For this test, unless otherwise specified:

- Coins are fair, and cannot land on the edge.
- Dice are fair, six-sided, and numbered 1 through 6.
- A deck of cards contains the standard 52 cards.
- Drawing *N* objects is done at random, and without replacement
- Random number generation has no memory.

1. 2 cards are drawn from a deck. Find the probability that they are of the same suit.

   A. $\frac{1}{17}$ B. $\frac{1}{16}$ C. $\frac{4}{17}$ D. $\frac{1}{4}$ E. NOTA

2. How many 3-digit even numbers have all digits distinct? 0 may not be the leading digit.

   A. 320 B. 328 C. 352 D. 360 E. NOTA

3. Two dice are rolled. What is the probability that the sum or the product of the outcomes is 4 (or both)?

   A. $\frac{1}{12}$ B. $\frac{1}{9}$ C. $\frac{5}{36}$ D. $\frac{1}{6}$ E. NOTA

4. On the Cartesian plane, a point is chosen at random within the region defined by $|x| + |y| \leq 8$. What is the probability it is within the region defined by $x^2 + y^2 \leq 8$?

   A. $\frac{\pi}{32}$ B. $\frac{\pi}{16}$ C. $\frac{\pi}{8}$ D. $\frac{\pi}{4}$ E. NOTA

5. Find the constant term in the expansion of $\left(2x + \frac{1}{x^2}\right)^6$

   A. 15 B. 30 C. 60 D. 240 E. NOTA

6. A positive integral factor of 360 is chosen at random. What is the probability that it is also a factor of 300?

   A. $\frac{1}{6}$ B. $\frac{1}{3}$ C. $\frac{1}{2}$ D. $\frac{2}{3}$ E. NOTA

7. Find the number of terms in the expansion of $(x + y + z)^7$.

A. 28 B. 36 C. 84 D. 120 E. NOTA

8. Find the constant term in the expansion of $\left(x^2 + 1 + \frac{1}{x}\right)^9$

A. 1 B. 84 C. 1596 D. 1597 E. NOTA

9. A committee of 4 is to be selected at random from 5 men and 4 women. What is the probability that both sexes are represented?

A. $\frac{20}{21}$ B. $\frac{121}{126}$ C. $\frac{125}{126}$ D. 1 E. NOTA

10. A bag contains 3 blue, 4 green, and 5 purple marbles. Three marbles are drawn from the bag. What is the probability that all three are of a different color?

A. $\frac{1}{22}$ B. $\frac{3}{22}$ C. $\frac{3}{11}$ D. $\frac{9}{22}$ E. NOTA

11. How many distinguishable permutations of the letters in OTHELLO have the property that the first and last letters are the same?

A. 60 B. 120 C. 240 D. 1260 E. NOTA

12. Three dice are rolled. If none show a 6, what is the probability that none show a 1?

A. $\frac{8}{27}$ B. $\frac{64}{125}$ C. $\frac{125}{216}$ D. $\frac{2}{3}$ E. NOTA

13. How many non-congruent triangles with integer side lengths have perimeter of 22?

A. 8 B. 9 C. 10 D. 11 E. NOTA

For questions 14-15: A rare genetic disorder affects 0.1% of the population. A test is developed to detect this disorder, and the test is correct 99.9% of the time. Marcellus and Bernardo both took the test.

14. Marcellus has the disorder. What is the probability that his test results are positive?

A. 0.001 B. 0.000999 C. 0.5 D. 0.999 E. NOTA

15. Bernardo's test results are positive. What is the probability that he has the disorder?

A. 0.001 B. 0.000999 C. 0.5 D. 0.999 E. NOTA

16. A 4-foot tall snowman is built by placing a sphere of radius 6 inches on top of a sphere of radius 8 inches on top of a sphere of radius 10 inches. A point is chosen at random on the surface of the snowman. Find the probability that it is farther off the ground than the center of the 8-inch sphere.

A. $\frac{59}{216}$ B. $\frac{1}{3}$ C. $\frac{17}{50}$ D. $\frac{5}{12}$ E. NOTA

17. Six people sit around a circular table. Romeo and Juliet wish to sit together. Julius does not wish to sit next to Brutus. Rosencrantz and Guildenstern do not care where they sit. In how many distinguishable ways can they be seated? Two seating arrangements are distinguishable if one cannot be rotated parallel to the floor to form the other.

A. 12 B. 24 C. 48 D. 120 E. NOTA

18. A circle of radius 1 is internally tangent to a circle with radius 2. A chord of the larger circle is selected at random. If the chord is tangent to the smaller circle, what is the probability that its length is no more than $\sqrt{15}$?

