For each question, "E) NOTA" indicates that none of the above answers is correct.
The phrase "DNE" is to be interpreted as "Does not exist."

1. Find every point on the surface of the ellipsoid $x^2 + 4y^2 + 9z^2 = 16$ at which the normal line at the point passes through the center $(0,0,0)$ of the ellipsoid. How many such points exist?

A) No Points B)1 C) 2 D) 4 E) NOTA

2. Find the volume of the region bounded by the paraboloid $z = 9 - \mathrm{x}^2 - y^2$, the xy-plane, and inside the cylinder $\mathrm{x}^2 + y^2 = 4$.

A) 0 B) 20π C) 28π D) 32π E) NOTA

3. The double integral $\int_0^4 \int_{x/2}^2 f(x,y)dydx$ can be expressed as a new integral with the order of integration reversed. This double integral is of the form $\int_A^B \int_C^{Dy} f(x,y)dxdy$. What is $A + B + C + D$?

A) 2 B) 4 C) 6 D) 8 E) NOTA

4. Evaluate the following limit:

$$\lim_{(x,y)\to(0,0)} \frac{x^4 + y^4}{(x^2 + y^2)^{\frac{3}{2}}}$$

A) DNE B) -1 C) 0 D) 1 E) NOTA

5. A multiple integral that gives the volume of the solid bounded by the surface with spherical equation $\rho = 1 - \cos\phi$ is of the form $\int_F^G \int_D^E \int_A^{Bf(x)+C} \rho^2 \sin\phi d\rho d\phi d\theta$. What is the closest integer to the sum $A + B + C + D + E + F + G$? Assume $0 \le \theta \le \pi$ and $0 \le \phi \le 2\pi$.

A) 5 B) 6 C) 7 D) 8 E) NOTA

6. Consider the following function of two variables: $f(x,y) = 5x^2 e^y \sqrt{10x^2y^3 + 1} + 5xye^x$.
Evaluate the following expression: $\left(f_{xy}(0,0)\right)^2 - 2\left(f_{xy}(0,0)\right)\left(f_{yx}(0,0)\right) + \left(f_{yx}(0,0)\right)^2$.

A) 0 B) 1 C) 12 D) 25 E) NOTA

7. Consider the three dimensional surface $z = x^2 + 2xy + 2y^2 - 6x + 8y$. There is only one point on the surface such that the tangent plane is horizontal. Find the sum of the x and y coordinates of such a point.

A) -3 B) 3 C) 5 D) 10 E) NOTA

8. Suppose that a drone is free to move in three dimensional space. This drone is designed so that it is most effective at higher temperatures. Suppose that it is known that the temperature at a given point (x, y, z) is given by the temperature function $T(x, y, z) = 92 + xyz$. Given this function and that the drone is at the point $(3,4,1)$, there is a particular direction that the drone should fly to be most effective (express this direction as a three dimensional vector). What is the sum of the coordinates of this unit vector?

A) $3/13$ B) $12/13$ C) $19/13$ D) 2 E) NOTA

9. Given the following information, find a vector $\vec{v}$ such that the following are satisfied. What is the sum of the absolute value of the coordinates?

- $\vec{v} \times < 3,2,1 > =< 1,1,-5 >$
- $\vec{v} \times < 2,3,2 > =< 3,-2,0 >$
- $\left||\vec{v}|\right| = \sqrt{14}$

A) 4 B) 6 C) 8 D) 9 E) NOTA

10. Suppose f is a function of x, y and z, with $x = u^2 + v$, $y = u + v^2$, and $z = uv$. In addition, this function has additional properties such that $\frac{\partial f}{\partial u} = 2$ at $(u, v) = (0,1)$ and $\frac{\partial f}{\partial z} = 5$ at $(x, y, z) = (1,1,0)$. What is the value of $\frac{\partial f}{\partial y}$ at $(x, y, z) = (1,1,0)$?

