"For all questions, answer choice "E. NOTA" means none of the above answers is correct." Also, DNE means "Does not exist".

1. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\pi}{2n} \sin(\frac{\pi n+\pi}{2n}) =$

A. -1 B. 0 C. 1 D. DNE E. NOTA

Answer C. The limit summation above represents $\int_{\frac{\pi}{2}}^{\pi} \sin(x)\, dx = 1.$

2. $\int tan^3x\ sec^3xdx$

A. $\frac{\sec^5 x}{5} + \frac{\sec^3 x}{3} + C$ B. $\frac{\sec^5 x}{5} - \frac{\sec^3 x}{3} + C$ C. $\frac{\sec^4 x}{4} + C$

D. $\frac{\tan^4 x \sec x}{4} + C$ E. NOTA

Answer B. One way to solve this integral is to substitute using the Pythagorean Identity $\tan^2 x = \sec^2 x - 1$. This allows a simple u-substitution where $u = \sec \mathrm{x}$ and $du = \sec x \tan x\, dx$. Integrating the resulting integral and re-substituting back in yields answer B.

3. $\frac{d^2y}{dx^2} = 2\sec^2 x \tan x$ and $y(\pi) = y(0) = y'(0)$ then y =

A. $\frac{\sec^3 x}{3} - \frac{1}{3}$ B. $\tan x$ C. $\tan x + 1$ D. $\frac{\tan^3 x}{3}$ E. NOTA

Answer C. Separation of variables together with the u-substitution above gives you $\frac{dy}{dx} = \sec^2 x + C_1$. Therefore $y'(0) = 1 + C_1$. Integrating again gives you $y = \tan x + C_1 x + C_2$ and therefore $(\pi) = \pi C_1 + C_2 = \ C_2 = y(0)$. This implies that $C_1 = 0$, therefore $y'(0) = 1$ and $C_2 = 1$. This gives us answer C.

4. If $\int_{-5}^{-2} f(x)dx = 6$, $\int_{-2}^{3} f(x)dx = 3$, $\int_{-1}^{-4} f(x)dx = -3$, and $\int_{3}^{-1} f(x)dx = -1$, what is $\int_{-4}^{-2} f(x)dx$?

A. -7 B. -1 C. 1 D. 7 E. NOTA

Answer C. Combining the last two integrals (after switching their limits to go from left to right) gives you $\int_{-4}^{3} f(x)dx = 4$. The second given integral indicates that 3 of those 4 units of area accrue in the interval (-2, 3). Therefore, on the interval (-4,-2) the net area is 1.

5. $\int \frac{2}{\sqrt{1-x^2}} dx =$

A. $-2arcsin(x) + C$ B. $-\arccos(x) + C$ C. $4\sqrt{1-\mathrm{x}^2} + C$

D. $\frac{2}{\sqrt{1+x}} - \frac{2}{\sqrt{1-x}} + C$ E. NOTA

Answer E. This is a constant multiple of $\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin(x) + C$.

6. $\int \frac{2x}{\sqrt{1-x^2}} dx =$

A. $-2xarccos(x) + C$ B. $2xarcsin(x) + C$ C. $-2\sqrt{1-\mathrm{x}^2} + C$

D. $-\ln\left(\sqrt{1-x^2}\right) + C$ E. NOTA

Answer C. The u-substitution $u = 1 - x^2$ and $du = -2xdx$ makes this a basic power rule integral, which after substituting back in yields answer C.

7. $\int \frac{2x^2}{\sqrt{1-x^2}} dx =$

A. $-\mathrm{x}\ln\left(\sqrt{1-x^2}\right) + C$ B. $-2\sqrt{1-\mathrm{x}^2} + C$ C. $2xarcsin(x) + C$

D. $\frac{1}{2}(arcsinx - x\sqrt{1-x^2}) + C$ E. NOTA

Answer E. After a trigonometric substitution using a right triangle with legs x and $\sqrt{1-x^2}$, a hypotenuse of 1, and an angle θ opposite the side of length x, you get that $x = sin\theta$ and $dx = cos\theta d\theta$, producing the integral $\int 2\sin^2\theta d\theta$. Using the power reduction formula for sine, this is easily integrated producing two times the answer D, after re-substitution using the triangle.

