For each question, "E) NOTA" indicates that none of the above answers is correct.

1. Let $f: \mathbb{R} \to \mathbb{R}$ be a differentiable function such that $f'(x) = f(1-x)$ for all x and $f(0) = 1$. Find $f'(1)$.

A) 1 B) 3 C) e D) $\cos 1$ E) NOTA

2. Consider the positive, first order differential equation with initial condition $y(1) = 1$:

$$y' = \frac{x^2+y^2}{2xy}.$$

Evaluate the $y(e)$. (HINT: Consider making a substitution!)

A) $e/2$ B) e C) $3e/2$ D) $3e$ E) NOTA

3. Suppose the function $g(x) = f(x) - f(2x)$ has derivative 5 at $x = 1$ and derivative 7 at $x = 2$. Find the derivative of $h(x) = f(x) - f(4x)$ at $x = 1$.

A) -9 B) 0 C) 12 D) 19 E) NOTA

4. Given the following differential equation: $(\sin t)y' + (\cos t)y = \sec^2 t$ with initial conditions $y\left(\frac{3\pi}{4}\right) = -\sqrt{2}$, evaluate $y(\pi)$.

A) -1 B) 0 C) 1 D) 3 E) NOTA

5. There exists a real-valued continuously differentiable function f with domain $\mathbb{R}$ such that for all $x \in \mathbb{R}$

$$\left(f(x)\right)^2 = 1 + \int_0^x \left(\left(f(t)\right)^2 + \left(f'(t)\right)^2\right)dt$$

and $f(0) = 1$. What non-negative integer is closest to the value of $f(1)$?

A) 1 B) 3 C) 5 D) 9 E) NOTA

Use the following information for questions 6-8.

There is a special type of differential equations known as exact differential equations. Differential equations that take the form: $M(x,y)dx + N(x,y)dy = 0$ and have the property such that $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$. Given the following is true about a differential equation, a solution F takes the form: $F(x,y) = \int M(x,y)dx + h(y)$. Additionally it is known about exact equations that $\frac{\partial F}{\partial y} = N(x,y)$.

6. Given the previous definition of exact equations determine which of the following most describe this equation.

$$y' = \frac{2xy - \sin x - 3y^2e^x}{6ye^x - x^2}$$

A) Exact B) Not Exact C) Separable D) Homogeneous E) NOTA

7. Solve the differential equation from problem 6 given the initial condition $y(0) = 2$.

A) $3y^2e^x - x^2y - \cos x = 9$
B) $y^2e^x — y - \cos x = 9$
C) $3y^2e^x - x^2y - \sin x = 11$
D) Not Exact
E) NOTA

8. Given the previous definition of exact equations determine which of the following most accurately describe this equation.

$$(2x\cos y - x^2)dx + (x^2\sin y)\,dy = 0$$

A) Exact B) Not Exact C) Separable D) Homogeneous E) NOTA

9. A radioactive substance decays at a rate proportional to the amount of the substance present. Suppose we start with 100 grams of radioactive material. After 1 hour, only 75 grams remain. How long in hours will it take until only 10 grams remain?

A) $\log_{3/4}(\frac{1}{10})$ B) $\log_{1/10}(\frac{3}{4})$ C) $\log_{3/4}(\frac{1}{100})$ D) $\ln\left(\frac{3}{4}\right)$ E) NOTA

Use the information in question 10 for questions 11 and 12.

10. The rate at which a student learns the material in a differential equations course is proportional to the difference between a maximum, M, and the amount the student already knows at time t, $A(t)$. This is called a learning curve. Write a differential equation to model the learning curve described. $\frac{dA}{dt}$ is equal to which of the following:

A) $\frac{dA}{dt} = kM - A(t)$
B) $\frac{dA}{dt} = A(t) - M$
C) $\frac{dA}{dt} = k(M - A(t))$
D) $\frac{dA}{dt} = k\frac{(M-A(t))}{M}$
E) NOTA

11. Given the information about the learning curve differential equation as well as your answer in problem 10, solve for such learning curve function, $A(t)$. Let k be the constant of proportionality in problem 10 and C be an additional constant for this general solution.

