

The acronym "NOTA" stands for "None Of The Above". You may find this information useful:

The standard form of a conic centered at the origin is given by: $A(x - 0)^2 + B(x - 0)(y - 0) + C(y - 0)^2 = 1$,

and if the conic is an ellipse, its enclosed area is given by $\frac{2\pi i}{\sqrt{B^2 - 4AC}}$ where $i = \sqrt{-1}$.

- Find the area bounded by the $x -$ axis, $y = x^{2017}$, $x = 1$, and $x = \sqrt[2017]{2017}$.
 A. 2016 B. 1 C. $\frac{2017}{2018}$ D. $\frac{1008}{1009}$ E. NOTA
- Find the area bounded by the $y -$ axis, $y = x^{2017}$, $y = 1$, and $y = \sqrt[2018]{2017}$.
 A. $\frac{1}{2018}$ B. $\frac{2017(\sqrt[2017]{2017})}{2018}$ C. $\frac{2017(1 - \sqrt[2017]{2017})}{2018}$ D. $\frac{2017(1 + \sqrt[2017]{2017})}{2018}$ E. NOTA
- Find the volume of the solid formed by rotating the region bounded by $y = \sin(x)$, $y = 2\sqrt{|\sin(x)|}$, $x = 0$, and $x = 2017\pi$ about the $x -$ axis.
 A. $32256\pi - 1008\pi^2$ B. $32260\pi - \frac{4033\pi^2}{4}$ C. $16132\pi - 504\pi^2$ D. $16136\pi - \frac{1009\pi^2}{2}$
 E. NOTA
- What is the area enclosed by a regular octagon with side length 2017?
 A. $\frac{2017^2(2+2\sqrt{2})}{8}$ B. $2017^2(2 + 2\sqrt{2})$ C. $2 \cdot 2017^2(2 + 2\sqrt{2})$ D. $2 \cdot 2017^2(2 + \sqrt{2})$
 E. NOTA
- Find the volume of the solid whose base is the region bounded by $y = \log_{2017}(x)$, $y = 0$, $y = 2017$, and $x = 0$, if the cross sections taken perpendicular to the y -axis are regular octagons?
 A. $\frac{(2+2\sqrt{2})(2017^{4034}+1)}{2\ln(2017)}$ B. $\frac{(2+2\sqrt{2})(2017^{4034})}{2\ln(2017)}$ C. $\frac{(2+2\sqrt{2})(2017^{4034}-1)}{2\ln(2017)}$ D. $\frac{(2+2\sqrt{2})(2017^{4034})}{16\ln(2017)}$
 E. NOTA
- Find the area enclosed by the curves $y = \frac{x}{\sqrt{4+x^2}}$, $y = \frac{x}{\sqrt{9-x^2}}$
 A. $\sqrt{26} - 5$ B. $2\sqrt{26} - 10$ C. $\sqrt{26}$ D. $2\sqrt{26}$ E. NOTA
- $\int_0^{\frac{2\pi}{3}} |\sin(4x) + \cos(2x)| dx$
 A. $\frac{21-2\sqrt{3}}{8}$ B. $\frac{19-2\sqrt{3}}{8}$ C. $\frac{21-\sqrt{3}}{8}$ D. $\frac{19-\sqrt{3}}{8}$ E. NOTA

