+j\omega)t}\right]_0^{\infty} = \frac{1}{a+j\omega}$. **C**

29. $\frac{2/3}{2a} = \frac{1}{2\pi}\int_{-W}^{W}|X(\omega)|^2 d\omega = \frac{1}{\pi}\int_0^{W}\frac{1}{a^2+\omega^2}d\omega = \left[\frac{1}{a\pi}\arctan\frac{\omega}{a}\right]_0^{W} = \frac{1}{a\pi}\arctan\frac{W}{a} = \frac{1}{3a} \to \frac{\square}{\square} = \tan\frac{\pi}{3} \to$ $W = a\sqrt{3}$. **D**

30. $f'(x) = -3x^2 + 1 \to f'(1) = -2 < 0 \to$ decreasing. $f''(x) = -6x \to f''(1) = -6 < 0 \to$ concave down. **D**