A) -3 B) -2 C) $2/5$ D) 3 E) NOTA

For questions 11 and 12 refer to the following information:

Let D be a solid described in the spherical coordinates by the inequalities

$$\sec\phi \le \rho \le 2\cos\phi$$

11. A triple integral formula to calculate $\iiint_D dV$ in spherical coordinates is of the form:

$$\int_E^F \int_C^D \int_{Ag(\phi)}^{Bh(\phi)} \rho^2 \sin\phi \, d\rho d\phi d\theta$$

$A + B + C + D + E + F$ is closest to which integer? Assume $0 \le \theta \le \pi$ and $0 \le \phi \le 2\pi$.

A)8 B)10 C)15 D)18 E) NOTA

12. Rewrite the triple integral formula with respect to Cartesian coordinates. This formula takes the form: $\int_B^C \int_{h(x)}^{f(x)} \int_A^{g(x,y)} dzdydx$. What is the z-coordinate of the maximum value attained by $f(x) + g(x,y) + h(x) + A + B + C$?

A) 2 B) 5 C) 9 D) 13 E) NOTA

13. Suppose there is an inward oriented surface S that takes the shape of a left handed glove with the fingers pointing up, the thumb pointing in the direction of the x-axis, and the opening at the wrist is the curve in the xy-plane with equation $4x^2 + 9y^2 = 36$. Compute the flux of the vector field through the surface of the glove.

$$\vec{F}(x,y,z) =< y^2z - z^2, 4 - xy, 3 + xz >$$

A) -18π B) -3π C) -3 D) 0 E) NOTA

14. Find the angle θ between the planes with equations.

$$2x + 3y - z = -3 \text{ and } 4x + 5y + z = 1$$

A) $\cos^{-1}(\frac{11}{21}\sqrt{3})$ B) 0 C) $\cos^{-1}(\frac{3}{27}\sqrt{3})$ D) $\pi/4$ E) NOTA

15. Compute the line integral below:

$$\int_C 7ydx + 3xdy$$

Where C is the closed path that consists from the line segment from $(0,0)$ to $(1,2)$ followed by the line segment from $(1,2)$ to $(2,0)$ followed by the line segment from $(2,0)$ to $(0,0)$.

A) -8 B)-5 C) 5 D) 8 E) NOTA

16. The following function of two variables has how many saddle points?

$$f(x,y) = x^4 + y^4 - 4xy$$

A) 0 B)1 C) 2 D) 3 E) NOTA

17. Given that $\vec{F}(x,y,z) = (y^2 + z^2)\hat{\imath} + (x^2 + z^2)\hat{\jmath} + (x^2 + y^2)\hat{k}$. Evaluate div $\vec{F}$ + $||\text{curl } \vec{F}||$ at the point $(x,y,z) = (1,1,1)$.

A) 0 B) 1 C) 4 D) 6 E) NOTA

18. Suppose that a circuit contains two resistors with resistances R_1 and R_2, respectively. When they are connected in parallel, the total resistance R of the resulting circuit obeys the equation $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$. Suppose that $R_!$ and R_2 are experimentally measured to be 300 and 600 Ω, respectively, with a maximum error of 1% in each measurement. Use differentials to estimate the maximum error (in Ω) in the calculated value of the total resistance.

A) No Error B) $\frac{2}{3}$ C) $\frac{4}{3}$ D) $\frac{5}{3}$ E) NOTA

19. Evaluate the following limit:

$$\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^4+y^4}$$

A) DNE B) -1 C) 0 D) 1 E) NOTA

20. Evaluate the double integral:

$$\int\int_R f(x,y)dA$$

where $f(x,y) = (x+1)(2y+2x-x^2)$ and R is region bounded by the two inequalities

$$-2 \le y - x^2 \le 1 \qquad 1 \le y + 2x \le 3$$

A) $-9/2$ B) -9 C) 9 D) $9/2$ E) NOTA

21. Let S be the surface of the cylinder formed by the planes $z = 0$ and $z = 3$ and the cylinder $x^2 + y^2 = 4$. Calculate the outward flux

$$\Phi = \int\int_S \vec{F} \cdot \vec{n} dS$$

Given that $\vec{F} = (x^2+y^2+z^2) < x, y, z >$.