8. Find the volume of the solid obtained by rotating the region bounded by $x = 1 + \sec(y)$, for $-\frac{\pi}{2} < y < \frac{\pi}{2}$, and $x = 3$ about the line $x = 1$.
- A. $\frac{8\pi^2}{3} - 2\pi\sqrt{3}$ B. $\frac{8\pi^2}{3} + 2\pi\sqrt{3}$ C. $\frac{4\pi^2}{3} - \pi\sqrt{3}$ D. $\frac{4\pi^2}{3} + \pi\sqrt{3}$ E. NOTA
9. Which of the following gives the volume of the solid obtained by rotating the region bounded by $x = \sqrt{2017\sin(y)}$, $0 \leq y \leq \pi$, $x = 0$ about $y = 2018$?
- A. $\int_0^\pi 2\pi y \sqrt{2017\sin(y)} dy$ B. $\int_0^\pi 2\pi y (2018 - \sqrt{2017\sin(y)}) dy$
 C. $\int_0^\pi 2\pi (2018 - y) \sqrt{2017\sin(y)} dy$ D. $\int_0^\pi 2\pi (y - 2018) \sqrt{2017\sin(y)} dy$ E. NOTA
10. Use Simpson's Rule with $n=4$ to approximate $\int_1^9 2^x dx$.
- A. 748 B. 374 C. $\frac{1634}{3}$ D. $\frac{3268}{3}$ E. NOTA
11. Find the area of the surface obtained by rotating $x = \frac{1}{3}(y^2 + 2)^{\frac{3}{2}}$ with $1 \leq y \leq 3$ about the x -axis.
- A. 192π B. 64π C. 48π D. 32π E. NOTA
12. Find the volume common to two spheres, each with radius 2017, if the center of each sphere lies on the surface of the other sphere.
- A. $2017^3 \cdot \frac{13}{12}\pi$ B. $2017^3 \cdot \frac{5}{12}\pi$ C. $2017^3 \cdot \frac{13}{24}\pi$ D. $2017^3 \cdot \frac{5}{24}\pi$ E. NOTA
13. Find the area of the region that lies inside both the polar curves: $r = 1 + 2\cos(\theta)$ and $r = 1 - 2\sin(\theta)$
- A. $-1 - 2\sqrt{2} + 3\sqrt{2} + \pi$ B. $-1 - 2\sqrt{2} + \frac{3\sqrt{3}}{2} + \frac{5\pi}{4}$ C. $-\frac{1}{2} - \frac{5\sqrt{2}}{2} + 2\sqrt{3} + \frac{5\pi}{4}$
 D. $-1 - 2\sqrt{3} + \frac{3\sqrt{2}}{2} + \frac{5\pi}{4}$ E. NOTA
14. Find the volume common to two circular cylinders, each with radius r and height $10r$, if the axes of the cylinders intersect at 45° .
- A. $\frac{4\sqrt{2}}{3}r^3$ B. $\frac{8}{3}r^3$ C. $\frac{8\sqrt{2}}{3}r^3$ D. $\frac{16}{3}r^3$ E. NOTA
15. What is the area enclosed by the polar curve given by $r = \left(\frac{1}{2}ie^{-\frac{3i\theta}{2}} - \frac{1}{2}ie^{\frac{3i\theta}{2}}\right)\left(\frac{1}{2}e^{-\frac{3i\theta}{2}} + \frac{1}{2}e^{\frac{3i\theta}{2}}\right)$? Let $i = \sqrt{-1}$ and $\theta \in \mathbb{R}$.
- A. $\frac{\pi}{4}$ B. $\frac{\pi}{8}$ C. $\frac{\pi}{24}$ D. $\frac{\pi}{48}$ E. NOTA

For questions 16-18 refer to the region bounded by $7 + 2x + 2x^2 - 11y + xy + 3y^2 \leq 0$

16. What is the centroid of this region?

- A. (1,2) B. (-1,-2) C. (1,-2) D. (-1,2) E. NOTA

17. What is the area enclosed by this region?

- A. $\frac{10\pi}{\sqrt{23}}$ B. $\frac{2\pi}{\sqrt{23}}$ C. $\frac{14\pi}{\sqrt{23}}$ D. $\frac{\pi}{\sqrt{23}}$ E. NOTA

18. What is the volume of the solid obtained by rotating the region about the exterior line $x = y$?

- A. $\frac{6\pi^2}{\sqrt{46}}$ B. $\frac{2\pi^2}{\sqrt{46}}$ C. $\frac{60\pi^2}{\sqrt{46}}$ D. $\frac{84\pi^2}{\sqrt{46}}$ E. NOTA

19. What is the area enclosed by the curve given by $x^4 + 2x^2y^2 + 2xy + y^4 = 0$?

- A. 1 B. π C. $\frac{1}{2}$ D. $\frac{\pi}{2}$ E. NOTA

20. Find the volume of the resulting solid when the region bounded by

$x = (y - 2)^2$ and $x + 4y = 5$ is rotated about $x = -1$.

- A. $\frac{112\pi}{15}$ B. $\frac{48\pi}{5}$ C. $\frac{224\pi}{15}$ D. $\frac{24\pi}{5}$ E. NOTA

21. What is the area of the region **above** the x -axis and **below** the parametric equations:

$y = 1 - t$ and $x = \sqrt{t}$?

