1. E. $(-2017!)$

$$\frac{d}{dx}\left(2017!^{2017!} - \log\left(\left(10^{2017!}\right)^x\right) + \ln 2017 - e^{2017} + 1^x\right) =$$
$$\frac{d}{dx}\left(2017!^{2017!} - x\log\left(10^{2017!}\right) + \ln 2017 - e^{2017} + 1\right) =$$
$$\frac{d}{dx}\left(2017!^{2017!} - \ln 2017 - e^{2017} + 1\right) - \frac{d}{dx}\left(x\log\left(10^{2017!}\right)\right) = -2017!$$

2. E. $(2017^{2017!})$

$$\lim_{x\to 0}\frac{\sin^{2017!}(2017x)}{x^{2017!}} = \left(\lim_{x\to 0}\frac{\sin(2017x)}{x}\right)^{2017!} = 2017^{2017!}$$

3. D.

Take the implicit derivative of the equation to get $\frac{1-2017!y^{2017!-1}y\prime}{2017!-1} = 0$.

Plugging in $(2017!, 1)$ gives $y' = \frac{1}{2017!}$.

4. B

Just the ratio of the coefficients of the $x^{2017!}$ Terms giving $\frac{2017!}{2016!} = 2017$.

5. A

The term with the highest order in the summation is x^{2016} meaning that taking 2017 derivatives will make the whole function 0.

6. C

Let us make the substitution $j = 2n$ to get $\lim_{j\to\infty}\left[\sum_{i=1}^{j}\frac{6}{j}\left(\frac{4i}{j}\right)^{\frac{4i}{j}}\left(\ln\left(\frac{4i}{j}\right)+1\right)\right] =$

$\lim_{j\to\infty}\left[\frac{3}{2}\sum_{i=1}^{j}\frac{4}{j}\left(\frac{4i}{j}\right)^{\frac{4i}{j}}\left(\ln\left(\frac{4i}{j}\right)+1\right)\right] = \frac{3}{2}\int_0^4 x^x(\ln(x)+1)$. With some intuition (or by trying integration by parts and taking the derivative of x^x) we see that $\frac{d}{dx}(x^x) = x^x(\ln(x)+1)$. Thus,

$\frac{3}{2}\int_0^4 x^x(\ln(x)+1) = \frac{3}{2}\left(4^4 - \lim_{x\to 0^+}x^x\right) = \frac{3}{2}\left(256 - e^{\lim_{x\to 0^+} x\ln(x)}\right) = \frac{3}{2}\left(256 - e^{\lim_{x\to\infty}\frac{\ln(x)}{x}}\right) =$

$$\frac{3}{2}(256 - e^0) = \frac{765}{2}$$

7. B

We start by calculating the vector each person is traveling at. Mercy is traveling in the direction $\langle -40,30\rangle$ or in the unit vector $\langle -\frac{4}{5},\frac{3}{5}\rangle$ while Roadhog is traveling in the direction $\langle \frac{7}{25},\frac{24}{25}\rangle$. Multiplying each unit vector by the rate of travel yields that Mercy is traveling at $\langle -32,24\rangle$ units/s while Roadhog is traveling at $\langle 7,24\rangle$ units/s. Let's call Mercy's position (x_m, y_m) and Roadhog's position (x_r, y_r). Then, the distance, d, between them can be expressed as $d^2 = (x_m - x_r)^2 + (y_m - y_r)^2$. Taking the derivative of both sides with respect to time and dividing both sides by 2 yields: $dd' = (x_m - x_r)(x_m{}' - x_r{}') + (y_m - y_r)(y_m{}' - y_r{}')$. After 1 second Mercy will be at $(30-32, 40+24) = (-2,64)$ and Roadhog will be at $(7,24)$. This means at 1 second, $d = 41$ hence 9,40,41 Pythagorean triple.

Thus, $d' = \frac{(x_m-x_r)(x_m{}'-x_r{}')+(y_m-y_r)(y_m'-y_r')}{d} = \frac{(-2-7)(-32-7)+(64-24)(24-24)}{41} = \frac{351}{41}$

8. A

$A = \lim\limits_{x\to -2} f(x) = -8, B = \lim\limits_{x\to 0} f(x) = DNE = 1, C = \lim\limits_{x\to 2^-} f(x) = 4\,, D = \lim\limits_{x\to 4^-} f(x) = -16$

$-8+1+4-16 = -19$

9. C

$\frac{d}{dx}\int_{-2x}^{x^2} 2\mathrm{e}^{-t^2}\sin(t)\,dt = \frac{d}{dx}\int_0^{x^2} 2\mathrm{e}^{-t^2}\sin(t)\,dt - \frac{d}{dx}\int_0^{-2x} 2\mathrm{e}^{-t^2}\sin(t)\,dt =$