A) -30π B) 30π C) 200π D) 300π E) NOTA

22. Find an equation for the plane through the points $(1,3,5)$, $(-1,2,4)$, and $(4,4,0)$.

A) $6x - 13y + z = -28$ B) $6x + 13y + z = 50$
C) $3x - 13y + z = -28$ D) $6x - 13 + z = -3$ E) NOTA

23. If $f(x,y)$ is a continuous throughout the rectangular region R $0 \le x \le 2$ and $-1 \le y \le 1$. Whose theorem guarantees this fact:

$$\int_0^2 \int_{-1}^1 f(x,y)dydx = \int_{-1}^1 \int_0^2 f(x,y)\, dxdy$$

A) Stokes B) Gauss C) Fubini D) Leibniz E) NOTA

24. Given that work is defined as follows:

$$W = \oint_C \vec{F} \cdot \hat{T} ds$$

Find the work done by the force field $\vec{F} = -y\hat{\imath} + x\hat{\jmath}$ in moving a particle counterclockwise once around the unit circle in the xy-plane.

A) 0 B) π C) $\pi/2$ D) 2π E) NOTA

25. Find the volume bounded by the paraboloids $z = x^2 + y^2$ and $z = 4 - 3x^2 - 3y^2$.

A) 0 B) π C) $\pi/2$ D) 2π E) NOTA

26. Suppose $F(x, y, z)$ is a smooth, continuous function of three variables for which $\nabla F(1, -1, \sqrt{2}) =< 1,2, -2 >$. Evaluate $\frac{\partial F}{\partial \phi}$ at the point $(\rho, \phi, \theta) = (2, \frac{\pi}{4}, -\frac{\pi}{4})$.

A)$-\sqrt{2}$ B) $-2\sqrt{2}$ C) $\sqrt{2}$ D) $2\sqrt{2}$ E) NOTA

27. Evaluate the following double integral:

$$\int_0^3 \int_{x^2}^9 x e^{-y^2} dy dx$$

A)$\frac{1}{4}(e^{-81} - 1)$ B) $\frac{1}{4}(1 - e^{-81})$ C) $e^{-81} - 1$ D) $1 - e^{-81}$ E) NOTA

28. Evaluate the single variable integral:

$$\int_0^\infty e^{-x^2} dx$$

(HINT: Let $X = \int_0^\infty e^{-x^2} dx$ and consider the double integral $\int_0^\infty \int_0^\infty e^{-x^2} e^{-y^2} dy dx$ and the polar coordinate system)

A) 0 B) $\frac{\sqrt{\pi}}{2}$ C)) $\frac{\pi}{2}$ D) $\sqrt{\pi}$ E) NOTA

29. The equation of the plane containing the point (1,1,1) that also intersects the xy −plane in the same line as the plane $3x + 2y - z = 6$ is in the form $Ax + By + Cz = D$ where $A > 0$ and $A, B, C,$ and D are relatively prime integers. What is $A + B + C + D$?

A) 5 B) 8 C) 12 D) 15 E) NOTA

30. There is a curve C in xyz-space parameterized by the function $\vec{x}(t) =< t^2 + 1, 2t - 3t^2, t^3 >$ and starting at $t = 0$ and finishing at $t = 1$. Compute the line integral $\int_C F \cdot \hat{T} ds$ along curve C of the vector field $\vec{F}(x, y, z) =< ye^{xy} + z, xe^{xy}, x >$.

A) e^{-2} B) e^{-4} C) $e^{-2} + 1$ D) $e^{-4} + 1$ E) NOTA