- A. $\frac{1}{3}$ B. $\frac{2}{3}$ C. $\frac{4}{3}$ D. $\frac{8}{3}$ E. NOTA

22. Find the area below the curve $y = \frac{\ln(x^2+2x+1)}{x^2+1}$ and above the x -axis from $x = 0$ to $x = 1$.

- A. $\frac{\pi}{2}\ln(2)$ B. $\frac{\pi}{4}\ln(2)$ C. $\frac{\pi}{8}\ln(2)$ D. $\frac{\pi}{16}\ln(2)$ E. NOTA

23. Find the smallest number N such that for any two squares of combined area 1, a rectangle of area N exists such that the two squares can be packed into the rectangle (without interior overlap). (Hint: You may assume that the sides of the squares are parallel to the sides of the rectangle.)

- A. 2 B. $\frac{2+3\sqrt{2}}{4}$ C. $\frac{1+\sqrt{2}}{2}$ D. $\frac{1+\sqrt{2}}{4}$ E. NOTA

For questions 24-25 refer to the following: Let A be the area of the region in the first quadrant bounded by the line $y = \frac{1}{2}x$, the x -axis, and the ellipse $\frac{1}{9}x^2 + y^2 = 1$.

24. Find the area of the region, A .

- A. $\frac{1}{2}\tan^{-1}\left(\frac{1}{2}\right)$ B. $\frac{1}{2}\tan^{-1}\left(\frac{3}{2}\right)$ C. $\frac{3}{2}\tan^{-1}\left(\frac{3}{2}\right)$ D. $\frac{3}{2}\tan^{-1}\left(\frac{1}{2}\right)$ E. NOTA

25. Find the positive number m such that A is equal to the area of the region in the first quadrant bounded by the line $y = mx$, the y -axis, and the ellipse $\frac{1}{9}x^2 + y^2 = 1$.

- A. $\frac{2}{9}$ B. $\frac{1}{3}$ C. $\frac{4}{9}$ D. $\frac{5}{9}$ E. NOTA

26. Find the converged area "bounded" by the curves $y = \left(\frac{x^{1513}}{1+x^{2018}}\right)^2$, $y = -\left(\frac{x^{1513}}{1+x^{2018}}\right)^2$, $x = -1$. Hint: Where do the first two curves approach as x gets big?

- A. $\frac{-1+3\pi}{4036}$ B. $\frac{-1+3\pi}{8072}$ C. $\frac{-2+3\pi}{4036}$ D. $\frac{-2+3\pi}{8072}$ E. NOTA

27. Let $f(x) = \ln(e \tan(x^2))$ and $g'(x) = \sqrt{\left(\ln(\operatorname{ecot}(x^2))\right)^2 - 1}$ be defined on the interval $[a, b]$, where $0 < a < b < \frac{\pi}{2}$. The area of the region bounded by $f(x)$, $x = a$, $x = b$, and the x -axis is p square units and the arc length of $g(x)$ on the interval $[a, b]$ is p units. What is p in terms of a and b ?

- A. $2a + 2b$ B. $a + b$ C. $2a - 2b$ D. Not enough information E. NOTA

28. Suppose that S is a finite set of points in the plane such that the area of triangle ΔABC is at most 1 whenever A , B , and C are in S . There exists a triangle of area K such that A , B , and C are on the sides of this triangle, and together with its interior, the triangle covers the set S . What is the maximum value of K ?

- A. 1 B. 2 C. 3 D. 4 E. NOTA

29. Triangle T is formed by connecting the points $(1,1,1)$, $(1,2,3)$, $(-2,0,2)$. The triangle is then rotated about the line given by: $l(t) = \langle 3,1,-1 \rangle t + \langle 4,1,-2 \rangle$. What is the volume of the resulting solid? (Hint: the line is coplanar to the triangle)

- A. $48\pi\sqrt{\frac{3}{5}}$ B. $72\pi\sqrt{\frac{1}{11}}$ C. $24\pi\sqrt{3}$ D. $96\pi\sqrt{\frac{3}{5}}$ E. NOTA

30. Which of the following is closest to the centroid of the region in the first quadrant bounded by $y = \sqrt[2017]{x}$ and $y = x^{2017}$.

- A. $\left(\frac{1}{2017}, \frac{2016}{2017}\right)$ B. $\left(\frac{2016}{2017}, \frac{1}{2017}\right)$ C. $\left(\frac{1}{2017}, \frac{1}{2017}\right)$ D. $\left(\frac{1008}{2017}, \frac{1008}{2017}\right)$ E. NOTA