1. C The first card is irrelevant. For the second card, there are 12 cards that match the suit of the first card out of 51. The probability is $\frac{12}{51} = \frac{4}{17}$

2. B We'll do case work based on whether the first digit is even or odd.
In the case the first digit is even, there are 4 choices for the first digit, 4 choices for the last digit, and 8 choices for the middle digit for a total of $4 \cdot 8 \cdot 4 = 128$
In the case the first digit is odd, there are 5 choices for the first digit, 5 choices for the last digit, and 8 choices for the middle digit for a total of $5 \cdot 8 \cdot 5 = 200$.
The total number of such integers is 328.

3. C There are 3 possible combinations of outcomes when the sum is 4 (13, 22, or 31).
There are 3 possible combinations of outcomes when the product is 4 (14, 22, or 41).
However, 22 overlaps between the two, for a probability of $\frac{5}{36}$

4. B The region defined by $|x| + |y| \leq 8$ is a square with diagonal of 16. The area is 128. The region defined by $x^2 + y^2 \leq 8$ is a circle with radius $2\sqrt{2}$. The area is 8π. So the probability of the point being in the circle is $\frac{8\pi}{128} = \frac{\pi}{16}$

5. D The constant term is $\binom{6}{2}(2x)^4\left(\frac{1}{x^2}\right)^2 = 15(16) = 240$.

6. C Since the greatest common factor of 300 and 360 is 60. Any common factor of 300 and 360 must also be a factor of 60. $360 = 2^3 3^2 5^1$ has $(3+1)(2+1)(1+1) = 24$ factors. $60 = 2^2 3^1 5^1$ has $(2+1)(1+1)(1+1) = 12$ factors. So the probability is $\frac{1}{2}$.

7. B Each term in the expansion is in the form $\frac{7!}{a!b!c!}x^a y^b z^c$ for nonnegative integers a, b, c with $a + b + c = 7$. So the number of ordered triple of solutions (a, b, c) is $\binom{9}{2} = 36$.

8. D Each term in the expansion is in the form of $\frac{9!}{a!b!c!}(x^2)^a (1)^b \left(\frac{1}{x}\right)^c$. The constant can be formed as long as $c = 2a$. There are 4 possibilities – when b is 0, 3, 6, and 9. So the constant term is $\frac{9!}{3!0!6!} + \frac{9!}{2!3!4!} + \frac{9!}{1!6!2!} + \frac{9!}{0!9!0!} = 84 + 1260 + 252 + 1 = 1597$.

9. A There are $\binom{9}{4} = 126$ ways to pick the committee. Counting by complement is simpler. There are $\binom{5}{4} = 5$ ways to pick 4 men, and $\binom{4}{4} = 1$ way to pick 4 women. The probability that both sexes are represented is $\frac{120}{126} = \frac{20}{21}$.

10. C There are $\binom{12}{3} = 220$ ways to draw 3 marbles. There are $5 \cdot 4 \cdot 3 = 60$ ways to draw one of each color. $\frac{60}{220} = \frac{3}{11}$.

11. B There are two choices for the first and last letters, either O_____O or L_____L. Of the 5 letters remaining, 2 are repeats. So the total number of such permutations is $2 \cdot \frac{5!}{2!} = 120$.

12. B There is a probability of $\frac{4}{5}$ for one die to not show a 1, since the roll cannot be a 6. For 3 dice it's $\left(\frac{4}{5}\right)^3 = \frac{64}{125}$.

13. C This is just the partition of 22 into 3 numbers that satisfy the triangle inequality. Simply list them all, organized by the longest side. The longest side is at most 10, or triangle inequality is not satisfied. It is also at least 8, or the perimeter is less than 22. In all the options are: $\{10, 10, 2\}, \{10, 9, 3\}, \{10, 8, 4\}, \{10, 7, 5\}, \{10, 6, 6\}$, $\{9, 9, 4\}, \{9, 8, 5\}, \{9, 7, 6\}, \{8, 8, 6\}, \{8, 7, 7\}$, for a total of 10.

