For all questions, answer choice "E) NOTA" means that none of the above answers is correct.

1. In what base b, with $b > 7$, is 3 times 7 equal to 18?

A) 9 B) 11 C) 13 D) 15 E) NOTA

2. How many positive integers divide 15!?

A) 2048 B) 3168 C) 4032 D) 4096 E) NOTA

3. How many different rectangles whose length and width are both integers have their area equal to three times their perimeter?

A) 1 B) 2 C) 4 D) 5 E) NOTA

4. Suppose a and b are nonnegative integers. What is the largest integer n such that n cannot be represented as $8a + 15b$?

A) 89 B) 97 C) 117 D) 119 E) NOTA

5. What is the largest prime factor of $3^{12} - 1$?

A) 7 B) 13 C) 31 D) 73 E) NOTA

6. When 2017 (in base 10) is expressed in base 7, what is the base-10 average of those base-7 digits?

A) 2.4 B) 2.6 C) 3 D) 3.25 E) NOTA

7. Let's call a positive integer *scrunchy* if it is equal to the sum of its digits plus the product of its digits. For example, 19 is scrunchy because $19 = 1 + 9 + 1 \cdot 9$. How many scrunchy numbers are there?

A) 9 B) 12 C) 18 D) infinite E) NOTA

8. In kindergarten, 17 children made an even number of total postcards. Any group of 5 children made no more than 25 postcards, while any group of 3 children made no less than 14. Determine the total number of postcards made.

A) 83 B) 84 C) 85 D) 86 E) NOTA

9. How many two-digit numbers double when the two digits are interchanged?

A) 1 B) 2 C) 3 D) 4 E) NOTA

10. Which one of the following positive integers is prime?

A) 20142018 B) 20172017 C) 20172018 D) 20172020 E) 20182021

11. Suppose $M = A + \theta$, where M, A, and θ are prime numbers with M greater than 3000 and A and θ each less than 3000. Compute $M + A + \theta$.

A) 6002 B) 6004 C) 6006 D) 6008 E) NOTA

12. You are in charge of designing a new system of coins for the independent country of Vestaviaville. You are allowed to design coins with any integral value, but the supreme leader of Vestaviaville demands that any integral amount from 1 to 99 cents be obtainable without using any denomination of coin twice. What is the smallest number of denomination of coins that you must have?

A) 5 B) 6 C) 7 D) 8 E) NOTA

13. If $4^3 \cdot 6^2 \cdot 12^5 = k^2 \cdot 2^8 \cdot 3^5$ then find the greatest possible value of k.

A) 32 B) 48 C) 96 D) 144 E) NOTA

14. How many different values of $a + b + c$ are there if a, b, and c are positive integers and $abc = 24$?

A) 4 B) 6 C) 8 D) 12 E) NOTA

15. N is the smallest positive integer for which $275 \cdot N$ (base 10) is a perfect cube. Write N in octal.

A) 100 B) 605 C) 752 D) 1075 E) NOTA

16. The square of an integer k has tens digit 7. What is the units digit of k^2?

A) 2 B) 4 C) 6 D) 8 E) NOTA

17. What is the largest prime factor of 27001?

A) 31 B) 67 C) 301 D) 871 E) NOTA

18. Compute 17^{2019} modulo 2017.

A) 1 B) 17 C) 289 D) 2016 E) NOTA

19. Of all positive factors of 18, how many are quadratic residues in modulo 19?

A) 1 B) 3 C) 4 D) 5 E) NOTA

20. What is the largest prime p such that p^2 divides $28! + 29! + 30!$?

A) 5 B) 13 C) 23 D) 31 E) NOTA

21. If x, y, and z are distinct primes, then what is the smallest value of $x^y + z$?

A) 5 B) 11 C) 13 D) 17 E) NOTA

22. Let N be the smallest positive number which satisfies each of $x \equiv 10 \pmod{13}$, $x \equiv 3 \pmod{9}$, and $x \equiv 5 \pmod{7}$. What is the sum of the digits of N?

A) 9 B) 10 C) 11 D) 12 E) NOTA

23. P is the smallest positive integer which has exactly 7 positive divisors. Q is the smallest positive integer which has exactly 9 positive divisors. Compute $P + Q$.

A) 60 B) 68 C) 80 D) 100 E) NOTA

24. Define the Fibonacci numbers recursively by $F_0 = F_1 = 1$ and $F_{n+2} = F_{n+1} + F_n$ for $n \geq 0$. Compute F_{2017} modulo 5.

A) 1 B) 2 C) 3 D) 4 E) NOTA

25. Given $1! + 2! + 3! + \cdots + x! = y^z$, where x, y, and z are each integers greater than 1, compute $x + y + z$.

A) 8 B) 9 C) 29 D) 81 E) NOTA

26. Suppose G is the greatest common divisor of 5913 and 7592, C is the greatest common divisor of 481 and 851, and D is the greatest common divisor of 1827 and 3248. Compute $G + C + D$.

A) 209 B) 210 C) 243 D) 297 E) NOTA

27. A pair of positive integers *P* and *Q* are called *amicable* if all the proper divisors of *P* sum to *Q*, and all the proper divisors of *Q* sum to *P*. The number 1184 is part of an amicable pair. Which of the following is its partner?

A) 1210 B) 1256 C) 1481 D) 2394 E) NOTA

28. Solve the following problem which was written by the Hindu mathematician Mahāvīra around the year 850: "Into the bright and refreshing outskirts of a forest, which was full of numerous trees with their branches bent down with the weight of flowers and fruits, trees such as jambu trees, lime trees, plantains, areco palms, jack trees, date palms, hintala trees, palmyras, punnāgo trees, and mango trees—outskirts, the various quarters whereof were filled with many sounds of crowds of parrots and cuckoos found near springs containing lotuses with bees roaming about them—into such a forest outskirts a number of weary travelers entered with joy. There were 63 numerically equal heaps of plantain fruit put together and combined with 7 more of those same fruits, and these were equally distributed among 23 travelers so as to have no remainder. You will tell me now the numerical measure of a heap of plantains." (give the smallest possible value)

A) 5 B) 7 C) 11 D) 14 E) 23

29. In 1848, French mathematician Alphonse de Polignac (1826-1863) asserted: "Every odd number can be expressed as the sum of a power of 2 and a prime." Unfortunately, his conjecture was found to be false! Which number below *cannot* be written as the sum of a power of 2 and a prime?

A) 3 B) 119 C) 509 D) 723 E) 1015

30. A positive, base-10 integer *N* is called a *9-13-base-double* if the base-10 number formed by the digits of the base-9 representation of *N* is twice the base-10 number formed by the digits of the base-13 representation of *N*, where the base-9 and base-13 representations have the same number of digits. For example, 177 is a 9-13-base-double because its base-9 and base-13 representations are 216_9 and 108_{13}, and 216 is double 108. What is the base-10 sum of the digits of the greatest 9-13-base-double?

A) 8 B) 9 C) 14 D) 18 E) NOTA