A) $A(t) = M + Ce^{-kt}$
B) $A(t) = -M - Ce^{-kt}$
C) $A(t) = M + Ce^{tk}$
D) $A(t) = -M - Ce^{tk}$
E) NOTA

12. If took a student 100 hours to learn 50% of the material in Math 353, Differential Equations, and would like to know 75% in order to get a B, how much longer should she study? You may assume that the student began knowing none of the material and that the maximum she might achieve is 100%.

A) 50 hours B) 100 hours C) 150 hours D) 200 hours E) NOTA

13. There exists a set of functions $f(x)$ such that $f: (0, \infty) \to (0, \infty)$ and there is a real, positive number a such that:

$$f'\left(\frac{a}{x}\right) = \frac{x}{f(x)}$$

for all $x > 0$. Which of the following most accurately describes this set of function $f(x)$?

A) Exponential B) Trigonometric C) Logarithmic D) Polynomial E) NOTA

14. Let d_n be the determinant of the n x n matrix whose entries, from top to bottom and then from left to right, are cos 1, cos 2, …, $\cos n^2$. An example of such matrix is:

$$d_3 = \det\begin{bmatrix} \cos 1 & \cos 4 & \cos 7 \\ \cos 2 & \cos 5 & \cos 8 \\ \cos 3 & \cos 6 & \cos 9 \end{bmatrix}$$

where the argument of cosine is always in radians. Evaluate $\lim_{n\to\infty} d_n$.

A) $-\infty$ B) 0 C) 1 D) ∞ E) NOTA

15. f, g, and h are real differentiable functions. Below are given values of functions f, g, and h at $x = 0$.

$$f(0) = 1;\ \ g(0) = 2;\ \ h(0) = 3;\ \ (gh)'(0) = 4;\ \ (hf)'(0) = 5;\ \ (fg)'(0) = 6$$

Find the value of $(fgh)'(0)$.

A) 0 B) 8 C) 15 D) 16 E) NOTA

16. Solve the following differential equation:

$$\frac{dy}{dx} = 1 + y + x^2 + yx^2$$

A) $y = Ce^{x+\frac{1}{3}x^3} - 1$ B) $y = Ce^{x+x^3} - 1$

C) $y = Ce^{x+\frac{1}{3}x^3}$ D) $y = Ce^{2x} - 1$ E) NOTA

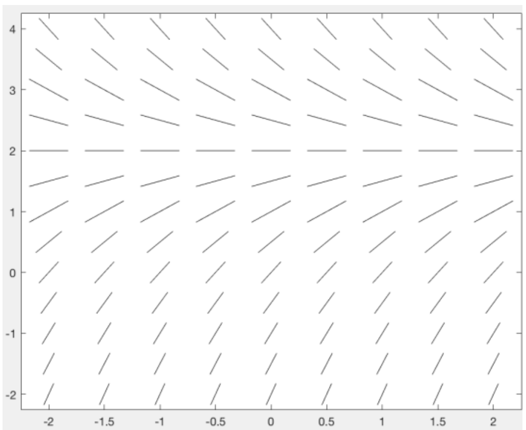
17. Find all solutions to the following differential equation:

$$x + y\frac{dy}{dx} = 2$$

A) $y^2 = 4x - x^2$ B) $y = 2x - \frac{1}{2}x^2$

C) $y^2 = 2x - \frac{1}{2}x^2$ D) $y^2 = 8x - 2x^2$ E) NOTA

18. The slope field below describes which of the following differential equations:



A) $\frac{dy}{dx} = 2 - x$

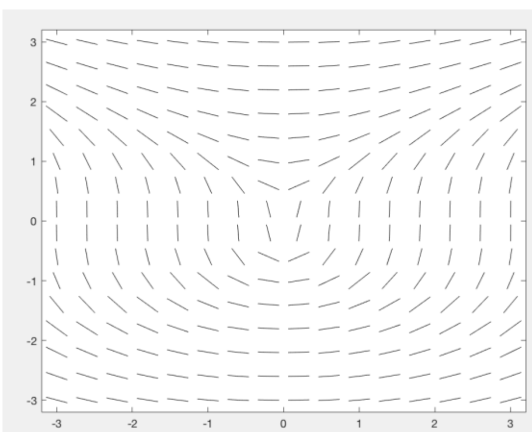
B) $\frac{dy}{dx} = 2 - y$

C) $\frac{dy}{dx} = 2y$

D) $\frac{dy}{dx} = y^2 + 2$

E) NOTA

19. Which of the following solutions has a slope field most closely resembling the following slope field?



A) $x^4 = \frac{4}{3}y^3$

B) $x^2 + y^2 = 4$

C) $y = x^2$

D) $y^2 = \frac{2}{5}x^3$

E) NOTA

20. A particle moving on a line is at position $s = t^3 - 6t^2 + 9t - 4$ at time t. At which time(s), if any, does it change direction?

A) $t = 0$ B) $t = 1$ C) $t = 1,3$ D) $t = 2,4$ E) NOTA

21. Use Euler's method with step size of $\Delta x = .1$ to approximate $y(1.2)$ for the differential equation $y' = x + y$ given that $y(1) = 0$.

A) $.21$ B) $.22$ C) $.23$ D) $.24$ E) NOTA

22. What is the order of the following differential equation:

$$t^2y'' + 4ty' + 5\cos t\, y''' + y = e^t \sin 3t$$

A) 1 B) 2 C) 3 D) 4 E) NOTA

23. A tank originally contains 100 L of pure water. Salt water with a concentration of 2 kg/L is pumped into the tank at 3 L/min. Additionally, the well-mixed solution is drained at the same rate. Find an equation for the amount of salt (in kg) in the tank at a given time t (in minutes), $A(t)$. What is $A(100)$?

A) 0
B) $200 - 200e^{-1}$
C) $200 - 200e^{-2}$
D) $200 - 200e^{-100}$
E) NOTA

24. It is determined that there is a limiting concentration of salt in the tank in problem 23. What is this concentration (kg/L)?

A) 0 B) 1 C) 2 D) 4 E) NOTA

25. If $y = f(x)$ is a differentiable function such that $f(2) = 0$ and such that $3x^2 - x^2y^3 + 4y = 12$. Use this value of $f(2)$ to determine the differential approximation for $f(1.97)$.

A) 0.09 B) -0.09 C) 0.97 D) -1.06 E) NOTA

26. Let $f(x) = \sin^6\left(\frac{x}{4}\right) + \cos^6\left(\frac{x}{4}\right) + 3\sin^2\left(\frac{x}{4}\right)\cos^2\left(\frac{x}{4}\right) - \frac{3}{4}\sin x$ for all real numbers x. Evaluate $f^{(2017)}(0)$.

A) 0 B) $3/8$ C) 1 D) 2^{2008} E) NOTA

27. Let $y(x)$ be a particular solution to the differential equation

$$\frac{dy}{dx} = \sum_{n=0}^{\infty} (-x)^n$$

where $y\left(\frac{1}{2}\right) = \ln 3$. Find $y\left(\frac{3}{4}\right)$.

A) $\ln\left(\frac{3}{2}\right)$ B) $\ln\left(\frac{7}{2}\right)$ C) $\ln\left(\frac{9}{2}\right)$ D) $\ln\left(\frac{11}{2}\right)$ E) NOTA

28. Given $g(x) = x + e^x - \sin x$. Find $g(0) + g'(0) + g''(0) + \cdots + g^{(2017)}(0)$.

A) 2016 B) 2017 C) 2018 D) 2019 E) NOTA

29. The graph of $x^2 - (y-1)^2 = 1$ has one tangent line with positive slope that passes through $(0,0)$. If the point of tangency is (x, y), find $\arcsin\left(\frac{x}{y}\right)$.

A) $-\frac{\pi}{6}$ B) $-\frac{\pi}{4}$ C) $\frac{\pi}{4}$ D) $\frac{\pi}{3}$ E) NOTA

30. For $x > 0$, let $f(x) = x^x$. Find the sum of all values of x such that $f(x) = f'(x)$.

A) 0 B) 1 C) 2 D) 4 E) NOTA