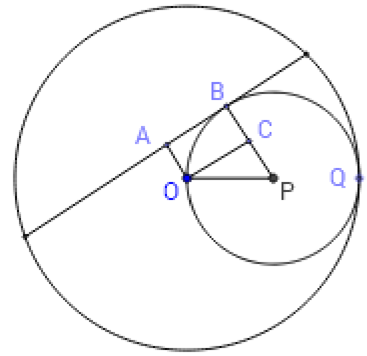
14. D Since the test is 99.9% accurate, the probability is simply 0.999.

15. C The probability a person tests positive is $(0.001)(0.999) + (0.999)(0.001)$. That is, 99.9% chance for the 0.1% of the population, and 0.1% chance for the 99.9% of the population. If Bernardo has the disorder, then he would fall under the 0.1% of population. The probability that he has the disorder and tests positive is $(0.001)(0.999)$. So given he tested positive, the conditional probability that he has the disorder is 0.5.

16. C The entire surface area of the snowman is $4\pi(6^2 + 8^2 + 10^2)$
Above the center of the middle sphere is the entire 6-inch sphere and half of the 8 inch sphere, which is $4\pi(6^2 + 0.5 \cdot 8^2)$.
So the probability is $\frac{36+32}{36+64+100} = \frac{68}{200} = \frac{17}{50}$.

17. B We can label the seats 1 through 6 in clockwise order Without loss of generality, we can label the seats Romeo and Juliet as 1 and 2. There are 2 ways for them to sit in those seats. Julius and Brutus are not in consecutive seats, so they can occupy seats 3 and 5, 3 and 6, or 4 and 6. For each of the 3 choices, there are 2 ways for them to arrange themselves. The remaining 2 seats are for Rosencrantz and Guildenstern, who have 2 ways to arrange themselves. This is a total of $2 \cdot 3 \cdot 2 \cdot 2 = 24$ valid seating arrangements.

18. E Let O be the center of the circle with radius 2, and P be the center of the circle with radius 1, as shown to the right. The point of tangency between the 2 circles is Q. The chord of circle O that is tangent to circle P is determined uniquely by a point on circle P. We'll call that point B, as shown. Then the length of the chord ranges from 4 to 0, as B ranges from point O to point Q. Thus it is sufficient to find $\angle OPB$ when the tangent has length $\sqrt{15}$, as every point between B and Q will result in a chord of length less than $\sqrt{15}$.



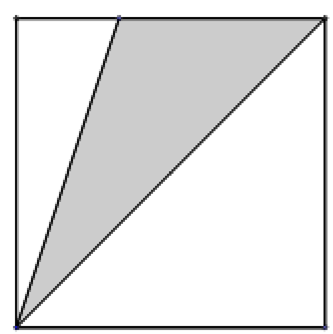
To help compute $\angle OPB$, we'll drop perpendiculars from point O to the tangent chord and to radius PB. Let A and C be the feet of those perpendiculars. Let $OA = x$, then $2^2 - x^2 = \left(\frac{\sqrt{15}}{2}\right)^2$ by Pythagorean Theorem (or by stretching OA into a diameter and using power of a point), so $x = \frac{1}{2}$. Then $BC = \frac{1}{2}$, since $OABC$ is a rectangle. Therefore, $CP = \frac{1}{2}$. We also have $OP = 1$, since it is also a radius of circle P. This makes ΔOCP a 30-60-90 triangle, with $\angle OPC = 60°$. So the probability a chord has length no more than $\sqrt{15}$ is $\frac{2}{3}$.

19. C There are 3 other people in Beatrice's group. Any of the 15 people other than Beatrice is equally likely to be among the 3 people. So the probability is $\frac{3}{15} = \frac{1}{5}$.

20. C The number is in the form 6ABC for distinct digits A, B, and C. Since 6 is divisible by 3, it is sufficient for $A + B + C$ to be divisible by 3. We'll divide the remaining 9 digits into 3 pools based on their remainder when divided by 3. That is, $\{0, 3, 9\}, \{1, 4, 7\}, \{2, 5, 8\}$. A, B, and C can be chosen 1 from each pool, or all 3 from a single pool. Regardless of selection of A, B, and C, there are 6 ways to arrange them, since they are all distinct. So the total number of such integers is $6(3 \cdot 3 \cdot 3 + 3) = 180$.