8. $\int \frac{2x^3}{\sqrt{1-x^2}} dx =$

A. $-\frac{2}{3}(1-x^2)^{\frac{3}{2}} + 2\sqrt{1-x^2} + C$ B. $-\frac{1}{3}(1-x^2)^{\frac{3}{2}} - \sqrt{1-x^2} + C$

C. $\frac{1}{3}(1-x^2)^{\frac{3}{2}} - \sqrt{1-x^2} + C$ D. $\frac{2}{3}(1-x^2)^{\frac{3}{2}} - 2\sqrt{1-x^2} + C$ E. NOTA

Answer D. This problem could be done in the same way as the one above or by using the same u-substitution as in problem #6. In the first case you would have to use the Pythagorean identity rather than the power reduction formula; in the second you would have to solve for x^2 in terms of u to complete the substitution.

9. Given that the density (in g/m^3) at any point of a spherical meteor is given by the function $\rho(r) = \frac{9}{r^3+1}$, where r is the distance (in m) from the center of the meteor, find the mass (in g) of the meteor if it is 8m in diameter.

A. $\frac{4}{3}\pi\ln(65)$ B. $12\pi\ln(65)$ C. $36\pi\ln(513)$ D. $36\pi\ln(513)$

E. NOTA

Answer B. The integral desired would be $\int_0^4 \rho dV = \int_0^4 \frac{36\pi r^2}{r^3+1} dr = 12\pi ln65$ by u-substitution ($dV = 4\pi r^2 dr$ for a sphere).

10. Let the region R be the area on the coordinate plane enclosed by the y-axis and the graphs of $f(x) = \sqrt{9-x^2}$ and $g(x) = x - 3$. Find the volume when R is rotated about the y-axis.

A. 18π B. 27π C. 36π D. 45π E. NOTA

Answer B. The solid is composed of two basic geometric shapes: a hemisphere of radius 3 on the top and a cone of the same radius with a height also of 3. Adding the two volumes results in the answer B.

11. $\int_{-3}^{1}(8x^3 + 2\sin(\pi x) - 6x^2 + 4x)dx + \int_1^3(8x^3 + 2\sin(\pi x) - 6x^2)dx$ =

A. 72 B. 92 C. 108 D. 128 E. NOTA

Answer B. The first two terms in each integral are odd functions so when taken from -3 to 3 they will be 0. The third term is even so we can do $2\int_0^3 6x^2 dx = 108$. Finally, the fourth term of the first integral is not present in the second integral, so we have to evaluate it separately and that integral is -16, giving the sum of 92.

12. If f(x) =$\int_{x^2}^{3} \sin(t^2)\, dt$, what is $f'(x)$?

A. $9 - \sin(x^4)$B. $\sin(x^4)$ C. $2xsin(x^2)$ D. $2xsin(x^4)$ E. NOTA

Answer E. Let the anti-derivative of the integrand be $F(x)$. Our integral is then $f(x) = F(3) - F(x^2)$. Since the first term is constant, the derivative would be $-2xsin(x^4)$.

13. If f(x) is odd and g(x) is even which of the following are true:

1) $\int_{-2}^{2} f(x)g(x)dx = 0$ 2) $\int_{1}^{-1} g(x)dx = 2\int_{0}^{1} g(x)dx$

3) $2\int_{0}^{2} g\big(f(x)\big)dx = \int_{-2}^{2} g(x)dx$ 4) $\int_{-2}^{2} [f(x) + g(x)]dx = \int_{-1}^{1} g(x)dx$

5) $\int_{-3}^{3} f\Big(g\big(f(x)\big)\Big)\,dx = 2\int_{3}^{0} f\Big(-g\big(f(x)\big)\Big)\,dx$

