For all questions, answer choice "E) NOTA" means that none of the above answers is correct.

1. In what base does $24^2 = 554$?

A) 6 B) 8 C) 12 D) 14 E) NOTA

2. Simplify $\log_7 36 \cdot \log_5 49 \cdot \log_6 25 \cdot \log\sqrt{1000}$.

A) 7.5 B) 9.5 C) 11.5 D) 13.5 E) NOTA

3. Solve the following equation.

$$\frac{x+5}{x-7} - 5 = \frac{4x-40}{13-x}$$

A) 8 B) 10 C) 12 D) No solution E) NOTA

4. The function $f(x) = -3\pi\sin(3x-3)$ has amplitude A, period P, and phase shift S. Compute $9PS/A$.

A) -6 B) $-\frac{9}{\pi}$ C) $\frac{9}{\pi}$ D) 6 E) NOTA

5. Let N be the smallest positive integer that is a multiple of 15 and, when written in base 10, uses only the digits 0 and 7. How many 7s are used to express N in base 10?

A) 3 B) 6 C) 9 D) 12 E) NOTA

6. If $f(x) = \ln(x) - k\sqrt{x}$ has a local maximum at $x = 4$, then the value of k is

A) -1 B) 0.5 C) 1 D) 4 E) NOTA

7. Suppose 50 is divided by a positive number. If the divisor is increased by 3, the quotient decreases by $3\frac{3}{4}$. What is the positive number by which 50 was originally divided?

A) $\frac{3}{5}$ B) $\frac{4}{5}$ C) 5 D) 8 E) NOTA

8. A king sent 30 men into his orchard to plant trees. If the men could plant 1000 trees in 9 days, in how many days could 36 men plant 4400 trees?

A) 24 B) 27 C) 30 D) 33 E) NOTA

9. Which of the following statements is necessarily true?

A) If $f''(0) = 0$, then the graph of f changes concavity at $x = 0$.
B) $\int_{-1}^{1} |x| dx = 0$.
C) If the graph of a function is always concave up, then the left-hand Riemann sums are always less than the right-hand Riemann sums, with the same subdivisions over the same interval.
D) If the function f is continuous on the interval $[a, b]$, where $a < b$, and if $\int_a^b f(x) dx = 0$, then f must have at least one zero between a and b.
E) NOTA

10. The contents of a purse are not revealed to us, but we are told that there are exactly 6 pennies and at least one nickel and at least one dime. We are further told that if the number of dimes were changed to the number of pennies, the number of nickels were changed to the number of dimes, and the number of pennies were changed to the number of nickels, the total amount of money would remain unchanged. Find the greatest possible number of coins the purse contains.

A) 13 B) 15 C) 17 D) 19 E) NOTA

11. The price, p, in dollars, and the quantity, x, sold of the latest Applesung Galaxyphone obey the demand equation $p = -\frac{1}{4}x + 120$ for $0 \le x \le 400$. The cost equation C, in dollars, of producing x smartphones is $C = \frac{1}{2}\sqrt{x} + 600$. Assuming that all items produced are sold, the cost C can be written as a function of p in the form $C = \sqrt{a - p} + b$. Compute $a + b$.

A) 480 B) 600 C) 720 D) 1080 E) NOTA

12. Suppose $A = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & t \\ 1 & -2 \end{bmatrix}$. Find the value of t so that $AB - BA = \begin{bmatrix} 3 & 0 \\ -3 & -3 \end{bmatrix}$.

A) 0 B) 1 C) 2 D) 3 E) NOTA

13. For how many values of θ, $0° \le \theta \le 360°$, is the equation $\sec(\theta) + \tan(\theta) + \cos(\theta) = 0$ a true statement?

A) 0 B) 1 C) 2 D) 4 E) NOTA

14. Triangle JKL has median JM. If $JM = KM$, $JK = 1$, and $m\angle JKL = x$, then the length of JL in simplest form is

A) $\tan(x)$ B) $\cos(x)$ C) $\sec(x)$ D) $\csc(x)$ E) NOTA

15. If $\frac{dy}{dx} = \cos(x)\cos^2(y)$ and $y = \frac{\pi}{4}$ when $x = 0$, then

A) $\tan(y) = \sin(x) + 1$
B) $\tan(y) = -\sin(x) + 1$
C) $\sec^2(y) = \sin(x) + 2$
D) $\tan(y) = \frac{1}{2}(\cos^2(x) + 1)$
E) NOTA

16. Find the sum of the integer values of a for which the equation $(a-3)x^2 + 4x + a = 0$ has more than one distinct real root.

A) 0 B) 1 C) 3 D) 6 E) NOTA

17. Let a and b be the legs of a right triangle, where $a > b$, whose hypotenuse is 5 and whose area is $3\frac{9}{25}$. Compute $a \div b$.

