1. D—The derivative of $f(x)-f(2x)$ is $f'(x)-2f'(2x)$. So, $f'(1)-2f'(2)=5$, $f'(2)-2f'(4)=7$. Thus, $f'(1)-4f'(4)=(f'(1)-2f'(2))+2(f'(2)-2f'(4)=5+2(7)=19$.

2. B—Let x indicate the distance the cow has yet to travel. Then the work for a distance dx is $(2x+200-\frac{1}{2}(100-x))dx$. So the total work is $\int_0^{100}(2.5x+150)dx=27500$ ft-lbs. foot-pounds.

3. A—$\lim_{x\to 0}\frac{\frac{d}{dx}\frac{\sin x}{x}}{x}=\lim_{x\to 0}\frac{x\cos x-\sin x}{x^3}=\lim_{x\to 0}\frac{\cos x-x\sin x-\cos x}{3x^2}=\lim_{x\to 0}\frac{-\sin x}{3x}=\frac{-1}{3}$.

4. E—$\frac{4\pi}{e^2}$ $V=\pi\int_0^2(\sqrt{2x-x^2}\cdot e^{\frac{-x}{2}})^2dx=\pi\int_0^2(2x-x^2)e^{-x}dx=\pi x^2e^{-x}\big|_0^2=\frac{4\pi}{e^2}$

5. C—Let $w(t)$ and $s(t)$ denote the amounts of water and salt, respectively, in the tank at time t. We can see that $w(t)=t+100$. Since the tank is constantly mixed, we know that $\frac{ds}{dt}=-2\frac{s(t)}{w(t)}\Rightarrow\frac{ds}{s}=-2\frac{dt}{t+100}$
So, $\ln(s)=-2\ln(C(t+100))\Rightarrow s=\frac{C}{(t+100)^2}$. Since $s(o)=50$, then $C=500000$ and $s(60)=\frac{625}{32}$.

6. B—$d=\int_0^{\infty}\frac{dt}{1+t^2}=\tan^{-1}(t)\big|_0^{\infty}=\frac{\pi}{2}$.

7. D—Let the circular island be a circle of radius 2 centered at the origin. Let the length of the rectangular base be from $-x$ to x and the width from $-y$ to y. By the equation of a circle, $x^2=4-y^2$. Then,
$V=\frac{1}{3}(2x)^2(2y)=\frac{8}{3}x^2y=\frac{8}{3}(4y-y^3)\Rightarrow\frac{dV}{dy}=\frac{8}{3}(4-3y^2)=0\Rightarrow y=\sqrt{\frac{4}{3}}$ and $V=\frac{128\sqrt{3}}{27}$.

8. A—This integral is equal to $\int_{-5}^{5}(x+x^2+x^3)dx=\frac{250}{3}$.

9. D—The set of points satisfying the equation form a closed curve that encloses a region. This curve is preserved if we transform $x\to -x$ and $y\to -y$, so it is symmetric in all 4 quadrants. In particular, we can find the area in the first quadrant. In this quadrant, we can rewrite our equation as $y=1-x^{2/5}$. This curve intersects the coordinate axes at (0, 1) and (1, 0), and it is continuous, so the area is $A=\int_0^1(1-x^{2/5})dx=\frac{2}{7}$. The total area is $4A=\frac{8}{7}$.

10. D—The number of theorems proven is $(s+\ln c)(24-s-\frac{c}{12})$. Differentiating with respect to s gives $24-\frac{c}{12}-2s-\ln c=0$, so $s=12-\frac{c}{24}-\frac{1}{2}\ln c$. This is a maximum since the second derivative is −2.
Plugging this back in and simplifying gives $(12-\frac{c}{24}+\frac{\ln c}{2})^2=f(c)^2$. This differentiates to $2f'(c)f(c)$, so the derivative will be zero when either $f(c)$ or $f'(c)$ is zero. $f(c)=0$ is too difficult to solve, but

$f'(c) = \frac{1}{2c} - \frac{1}{24}$, so $c = 12$ is a solution. Testing shows that it is a maximum.

11. B—The graphs do not need to intersect or they could intersect. However, if they do intersect, then they will intersect no more than once because $f(x)$ grows faster than $g(x)$.

12. A— $a(t) = 24t^2, v(t) = 8t^3 + c$ and $v(0) = 0 \Rightarrow c = 0$. The particle is always moving to the right, so the distance = $\int_0^2 8t^3 dt = 32$.