A. 1 only B. 1, 2, and 4 only C. 2, 3, and 5 only D. 1 and 5 only E. NOTA

Answer D. First, the major properties at work here are that if f(x) is odd, $\int_{-a}^{a} f(x)dx = 0$ and if g(x) is even $\int_{-a}^{a} g(x)dx = 2\int_{0}^{a} g(x)dx$. In 1, the product of an odd and even function is itself odd, so it is true. In 2, we almost have the second property except the limits are reversed on the left hand side; so it is false. In 3, we again almost have the second property, but the integrands do not match, so it is also false. In 4, the first property is applied correctly, but the second is not; so it too is false. Finally, in 5, the composition of any number of even and odd functions is itself even if at least one of the functions is even. The second property is applied, but the limits are reversed, but this is offset by the negative that can be brought out of f(x) (since it's odd); so the property is true. Therefore, only 1 and 5 are true.

14. If $\int_{0}^{k} (2kx - x^2)dx = 18$ then k =

A. -3 B. $-\frac{3\sqrt{2}}{2}$ C. $\frac{3\sqrt{2}}{2}$ D. 3 E. NOTA

Answer D. The integral produces the expression $k^3 - \frac{1}{3}k^3 - 0$ which yields the equation $\frac{2}{3}k^3 = 18$. Solving for k gives you answer D.

15. Let R be the region bounded by the functions $y = x^2$ and $y = 9$. The line $y = c$ splits the region R in such a way that when the portion of R below $y = c$ is rotated about the y-axis it yields a solid of volume equal to that of the solid produced when the portion of R above the line $y = c$ is rotated about the same axis. Find the value of c.

A. $\frac{9}{2}$ B. 6 C. $\frac{9\sqrt{2}}{2}$ D. $\frac{9\sqrt{2\pi}}{2\pi}$ E. NOTA

Answer C. Using the disk method in dy the constraints of the problem produce the following equation: $\int_{0}^{c} \pi y dy = \int_{c}^{9} \pi y dy$. After integration we get the equation $\frac{c^2}{2} = \frac{81}{2} - \frac{c^2}{2}$ which produces a solution of $\frac{9\sqrt{2}}{2}$.

16. A particle moves along a path modeled by the parametric equations $x(t) = t^2$ and $y(t) = \frac{2}{3}(2t+1)^{\frac{3}{2}}$, where $t \geq 0$ is measured in seconds and x and y are in meters. Find the total distance traveled by the particle in the first 4 seconds of its movement.

A. 16m B. 24m C. $\frac{\sqrt{5008}}{3}$m D. 48m E. NOTA

Answer B. The equation for the arc length of a parametric curve (which would represent the total distance traveled by the particle) is $s = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$. In our case, this becomes $\int_0^4 \sqrt{(2t)^2 + \left[2(2t+1)^{\frac{1}{2}}\right]^2}\, dt = \int_0^4 2t + 2dt = 24.$

17. $\int \frac{x^3}{x+1} dx =$

A. $\frac{x^3}{3} - \frac{x^2}{2} - x + \ln|x+1| + C$ B. $\frac{x^3}{3} - \frac{x^2}{2} + x - \ln|x+1| + C$
C. $\frac{x^3}{3} + \frac{x^2}{2} + x - \ln|x+1| + C$ D. $\frac{x^3}{3} + \frac{x^2}{2} + x + \ln|x+1| + C$ E. NOTA

Answer B. Rewriting the top as $x^3 + 1 - 1$ allows us to use the sum of cubes formula to easily divide the polynomials yielding the result $\int x^2 - x + 1 - \frac{1}{x+1} dx$ which is an elementary integral.

18. What is the area contained within one petal of the graph of $r = 2\cos(7\theta)$?

A. $\frac{\pi}{14}$ B. $\frac{\pi}{7} - \frac{1}{14}$ C. $\frac{\pi}{7}$ D. $\frac{\pi}{7} + \frac{1}{14}$ E. NOTA

Answer C. This area is represented by the integral $\int_{-\frac{\pi}{14}}^{\frac{\pi}{14}} 2\cos^2(7\theta)\, d\theta$. Using the power reduction formula the integrand may be re-written as $1 - \cos(14\theta)$. Furthermore, since the integrand is even we may rewrite the integral as $2\int_0^{\frac{\pi}{14}} 1 - \cos(14\theta)\, d\theta = \frac{\pi}{7}$

19. What is the volume of the solid produced by rotating the region between the x-axis and the function $y = 4e^{-x^2}$ about the y-axis?