21. B Since $|\tan\theta| = \frac{|\sin\theta|}{|\cos\theta|} \geq |\sin\theta|$, $\sin\theta > \tan\theta$ when $\tan\theta < 0$, which occurs in quadrant 2 and 4. $\text{Sin}\,\theta > \cos\theta$ when θ is above the line $y = x$. The overlap between the 2 regions is the entirety of quadrant 2. Therefore, the probability is $\frac{1}{4}$.

22. D We can call the two random numbers x and y. They can be plotted on a square of length 1, with one side representing x ranging from 0 to 1, the other side representing y. Without loss of generality, we only consider $y > x$, which is the upper half of the square. Then the condition given is $x + y \geq 2(y - x)$, or



$y \le 3x$. This represents the shaded region as shown. The area of the shaded region is $\frac{1}{3}$, and area of the upper half of the square is $\frac{1}{2}$, so the probability is $\frac{2}{3}$.

23. D There are $2^6 = 64$ total possible outcomes. Of those, $\binom{6}{3} = 20$ have 3 heads and 3 tails. Half of the remaining, or 22 have more heads than tails. Of those, only 1 outcome is all heads. So the probability is $\frac{1}{22}$

24. B The probability that the marble is red is $\frac{1}{3} \cdot \frac{3}{8} + \frac{1}{3} \cdot 0 + \frac{1}{3} \cdot \frac{2}{3} = \frac{25}{72}$. The probability the marble is a red one from bag A is $\frac{1}{3} \cdot \frac{3}{8} = \frac{1}{8} = \frac{9}{72}$. So the conditional probability is $\frac{9}{25}$.

25. A Consider fraction $\frac{m}{n}$. For the sake of simplicity, we'll call *m* and *n* relatively prime with $m < n$. If they are not relatively prime, the fraction can be reduced. If $m > n$, then only the integer part can be subtracted out. For $\frac{m}{n}$ to be a purely repeating decimal of length k, then $\frac{10^k m}{n}$ must have the same decimal part. So n must divide $(10^k m - m)$. Since *m* and *n* are relatively prime, we must have $n|(10^k - 1)$. If that condition is not satisfied, then $\frac{m}{n}$ cannot be a purely repeating decimal.
Therefore, the decimal expansion of $\frac{1}{a}$ is a purely repeating decimal with a cycle length (but not necessarily minimal) of 6 if a is a factor of 999999. $999999 = 999 \cdot 1001 = 3^3 \cdot 37 \cdot 7 \cdot 11 \cdot 13$. Since 27 and 37 divide into 999, the decimal expansions of $\frac{1}{27}$ and $\frac{1}{37}$ cycle every 3 digits. 11 divides into 99, so the decimal expansion of $\frac{1}{11}$ cycles every 2 digits. $\frac{1}{3}$ and $\frac{1}{9}$ cycle every 1 digit.
Thus $\frac{1}{7}$ and $\frac{1}{13}$ are the only unit fractions with repeating decimal with minimum cycle of length 6. However, there is the freedom to choose the numerator m in $\frac{m}{n}$. Any multiple of 7 or 13 can achieve the given requirement. For example, if $n = 7b$ for some positive integer *b*, we only need to set $m = b$, then $\frac{m}{n} = \frac{b}{7b} = \frac{1}{7}$.
The problem remains to compute the number of multiples of 7 and 13 less than 1000. There are 142 multiples of 7, 76 multiples of 13, and 10 multiples of 91, for a total of $142 + 76 - 10 = 208$.

26. B For Falstaff to move from (0, 0) to (12, 12) in 8 seconds, he must make 4 (1, 2) moves, and 4 (2, 1) moves. So the number of possible ways for him to take those moves in order is $\binom{8}{4} = 70$.

