1. Given that the sum converges, evaluate $\sum_{n=0}^{\infty}\left(\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{-2nx}\right)$.

A) $\frac{e}{e-1}$ B) $\frac{e^2}{e^2-1}$ C) $\frac{e^3}{e^3-1}$ D) $\frac{e^4}{e^4-1}$
E) NOTA

2. Find the next term in the sequence: $a_0=1,\ a_1=11,\ a_2=21,\ a_3=1211$. (Hint: Speak-and-say the terms out loud. Internal monologue everyone!)

A) 111221 B) 112112 C) 112121 D) 112221
E) NOTA

3. You're playing a popular turn-based card game. As per the rules of the game, at the end of each of your turns, you put X zombie tokens into play, where X is two plus the number of zombie tokens you already have in play. Given that you had two zombie tokens in play at the beginning of your fifth turn (assuming you don't lose any zombie tokens throughout the game), how many tokens will you have at the beginning of your fifteenth turn?

A) 1022 B) 2046 C) 4094 D) 8190
E) NOTA

4. Find the number of sequences of length n that can be formed from a k-element set. For example, if my set is $\{a,b,c\}$ then one such sequence of length 4 is $aabc$.

A) n^k B) k^n C) $\frac{n!}{k!}$ D) $\frac{n!}{k!(n-k)!}$
E) NOTA

5. Let $S_n=a_1+a_2+\cdots+a_n$. Suppose that $S_n=(1+2n)^{\frac{-4}{n}}$. Find $\sum_{n=1}^{\infty}a_n$.

A) 0 B) 1 C) e^{-6} D) e^{-8}
E) NOTA

6. Find the interval of convergence for $\sum_{n=3}^{\infty}\frac{(x-3)^{2n}}{n^n}$.

A) $[-3,3]$ B) $[-3,3)$ C) $(-3,3]$ D) $(-3,3)$
E) NOTA

7. Let a_n be an infinite sequence such that $a_{n+1}=a_n+\frac{1}{2^n}$, $a_1=1$. Find $\lim_{n\to\infty}a_n$.

A) 1 B) 2 C) 4 D) 8
E) NOTA

8. How many of the following statements is/are true for $a_n \neq 0$:
   a. If $\{a_n\}$ is convergent, then $\sum a_n$ is convergent.
   b. If $\{a_n\}$ is divergent, then $\sum a_n$ is divergent.
   c. If $\sum (a_n + b_n)$ is convergent, then $\sum a_n$ is convergent.
   d. If $\sum a_n$ is convergent, then $\sum\left(a_n + \frac{1}{n}\right)$ is divergent.

A) 1 B) 2 C) 3 D) 4
E) NOTA

9. Evaluate $\sum_{n=1}^{\infty}\left(\cos\left(\frac{1}{n}\right) - 2\cos\left(\frac{1}{n+1}\right) + \cos\left(\frac{1}{n+2}\right)\right)$

A) $\cos\left(\frac{1}{2}\right) - \cos(1)$ B) $\cos(1) - \cos\left(\frac{1}{2}\right)$ C) $\cos(1) - \cos\left(\frac{1}{2}\right) + 1$
D) $\cos\left(\frac{1}{2}\right) - \cos(1) + 1$ E) NOTA

10. Consider the alternating series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n(n+1)}$. Find the least number of terms necessary to ensure the difference between the actual sum, s, and the nth partial sum, S_n, is less or equal to $\frac{1}{49}$.

A) 3 B) 4 C) 5 D) 6
E) NOTA

11. Determine how many of the following sequences <u>diverges</u>?

a. $\{\ln(2n) - \ln(n+1)\}$ b. $\left\{\frac{\cos\left(\sqrt{n}\right)}{n^2}\right\}$ c. $\left\{\frac{n\sin n}{n^2 - 1}\right\}$ d. $\left\{\frac{3^n}{n^{100}}\right\}$ e. $\left\{\frac{2^n}{n^n}\right\}$

A) 2 B) 3 C) 4 D) 5
E) NOTA

12. How many of the following series are <u>convergent</u>?

a. $\sum_{n=3}^{\infty}\frac{1}{n\ln n}$ b. $\sum_{n=3}^{\infty}\frac{1}{n^5\ln n}$ c. $\sum_{n=3}^{\infty}\frac{(-1)^n}{n(\ln n)^2}$ d. $\sum_{n=3}^{\infty}\frac{1}{n(\ln n)^{1/3}}$

A) 1 B) 2 C) 3 D) 4
E) NOTA

13. The sequence $\sqrt{5}, \sqrt{5\sqrt{5}}, \sqrt{5\sqrt{5\sqrt{5}}}, \ldots$ is monotonically increasing and bounded above. Find the limit of the sequence.

A) $\sqrt{5}$ B) 5 C) 25 D) 125 E) NOTA

14. Suppose $\sum_{n=1}^{\infty} a_n = 4$ and S_n is the *nth* partial sum of the series. Let $L = \lim_{n\to\infty} a_n$ and $s = \lim_{n\to\infty} S_n$. What is the value of $L + s$?