A. 2π B. 4π C. 8π D. 16π E. NOTA

Answer B. If we use the shell method we get the integral $\int_0^\infty 8\pi x e^{-x^2} dx = \lim_{a\to\infty} -4\pi e^{-a^2} + 4\pi = 4\pi$

20. Which of the following integrals have no discovered solution outside of infinite series?

1) $\int \sin(x^2)\,dx$ 2) $\int \sin^2 x\,dx$ 3) $\int e^{-x^2}dx$

A. 1 only B. 1 and 2 only C. 1 and 3 only D. 3 only E. NOTA

Answer C. Number 2 is easily solvable with the power reduction formula

21. Which of the following integrals converge?

1) $\int_1^\infty \frac{\sin(x)+2}{x}dx$ 2) $\int_2^\infty \frac{x+1}{x^2-3}dx$ 3) $\int_1^\infty e^{-x^2}dx$

A. 1 only B. 2 only C. 3 only D. 1, 2, and 3 E. NOTA

Answer C. All of these integrals can be compared to basic integrals which very obviously converge or diverge. The first two can be compared with $\int_1^\infty \frac{1}{x}dx$ which diverges. Since both are larger on the given interval, they also diverge. The third can be compared to $\int_1^\infty 2xe^{-x^2}dx$ which converges (this can be shown by a simple u-substitution). Since it is greater than the third integral everywhere on the given interval, the third also converges.

22. $\int x^3\mathrm{e}^{-\mathrm{x}}\mathrm{dx} - \int x^4\mathrm{e}^{-\mathrm{x}}\mathrm{dx} =$

A. $e^{-x}(x^4+3x^3+9x^2+18x+18)+C$ B. $e^{-x}(x^4+4x^3+12x^2+24x+24)+C$
C. $e^{-x}(x^4+5x^3+15x^2+30x+30)+C$ D. $x^4e^{-x}+C$ E. NOTA

Answer A. Both integrals can be evaluated quickly using tabular integration. The normal alternation of sign that occurs in the tabular method is undone by having e^{-x} in the anti-derivative column (which itself will alternate signs). Therefore all the terms produced by the first integral are negative, while those produced by the second are all positive. However, the like terms in the second integral are 4 times that of those in the first so they do not entirely cancel out. The result is answer A.

23. What is the average value of y for the part of the curve y = 3x – x^2 which is in the first quadrant?

A. $\frac{1}{2}$ B. $\frac{9}{8}$ C. $\frac{3}{2}$ D. $\frac{9}{2}$ E. NOTA

Answer C. $\frac{1}{3}\int_0^3 3x - x^2dx = \frac{1}{3}\left(\frac{3^3}{2}-\frac{3^3}{3}\right) = \frac{3}{2}$

24. $\int e^x \cos(x)\,dx =$

A. $e^x \sin(x) + C$ B. $\frac{e^x(\cos(x)-\sin(x))}{2} + C$ C. $\frac{e^x(\sin(x)-\cos(x))}{2} + C$ D. $e^x(\cos(x)+\sin(x)) + C$
E. NOTA

Answer E. This integral requires integration by parts twice, followed by a substitution after the original integral is produced from the second integration by parts. Using the substitution $u = \cos(x)$ and $dv = e^x dx$ (and a similar substitution for the second iteration) we get $\int e^x \cos(x)\,dx = e^x \cos(x) + \int e^x \sin(x)\,dx = e^x \cos(x) + e^x \sin(x) - \int e^x \cos(x)\,dx$. If we then let the original integral be I, we get the equation $I = e^x(\cos(x)+\sin(x)) - I$, solving for I gives us $\frac{e^x(\cos(x)+\sin(x))}{2} + C$.