27. D Let Falstaff makes a $(\pm 1, \pm 2)$ moves and b $(\pm 2, \pm 1)$ moves. Then $a + b = 10$. Additionally, both *a* and *b* must be even, or Falstaff will end at a point with odd x and y coordinates. Finally, *a* and *b* must be at least 2, or Falstaff cannot move

beyond 10 in one of the coordinates. So $(a, b) \in \{(2,8),(4,6),(6,4),(8,2)\}$. By symmetry, it is sufficient to only consider $(2,8)$ and $(4,6)$, then double the results.
Case 1: $a = 2, b = 8$
First consider only the x coordinate. Since $a + 2b = 18$, Falstaff moves too far by 6 units if he purely moves in the positive direction. So 3 units of movement need to be reversed. He must make one each of -1 and -2 moves in the x direction.
Next consider only the y direction. Since $2a + b = 12$, Falstaff moves the exact amount if he purely moves in the positive direction, which is what we want.
Therefore, the moves Falstaff makes are $1\ (-1,2), 1\ (1,2), 1\ (-2,1), 7\ (2,1)$. The number of ways he can make these moves is $\frac{10!}{7!} = 720$.
Case 2: $a = 4, b = 6$
In the x direction, $a + 2b = 16$, 2 units of movement need to be reversed. This can be achieved by reversing 2 of *a* movements, or 1 of *b* movements.
In the y direction, $2a + b = 14$, 1 unit of movement needs to be reversed. This can only be achieved by reversing 1 of *b* movements.
Keep in mind that the reversal of movements is in a single direction, so there are a few combinations:
Subcase I: reversing 2 of *a* movements in the x direction, 1 of *b* movements in the y direction. That is to say the movements are: $2\ (-1,2), 2\ (1,2), 1\ (2,-1), 5\ (2,1)$.
The number of ways he can make these moves is $\frac{10!}{2!2!1!5!} = 7560$.
Subcase II: reversing 1 of *b* movements in the x direction, 1 of *b* movements in the y direction, reversing on different moves. That is to say the movements are:
$4\ (1,2), 1\ (-2,1), 1\ (2,-1), 4\ (2,1)$. The number of ways he can make these moves is $\frac{10!}{4!1!1!4!} = 6300$.
Subcase III: reversing 1 of *b* movements in the x direction, 1 of *b* movements in the y direction, but reversing on the same move. That is to say the movements are:
$4\ (1,2), 1\ (-2,-1), 5\ (2,1)$. The number of ways he can make these moves is $\frac{10!}{4!1!5!} = 1260$.
So the total number of ways he can make it to (12, 12) in 10 seconds is
$2(720 + 7560 + 6300 + 1260) = 31680$.

28. C In a 3-person RPS, there are $3^3 = 27$ possible combinations of gestures from the 3 players. Of those 3 have all the same, 6 have all different, so $\frac{1}{3}$ chance the game continues to second round with all 3 people in place. The remaining $\frac{2}{3}$ is when only 2 gestures are present, evenly split between 2 people winning and 1 person winning.
In a 2-person RPS, there is $\frac{1}{3}$ probability both players choose the same gesture. $\frac{2}{3}$ probability of someone winning.
So there is a $\frac{1}{3}$ probability the game ends in 1 round.

There is a $\frac{1}{3}$ probability the game goes to the second round with 2 players, which then has a $\frac{2}{3}$ chance of the game ending, for a combined probability of $\frac{2}{9}$.
There is a $\frac{1}{3}$ probability the game goes to the third round with all 3 players, which then has a $\frac{1}{3}$ chance of the game ending, for a combined probability of $\frac{1}{9}$.
So the probability it takes no more than 2 rounds to declare the winner is $\frac{2}{3}$.

29. A Let $E(X)$ represents the expected number of rounds a game with X player takes.
Clearly, $E(1) = 0$, since no rounds are needed to find a winner.
Using the information from above, with 3 players, one round must be played, then there is a $\frac{1}{3}$ probability each that the game continues with 3 and 2 players. Therefore, $E(3) = 1 + \frac{1}{3}E(2) + \frac{1}{3}E(3)$. Similarly, $E(2) = 1 + \frac{1}{3}E(2)$.
From the second equation, $\frac{2}{3}E(2) = 1$, so $E(2) = \frac{3}{2}$.
Substituting into the first equation, $\frac{2}{3}E(3) = \frac{3}{2}$, so $E(3) = \frac{9}{4}$.

30. B For all 3 gestures to be present in 4-player RPS, the distribution must be 2 of one gesture, and 1 each of the other two. There are 3 ways to choose the double gesture, and $\frac{4!}{2!} = 12$ ways to distribute the 4 gestures among the 4 players. There are a total of $3^4 = 81$ ways the gestures can be selected. So the probability is $\frac{3 \cdot 12}{81} = \frac{4}{9}$.