13. D— $y'' - y' - 2y = 0,\ y'(0) = -2,\ y(0) = 2$. The characteristic equation is $r^2 - r - 2 = 0 \ \Rightarrow r = -2, r = 1$. So, the general solution to the differential equation is $y = c_1 e^{-x} + c_2 e^{2x}$ with $y' = -c_1 e^{-x} + 2c_2 e^{2x}$, Using the initial conditions, one can solve $c_1 = 2, c_2 = 0$. So, the solution is $f(x) = 2e^{-x} \Rightarrow f(1) = 2e^{-1}$.

14. D—The ratio test shows that the series is convergent for any value of x that makes $|x+1| < 1$. This gives you $-2 < x < 0$. Checking endpoints shows that both series are convergent, so the interval is $-2 \le x \le 0$.

15. D—For x in the interval (-1, 1), $g(x) = |x^2 - 1| = -(x^2 - 1)$ and so $y = \ln g(x) = \ln(-(x^2 - 1))$. So,

$$y' = \frac{2x}{x^2 - 1} \Rightarrow y'' = \frac{-2x^2 - 2}{(x^2 - 1)^2} < 0 \Rightarrow \text{ concave down}$$

16. C— $A = \frac{1}{2}\int_0^{2\pi} (1 - \cos\theta)^2 d\theta = \frac{3\pi}{2}$.

17. E— $F'(x) = xg'(x)$ with $x \ge 0$ and $g'(x) < 0 \Rightarrow F'(x) \le 0 \Rightarrow F$ is not increasing, F is differentiable (therefore, continuous, so a, b, and d are true). It is easy to check that c works as well.

18. B— $\int_0^{\infty} (4x - 2xf(x))dx = -3$.

19. E— $\frac{dy}{dt} = ky(1 - y)$

20. A— $A = \pi r^2$ and $A = 64\pi$ when $r = 8$. $\frac{dA}{dt} = 2\pi\frac{dr}{dt} \Rightarrow 96\pi = 2\pi(8)\frac{dr}{dt} \Rightarrow \frac{dr}{dt} = 6$.

21. D—The solution is of the form $y = y_h + y_p$ where y_h is the solution to $y' - y = 0$ and the form of y_p is $Ax^2 + Bx + K$. So, $y_h = Ce^x$. Substitute y_p into the original differential equation to determine $A, B,$ and K.

22. D— $\int_{-4}^{4} f(x)dx - 2\int_{-1}^{4} f(x)dx = (A_1 - A_2) - 2(-A_2) = A_1 + A_2$.

23. D—Recall from Taylor series that if $f(0) = 0$, then $f(x) \approx f'(0)x$ when x is small. So, this means that

$$\lim_{x\to 0}\frac{\sin^2(5x)\tan^3(4x)}{(\ln(2x+1))^5} = \lim_{x\to 0}\frac{(5x)^2(4x)^3}{(2x)^5} = 50.$$

24. C—Profit (P) is revenue – cost. So, $P = n \cdot D - (600 + 10n + n^2 0 = -3n^2 + 100n - 600.$ The critical point is $16\frac{2}{3}$, which means $n = 16$ or $n = 17$ is the largest profit. Checking yields $n = 17$.

25. B—In general, for even functions, $\int_{-a}^{a} \frac{f(x)dx}{1+b^x} = \frac{1}{2}\int_{-a}^{a} f(x)dx.$ So, $\int_{-2}^{2} \frac{1+x^2}{1+2^x} = \frac{1}{2}\int_{-2}^{2}(1+x^2)dx = \frac{14}{3}.$

26. E—II and III only$\Rightarrow$I is FALSE (max could be at a cusp). II is TRUE (there is a critical pt at $x = c$ where $f'(c)$ exists). III is TRUE (if 2nd deriv is >0, then there would be a relative min, not max).

27. D—If $f(x) = \sum_{n=0}^{\infty} a_n x^n$, then $f'(x) = \sum_{n=0}^{\infty} n a_n x^{n-1}, = \sum_{n=1}^{\infty} n a_n x^{n-1}.$ So, $f'(1) = \sum_{n=1}^{\infty} n a_n 1^{n-1} = \sum_{n=1}^{\infty} n a_n.$

28. B— $\int_3^5 [f(x) + g(x)]dx = \int_3^5 [2g(x) + 7]dx = 2\int_3^5 [g(x)]dx + 7(2) = 2\int_3^5 [g(x)]dx + 14.$

29. D—I is TRUE… $\frac{f(3) - F(1)}{3-1} = \frac{5}{2}$. II is FALSE…there is not enough info to determine the average value. III is TRUE…the average value of f' is the average rate of change of f.

30. A— $s_n = \frac{1}{5}\left(\frac{5+n}{4+n}\right)^{100}$, $\lim_{n\to\infty} s_n = \frac{1}{5}(1) = \frac{1}{5}.$