A. $\frac{1}{4}$ B. $\frac{1}{2}$ C. $\frac{3}{4}$ D. 1 E. NOTA

19. A group of 16 people, Beatrice and Benedict among them, are divided into 4 groups of 4 people each. What is the probability that Beatrice and Benedict are in the same group?

A. $\frac{1}{20}$ B. $\frac{1}{16}$ C. $\frac{1}{5}$ D. $\frac{1}{4}$ E. NOTA

20. How many 4 digit numbers are there that start with 6, are divisible by 3, and have all four digits distinct? The leading digit cannot be 0.

A. 162 B. 168 C. 180 D. 504 E. NOTA

21. $\theta$ is randomly chosen on $[0, 2\pi)$. Find the probability that $\sin\theta > \tan\theta$ and $\sin\theta > \cos\theta$.

A. $\frac{1}{8}$ B. $\frac{1}{4}$ C. $\frac{3}{8}$ D. $\frac{1}{2}$ E. NOTA

22. 2 random real numbers are generated on the interval $[0, 1)$. Find the probability that their sum is at least twice the absolute value of their difference.

A. $\frac{1}{6}$ B. $\frac{1}{3}$ C. $\frac{1}{2}$ D. $\frac{2}{3}$ E. NOTA

23. A coin is flipped 6 times. If more heads are flipped than tails, what is the probability that all 6 are heads?

A. $\frac{1}{35}$ B. $\frac{1}{32}$ C. $\frac{1}{29}$ D. $\frac{1}{22}$ E. NOTA

24. Each of 3 identical looking bags contains some marbles. Bag A contains 3 red and 5 blue marbles. Bag B contains 4 blue and 7 white marbles. Bag C contains 2 white and 4 red marbles. Ophelia selects a bag and random, then draws a marble from the bag. If she drew a red marble, what is the probability that she chose bag A? Each bag is equally likely to be selected.

A. $\frac{1}{3}$ B. $\frac{9}{25}$ C. $\frac{3}{7}$ D. $\frac{1}{2}$ E. NOTA

25. A positive integer *n* is selected at random such that $n \leq 1000$. What is the probability that there exists some integer *m* such that the decimal representation of $\frac{m}{n}$ is a purely repeating decimal where the repeating cycle is of length 6 but could not be considered repeating of a shorter length? For example, $.\overline{123456}$, not $.\overline{333333} = .\overline{3}$.

A. 0.208 B. 0.209 C. 0.218 D. 0.219 E. NOTA

26. Falstaff the knight is currently at $(0, 0)$. Every second, he can move a total of 1 in the x direction and 2 in the y direction, or a total of 2 in the x direction and 1 in the y direction. For example, from $(0, 0)$, he can move to one of 8 locations – $(\pm 1, \pm 2)$ and $(\pm 2, \pm 1)$. How many ways can Falstaff move to $(12, 12)$ in exactly 8 seconds?

A. 56 B. 70 C. 112 D. 256 E. NOTA

27. In how many ways can Falstaff (from the previous question) move from $(0, 0)$ to $(12, 12)$ in exactly 10 seconds?

A. 3960 B. 19080 C. 30240 D. 31680 E. NOTA

For questions 28-30: In 2-player Rock-Paper-Scissors (RPS), each player throws one of three gestures, Rock (R), Paper (P), or Scissors (S). R beats S; S beats P; P beats R. In a game of n-player RPS, each player throws a gesture. If only one gesture is present, or if all three gestures are present, the game goes to the next round with all n players. If exactly two gestures are present, then anyone who threw the losing gesture is eliminated. The game continues until only one player remains, who is declared the winner. For the purpose of the next three problems, assume each player chooses his/her gesture at random.

28. In a game of 3-player RPS, what is the probability it takes no more than 2 rounds to declare the winner?

A. $\frac{4}{9}$ B. $\frac{5}{9}$ C. $\frac{2}{3}$ D. $\frac{7}{9}$ E. NOTA

29. In a game of 3-player RPS, what is the expected number of rounds for a winner to be declared?

A. $\frac{9}{4}$ B. $\frac{11}{4}$ C. $\frac{13}{4}$ D. $\frac{15}{4}$ E. NOTA

30. In a round of RPS with 4 players, what is the probability that all three gestures are present?

A. $\frac{2}{9}$ B. $\frac{4}{9}$ C. $\frac{2}{3}$ D. $\frac{8}{9}$ E. NOTA