A) $\frac{31}{17}$ B) $\frac{961}{289}$ C) $\frac{24}{7}$ D) $\frac{168}{25}$ E) NOTA

18. Suppose three standard fair dice are rolled, and the numbers showing face up are added. Let N be the number of ways a roll results in a sum of nine and let T be the number of ways a roll results in a sum of ten. Compute $|T - N|$.

A) 0 B) 1 C) 2 D) 3 E) NOTA

19. For what nonnegative value of k is the line $y = \frac{k-x}{32}$ normal to the graph of $y = x^4$?

A) 514 B) 1022 C) 1024 D) 2046 E) NOTA

20. Let $P(x)$ be a fourth-degree polynomial with nonnegative integer coefficients. Given that $P(1) = 11$ and $P(10) = 12{,}134$, compute $P(3)$.

A) 33 B) 110 C) 134 D) 157 E) NOTA

21. Suppose A, B, and C are positive integers such that $A^2 + B - C = 100$ and $A + B^2 - C = 124$. Find the value of C.

A) 12 B) 13 C) 24 D) 57 E) NOTA

22. Evaluate the limit below.

$$\lim_{n\to\infty} \sum_{k=1}^{n} 2\left(-1 + \frac{4k}{n}\right)\left(\frac{4}{n}\right)$$

A) 8 B) 9 C) 10 D) does not exist E) NOTA

23. Evaluate the definite integral below.

$$\int_0^{\frac{1}{2}\ln 3} \frac{1}{e^x + e^{-x}}\, dx$$

A) $\frac{\pi}{12}$ B) $\ln\left(\frac{2}{\sqrt{3}}\right)$ C) $\frac{1}{\ln 3}$ D) $\ln\sqrt{2}$ E) NOTA

24. The curve C is defined by the parametric equations $x(t) = t^2 - t$ and $y(t) = t^3 - 3t$ for $t > 0$. Which of the following are true?

I. The point $(0, -2)$ lies on C.
II. $\frac{dy}{dx} = 3$ at $t = 2$.
III. There is a vertical tangent on C at $t = 1$.
IV. C is concave up at $t = 3$.

A) I, III, IV only B) I, II, IV only C) II, III, IV only D) I, II, III only E) NOTA

25. Evaluate the limit below in terms of a, where a is a real constant.

$$\lim_{x\to a} \frac{\sqrt{2a^3x - x^4} - a\sqrt[3]{a^2x}}{a - \sqrt[4]{ax^3}}$$

A) does not exist B) $\frac{8}{9}a$ C) $\frac{16}{9}a$ D) $\frac{8}{3}a$ E) NOTA

26. Given that $f(x) = x^{2017} + x - 13^{2017} + \cos\left(\frac{x}{2017}\right)$ has exactly one real root at $x = c$, then find the integer value of k so that c is in the interval $(k, k+1)$.

A) 12 B) 13 C) 1008 D) 2016 E) NOTA

27. The four distinct solutions to $x^4 + 2x^3 - 13x^2 + 2x + 1 = 0$ can be written in the forms $\frac{a \pm \sqrt{b}}{c}$ and $\frac{u \pm \sqrt{v}}{w}$, where all letters represent integers, b and v are positive and not divisible by the square of any prime and $\gcd(a, c) = \gcd(u, w) = 1$. Compute $a + b + c + u + v + w$.

A) 14 B) 18 C) 22 D) 27 E) NOTA

28. $\csc^{-1}\sqrt{5} + \cot^{-1} 3 =$

A) $\frac{\pi}{8}$ B) $\frac{\pi}{6}$ C) $\frac{\pi}{5}$ D) $\frac{\pi}{4}$ E) NOTA

29. Duncan thinks of a positive real number N in the interval $[0, 55]$. He challenges Valentin to guess the integer m such that N lies in the interval $[m, m+1]$. With Valentin's each guess, Duncan responds with either "N is in this interval or below it on the number line" or "N is above this interval on the number line", depending on if N is . Each guess costs Valentin one penalty point, and if Duncan's response is "…above…", it costs Valentin an additional penalty point. If Valentin plays optimally, at most how many penalty points will he incur in order to know the correct value of m?

A) 5 B) 6 C) 8 D) 13 E) NOTA

30. There are 128 teams competing in a tournament. In the first round, the 128 teams are paired in 64 games. The losers of the games are out of the tournament, and the winners are then paired into 32 games, and so on, until only one team remains (in a bracket-style tournament). Suppose the teams are ranked from #1 to #128 (where #1 is the best) and suppose that the better team always beats the worse team. Clearly, Team #1 is guaranteed to win. If the teams are paired up at random, what is the probability that Team #3 makes it to the final round against Team #1?

A) $\frac{32}{127}$ B) $\frac{16}{63}$ C) $\frac{63}{128}$ D) $\frac{32}{61}$ E) NOTA