$2x * 2e^{-(x^2)^2}\sin(x^2) - (-2) * 2e^{-(-2x)^2}\sin(-2x) = -4e^{-4x^2}\sin(2x) + 4xe^{-x^4}\sin(x^2)$

10. C

There are $\frac{(n-1)}{2}$ sets of n chords of equal lengths. For any chosen chord, all points in the smaller subtended arc of the chord will form an obtuse triangle with the chord. Thus, the number of obtuse triangles is equal to $n*0 + n*1 + n*2 + \cdots + n*\frac{n-1}{2} = n\left(\frac{n-1}{2}\right)\left(\frac{n+1}{2}\right)$. Thus the probability is equal to $\lim\limits_{n\to\infty}\frac{n\left(\frac{n-1}{2}\right)\left(\frac{n+1}{2}\right)*\frac{1}{2}}{\binom{n}{3}} = \lim\limits_{n\to\infty}\frac{n\left(\frac{n-1}{2}\right)\left(\frac{n+1}{2}\right)*\frac{1}{2}}{\frac{n!}{3!(n-3)!}} = \lim\limits_{n\to\infty}\frac{\frac{1}{8}n(n-1)(n+1)}{\frac{n(n-1)(n-2)}{6}} = \frac{6}{8} = \frac{3}{4}$

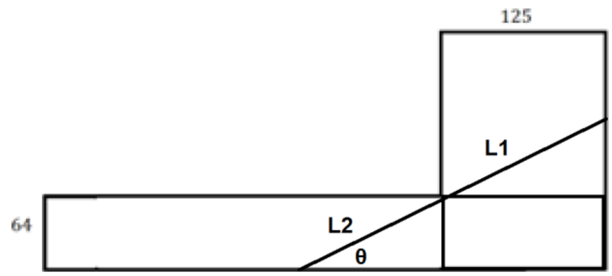
11. A

$\nabla f(x,y,z)\cdot \boldsymbol{u} = |\nabla f(x,y,z)||1|\cos(\theta)$ since $|\boldsymbol{u}| = 1$. In order to minimize the expression we choose $\cos(\theta) = -1$ which means that $\boldsymbol{u}$ is a unit vector pointing in the opposite direction of $\nabla f(x,y,z)$. Thus for a unit vector, we will have $\boldsymbol{u} = -\frac{\nabla f(x,y,z)}{|\nabla f(x,y,z)|} = -\frac{\langle a,b,c\rangle}{|\langle a,b,c\rangle|}$.

12. D

$\frac{dI}{d\sqrt{V_T}} = \frac{\frac{dI}{dV_t}}{\frac{d\sqrt{V_T}}{dV_t}} = \frac{k\left(\frac{W}{L}\right)(V_{GS}-V_T)(1+\lambda V_{DS})}{\frac{1}{2\sqrt{V_T}}} = k\left(\frac{W}{L}\right)2\sqrt{V_T}(V_{GS}-V_T)(1+\lambda V_{DS})$

13. D



We start by writing $L1 = 125\sec(\theta)$, $L2 = 64\csc(\theta)$. The longest ladder that can fit through the hallway is when $L = L1 + L2$ is minimized. Thus, we will take the derivative and set it equal to zero and solve for $\sec(\theta)$ and $\csc(\theta)$, then $L1 + L2$. $125\sec(\theta)\tan(\theta) - 64\csc(\theta)\cot(\theta) = \frac{125\sin^3(\theta) - 64\cos^3(\theta)}{\cos^2(\theta)\sin^2(\theta)} = 0$. So $125\sin^3(\theta) = 64\cos^3(\theta)$ and $\tan^3(\theta) = \frac{64}{125}$ and $\tan(\theta) = \frac{4}{5} = T$ so $\sec(\theta) = \frac{\sqrt{41}}{5}$ and $\csc(\theta) = \frac{\sqrt{41}}{4}$. $L1 + L2 = 25\sqrt{41} + 16\sqrt{41} = 41\sqrt{41} = L$ so $LT = \frac{164\sqrt{41}}{5}$.

14. A

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} e^{-\frac{1}{x}} = 0 = \lim_{x\to 0^-} 0$$

15. A

$$f''(x) = \begin{cases} 0 & , \ x \le 0 \\ -\frac{e^{-1/x}(-1+2x)}{x^4} & , \ x > 0 \end{cases}$$

The $\lim_{x\to 0^+} f''(x) = 0$ due to the $e^{-1/x}$ term decreasing faster than the $1/x^4$ increases. Thus the answer is 0.