25. $\int \frac{3x+5}{(x-1)(x+6)}\,dx =$

A. $-\frac{8}{7}\ln|x-1| - \frac{13}{7}\ln|x+6| + C$ B. $\frac{13}{7}\ln|x-1| - \frac{8}{7}\ln|x+6| + C$
C. $\frac{8}{7}\ln|x-1| + \frac{13}{7}\ln|x+6| + C$ D. $\frac{13}{7}\ln|x-1| + \frac{8}{7}\ln|x+6| + C$ E. NOTA

Answer C. Partial fraction decomposition gives us the integral $\int \frac{8}{7(x-1)} + \frac{13}{7(x+6)}\,dx = \frac{8}{7}\ln|x-1| + \frac{13}{7}\ln|x+6| + C$

26. Which of the following functions is such that $\int_1^\infty f(x)dx$ diverges, but $\int_1^\infty [f(x)]^2 dx = 1$?

A. $\frac{1}{x}$ B. $\frac{1}{\sqrt{x}}$ C. $\sqrt{x}$ D. x E. NOTA

Answer A.

27.What is wrong with the following proof?

1. $\int 2\sin(x)\cos(x)\,dx = \int 2\sin(x)\cos(x)\,dx$	Reflexive property of equality
2. $\int 2u du = \int 2\sin(x)\cos(x)\,dx$	Substitution, $u = \sin(x)$
3. $\int 2u du = \int -2v dv$	Substitution, $v = \cos(x)$
4. $u^2 = -v^2$	Integration
5. $u^2 + v^2 = 0$	adding v^2 to both sides
6. $\sin^2 x + \cos^2 x = 0$	substitution
7. $1 = 0$	Pythagorean Identity

A. Incorrect use of the Pythagorean Identity, line 7 B. Incorrect substitution, line 2
C. Incorrect substitution, line 3 D. Incorrect substitution, line 6 E. NOTA

Answer E. The problem is actually in line 4, forgetting the constant of integration (+ C).

28. Given the initial condition $y(1) = 3$, which of the following is a solution to the differential equation $\frac{dy}{dx} = x \cdot \sqrt{\frac{9-y^2}{x^2-1}}$?

A. $y = \frac{3\sqrt{x^2-1}}{x}$ B. $y = 3sin(\sqrt{x^2-1} + \frac{\pi}{2})$ C. $y = 3sin(3\sqrt{x^2-1} + \frac{\pi}{2})$

D. $y = 3\sin(3\,\text{arcsec}(x) + \frac{\pi}{2})$ E. NOTA

Answer B. The equation separates as follows: $\int \frac{dy}{\sqrt{9-y^2}} = \int \frac{x}{\sqrt{x^2-1}} dx$. Integration then produces $arcsin\left(\frac{y}{3}\right) = \sqrt{x^2-1} + C$. Solving for y and using our initial condition we get $y = 3sin(\sqrt{x^2-1} + \frac{\pi}{2})$ (other values for C are also possible, but this is a solution).

29. If a particle's position along a line is given by $x(t) = 3\sin(\frac{\pi}{4}t)$, where $t \geq 0$ is in seconds, what is the total distance traveled by the particle during the first six seconds of motion?

A. $\frac{4}{\pi}$ B. $\frac{9}{\pi}$ C. $\frac{12}{\pi}$ D. $\frac{36}{\pi}$ E. NOTA

Answer D. The total distance traveled is given by $\int_0^6 |3\sin(\frac{\pi}{4}t)|dt = \int_0^4 3\sin(\frac{\pi}{4}t)dt - \int_4^6 3\sin(\frac{\pi}{4}t)dt = \frac{24}{\pi} + \frac{12}{\pi} = \frac{36}{\pi}$

30. If $f(x) = Asin\big(B(x-C)\big) + D$ and $g(x) = Acos\big(B(x-C)\big) + D$, what is the average value of $h(x) = f(x) + g(x)$ on the interval $[C, C + \frac{2\pi}{B}]$, where $A, B, C, D > 0$?

A. 0 B. D C. 2AD D. 2D E. NOTA

Answer D. Each of the functions f and g have an average value of D on the given interval, therefore the sum has an average value of 2D.