A) 4 B) 8 C) 16 D) 32
E) NOTA

15. For what values of x does the series $\sum_{n=1}^{\infty}\left(\frac{5x-2}{3}\right)^n$ converge?

A) $-\frac{1}{5} \le x \le 1$ B) $-\frac{1}{5} \le x < 1$ C) $-\frac{1}{5} < x \le 1$ D) $-\frac{1}{5} < x < 1$
E) NOTA

16. Determine all integral values of k such that the series $\sum_{n=5}^{\infty} \frac{n^3+2n^2}{\sqrt[4]{n^k-1}}$ will converge:

A) $k > 7$ B) $k > 12$ C) $k > 16$ D) $k > 27$
E) NOTA

17. Evaluate $\sum_{k=1}^{\infty} \frac{k^2}{2^k}$

A) $\ln 2$ B) 1 C) e D) 6
E) NOTA

18. How many of the following series converge?

a. $\sum_{n=3}^{\infty} \frac{\tan(1/n)}{n}$ b. $\sum_{n=3}^{\infty} n^2 \sin(1/n^3)$ c. $\sum_{n=3}^{\infty} \frac{\sin(1/n)}{n^2}$ d. $\sum_{n=3}^{\infty} \cos(1/n)$ e. $\sum_{n=3}^{\infty} \frac{\sin(1/n)}{n\cos(1/n^2)}$

A) 2 B) 3 C) 4 D) 5
E) NOTA

19. Find the Taylor series for $f(x) = \ln(4x^2 - 12x + 13)$ centered at $a = \frac{3}{2}$ (you may use $\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}x^n}{n}$, IOC $(-1,1]$.)

A) $\ln(4) + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}\left(x-\frac{3}{2}\right)^{2n}}{n}$ B) $\ln(4) + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}\left(x-\frac{3}{2}\right)^{2n}}{2n}$

C) $\ln(4) + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}\left(x-\frac{3}{2}\right)^{2n}}{2n-1}$ D) $\ln(4) + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}\left(x-\frac{3}{2}\right)^{2n}}{2n+1}$ E) NOTA

20. Express the integral $\int_0^x \frac{e^{t^2}-1-t^2}{t^4}\,dt$ in terms of a power series.

A) $\sum_{n=2}^{\infty} \frac{x^{2n-1}}{n!(2n-1)}$ B) $\sum_{n=2}^{\infty} \frac{x^{2n-2}}{n!(2n-2)}$ C) $\sum_{n=2}^{\infty} \frac{x^{2n-3}}{n!(2n-3)}$ D) $\sum_{n=2}^{\infty} \frac{x^{2n-4}}{n!(2n-4)}$
E) NOTA

21. Find the 42nd derivative of $f(x)=\sin\left(\frac{x^2}{2}\right)$ at $x = 0$.

A) $\frac{42!}{2^{21}21!}$ B) $\frac{40!}{2^{20}20!}$ C) $\frac{38!}{2^{19}19!}$ D) $\frac{36!}{2^{18}18!}$
E) NOTA

22. Find the sum of the series $\frac{8}{3!}-\frac{32}{5!}+\frac{128}{7!}-\cdots$.

A) $\sin 2 - 2$ B) $2 - \sin 2$ C) $2 - \cos 2$ D) $\cos 2 - 2$
E) NOTA

23. Find a power series representation for $\frac{x^3}{(3-x)^2}$.

A) $\sum_{n=0}^{\infty} \frac{(n+1)x^n}{3^n}$ B) $\sum_{n=0}^{\infty} \frac{(n+1)x^{n+2}}{3^{n+1}}$ C) $\sum_{n=0}^{\infty} \frac{(n+1)x^{n+3}}{3^n}$ D) $\sum_{n=0}^{\infty} \frac{(n+1)x^{n+3}}{3^{n+2}}$
E) NOTA

24. Evaluate the sum $\sum_{n=2}^{\infty} \frac{(-1)^n \pi^{2n}}{3^{2n+1}(2n)!}$ using a known Maclaurin series.

A) $\frac{\pi}{2}+\frac{1}{6}$ B) $\frac{\pi^2}{54}-\frac{1}{6}$ C) $\frac{\pi^4}{162}-\frac{\pi^2}{54}+\frac{1}{6}$ D) $\frac{\pi^4}{162}+\frac{\pi^2}{54}-\frac{1}{6}$
E) NOTA

25. Find the sum of the squares of the least ten distinct positive even integers.

A) 1337 B) 1624 C) 1540 D) 1729
E) NOTA

26. Let $A = \sum_{n=0}^{\infty}\left(\frac{3}{10}\right)^n$. Evaluate the error if the first one hundred terms are used to approximate the summation.

A) $\frac{3^{103}}{10^{102}\times 7}$ B) $\frac{3^{102}}{10^{101}\times 7}$ C) $\frac{3^{101}}{10^{100}\times 7}$ D) $\frac{3^{100}}{10^{99}\times 7}$
E) NOTA

27. Suppose that $\sum c_n(x-2)^n$ converges for $x=4$ and diverges for $x=-2$. Which of the following must be correct?

A) $\sum c_n(-2)^n$ converges B) $\sum c_n 4^n$ diverges C) $\sum c_n 3^n$ converges
D) $\sum c_n(-1)^n$ converges E) NOTA

28. Evaluate the sum $\ln\left(1+i\pi-\frac{\pi^2}{2!}-\frac{i\pi^3}{3!}+\frac{\pi^4}{4!}+\ldots\right)$, allowing for complex logarithms.

A) -1 B) $i\pi$ C) $\frac{i\pi}{2}$ D) $\frac{i\pi}{4}$
E) NOTA

29. Determine the common ratio of a geometric series with first term $1+i$ and sum $3-i$.

A) $\frac{3+4i}{10}$ B) $\frac{3-4i}{10}$ C) $\frac{4+2i}{5}$ D) $\frac{4-2i}{5}$
E) NOTA

30. Evaluate $\lim_{n\to\infty}\sum_{i=0}^{n}\frac{i^{0.5}}{n^{1.5}}$

A) 0 B) $\frac{1}{3}$ C) $\frac{2}{3}$ D) 1
E) NOTA