16. A

The n^th derivative will always contain some form of $f''(x) = \begin{cases} 0 & , \ x\le 0 \\ -\frac{e^{-\frac{1}{x}}(P(x))}{x^a} & , \ x>0 \end{cases}$ where $P(x)$ is a polynomial. Thus $\lim_{x\to 0^+} f^{(n)}(x) = 0$ due to the $e^{-1/x}$ term decreasing faster than the $1/x^a$ increases so the answer is 0.

17. D

Solve for when $x^{\frac{1}{3}} - 4 = -3 \pm \varepsilon$ gives $x = 1.1^3$ or $0.9^3 = 1.331$ or 0.729 which is 0.331 and 0.271 away from 1. Thus, we choose the smaller value for δ: 0.271.

18. E (-1/4)

$$\frac{\sqrt{2+x}-2}{2-x} = \frac{\sqrt{2+x}-2}{2-x} * \frac{\sqrt{2+x}+2}{\sqrt{2+x}+2} = \frac{x-2}{(2-x)\sqrt{2+x}+2} = -\frac{1}{\sqrt{2+x}+2}. \lim_{x\to 2} -\frac{1}{\sqrt{2+x}+2} = -\frac{1}{4}$$

19. C

We can rewrite $y = x^{x^{x^{\cdot^{\cdot^{\cdot}}}}}$ as $y = x^y$. Take the natural log of both sides gives $\ln(y) = y\ln(x)$. Now doing an implicit differentiation gives $\frac{y'}{y} = y'\ln(x) + \frac{y}{x}$. Solving for $y' = \frac{\frac{y}{x}}{\frac{1}{y} - \ln(x)} = \frac{y^2}{x(1 - y\ln(x))}$.

20. B

$P'(x) = (H'(x) - 1)G'(H(x) - x)F'\big(G(H(x) - x)\big)$ so

$P'(2) = (H'(2) - 1)G'(H(2) - 2)F'\big(G(H(2) - 2)\big) = (-1 - 1)G'(2 - 2)F'\big(G(0)\big)$

$= -2 * 1 * F'(2) = -2 * 1 * 1 = -2$

21. D

$$Q'(x) = \frac{\big(2xF'(x^2)G(x-1) + F(x^2)G'(x-1)\big)H(x) - H'(x)F(x^2)G(x-1)}{H(x)^2}$$

$$Q'(1) = \frac{\big(2F'(1)G(0) + F(1)G'(0)\big)H(1) - H'(1)F(1)G(0)}{H(1)^2}$$

$$= \frac{(2 * 1 * 2 + 2 * 1) * 2 - (-1) * 2 * 2}{4} = \frac{12 + 4}{4} = 4$$

22. B

$$\frac{d^3y}{dx^3} = \frac{d}{dx}\left(\frac{d}{dx}\left(\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right)\right) = \frac{\frac{d}{dt}\left(\frac{\frac{d}{dt}\left(\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right)}{\frac{dx}{dt}}\right)}{\frac{dx}{dt}} = \frac{\frac{d}{dt}\left(\frac{\frac{d}{dt}(\cos(t)\,e^{-t})}{e^t}\right)}{e^t}$$

$$= \frac{\frac{d}{dt}(-\cos(t)\,e^{-2t} - \sin(t)\,e^{-2t})}{e^t} = \frac{\sin(t)\,e^{-2t} + 2\cos(t)\,e^{-2t} - \cos(t)\,e^{-2t} + 2\sin(t)\,e^{-2t}}{e^t}$$

$$= 3e^{-3t}\sin(t) + e^{-3t}\cos(t)$$

23. B

Using the formula:

If $a = c$,

$$\lim_{x\to\infty} \sqrt[n]{ax^n + bx^{n-1} + \ldots} - \sqrt[n]{cx^n + dx^{n-1} + \ldots}$$

$$= \frac{b - d}{na^{(n-1)/n}}$$

We get $\frac{5+2}{\frac{5}{2}(2)^{\frac{\frac{3}{2}}{\frac{5}{2}}}} = \frac{7*2}{5*2^{\frac{3}{5}}} = \frac{7*2^5}{5}$

24. A

Find the derivative by taking the implicit differentiation: $y + xy' + 2xy^3 + 3x^2y^2y' = 0$. Solve for y' and plug in the point (1,2) to get $2 + y' + 16 + 12y' = 0$ so $y' = -\frac{18}{13}$. This means that the slope of the perpendicular line will be $\frac{13}{18}$. Using point slope form we get that the line is $y - 2 = \frac{13}{18}(x-1)$ so $-23 - 13x + 18y = 0$.

25. E $(-1)^n a^n (n)!$

The $(n+1)^{\text{th}}$ derivative can be written as $\frac{d}{dx}\left(\frac{P_n(x)}{(x^a-1)^{n+1}}\right) = \frac{P_n'(x)(x^a-1)^{n+1} - P_n(x)(n+1)(x^a-1)^n a x^{n-1}}{(x^a-1)^{2n+2}}$ $= \frac{P_n'(x)(x^a-1) - P_n(x)(n+1)ax^{n-1}}{(x^a-1)^{n+2}} = \frac{P_{n+1}(x)}{(x^a-1)^{n+2}}$. This means $P_n'(x)(x^a - 1) - P_n(x)(n+1)ax^{n-1} = P_{n+1}(x)$. Plugging in $x = 1$ gives $-P_n(1)(n+1)a = P_{n+1}(1)$. Starting with $n = 0$ we have $-P_0(1)(0+1)a = P_1(1) = -1$ since $P_0(1) = 1$ from $\frac{1}{x^a-1}$. $-P_1(1)(1+1) = P_2(1) = 2$. We then see a pattern which gives us $P_n(1) = (-1)^n a^n (n)!$

26. B

Using Stirling's approximation given on the front page, $\lim_{n\to\infty} \frac{P_{n+1}(1)}{(-a)^{n+2}\left(\frac{n^{n+1}}{e^{n+1}}\right)\sqrt{2\pi(n+1)}} =$

$$\lim_{n\to\infty} \frac{(-1)^{n+1}a^{n+1}(n+1)!}{(-a)^{n+2}\left(\frac{n^{n+1}}{e^{n+1}}\right)\sqrt{2\pi(n+1)}} = \frac{(-1)^{n+1}a^{n+1}\sqrt{2\pi(n+1)}\left(\frac{(n+1)}{e}\right)^{n+1}}{(-a)^{n+2}\left(\frac{n^{n+1}}{e^{n+1}}\right)\sqrt{2\pi(n+1)}} =$$

$$\frac{-\left(\frac{n+1}{n}\right)^{n+1}}{a} = \frac{-\left(1+\frac{1}{n}\right)^{n+1}}{a} = -\frac{e}{a}$$

27. C

$\frac{d}{dx}\cosh(x) = \sinh(x)$, $\frac{d}{dx}\sinh(x) = \cosh(x)$. $\cosh(0) = 1, \sinh(0) = 0$. The first few terms of $\cosh(x) = 1 + \frac{1}{2}x^2 + \frac{1}{4!}x^4 + \cdots = \sum_{k=0}^{\infty}\frac{x^{2k}}{(2k)!}$

28. B

Let's make the substitution that $x^{1009} = \tan(u)$. $1009x^{1008}dx = \sec^2(u)du$. Then, our integral $\lim_{b\to\infty}\int_{-1}^{b}\left[\left(\frac{x^{1513}}{1+x^{2018}}\right)^2\right]dx$ becomes $\int_{-\frac{\pi}{4}}^{\frac{\pi}{2}}\left[\frac{x^{3026}}{(1+\tan^2(u))^2}\right]dx = \int_{-\frac{\pi}{4}}^{\frac{\pi}{2}}\left[\left(\frac{x^{1008}x^{2018}dx}{\sec^4(u)}\right)\right] = \int_{-\frac{\pi}{4}}^{\frac{\pi}{2}}\frac{\tan^2(u)}{1009\sec^2(u)}du$

$$= \frac{1}{1009}\int_{-\frac{\pi}{4}}^{\frac{\pi}{2}}\sin^2(u)\,du = \frac{1}{1009}\int_{-\frac{\pi}{4}}^{\frac{\pi}{2}}\left(\frac{1-\cos(2u)}{2}\right)du = \frac{1}{1009}\left[\frac{2u+\sin(2u)}{4}\right]_{-\frac{\pi}{4}}^{\frac{\pi}{2}} = \frac{-2+3\pi}{8072}$$

29. C

We can rewrite the differential equation as $(e^{-x}\sin(y) - e^{-x}\cos(y)\,y') + (y + xy') + y' = 0$. Then we see that $(e^{-x}\sin(y) - e^{-x}\cos(y)\,y')$ is the implicit differentiation of $e^{-x}\sin(y) = C$ and $(y + xy') + y'$ is the implicit differentiation of $xy + y = C$. Thus the solution is

$-e^{-x}\cos(y) + \mathrm{xy} + y = C$. Finding **a** solution we can have $-e^{-x}\cos(y) + \mathrm{xy} + y = 3$.

30. E

The left hand limit is undefined since $\arctan(x)$ will be negative. Thus, the limit does not exist.