MAΘ Nationals Buffalo
Theta Apps Solutions

1) C
2) B
3) D
4) D
5) B
6) E
7) D
8) C
9) E
10) B
11) D
12) A
13) A
14) B
15) C
16) D
17) C
18) D
19) D
20) A
21) B
22) C
23) B
24) E
25) A
26) C
27) D
28) E
29) D
30) C

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1) The possible configurations of roots are $\pm 1, \pm 2, \pm 2$, meaning there would be 8 configurations of roots. However +2 and -2 would produce the same functions as -2 and +2, so we throw out two configurations (one for +1 and one for -1), and are left with 6.

2) Take the discriminant and set equal to odd perfect squares.

$225 - 16k = 1 \to k = 14$

$225 - 16k = 9 \to k = \frac{27}{2}$

$225 - 16k = 25 \to k = \frac{25}{2}$

$225 - 16k = 49 \to k = 11$.

$225 - 16k = 81 \to k = 9$

$225 - 16k = 121 \to k = \frac{13}{2}$

$225 - 16k = 169 \to k = \frac{7}{2}$

3) Since gcd(51,21)=3, no matter what you are always going to obtain some multiple of 3. Therefore, the smallest possible multiple of 3 is 3.

4) There are 2525-2017=508 years until the mentioned date. Writing this population as $P=3*2^{508}=2^{509.59}$.

5) The plant's total energy, accounting for energy usage, is $s(t) - u(t) = \sqrt{t} - \sqrt[3]{t}$. Making the substitution $x^6 = t$ we get $x^3 - x^2 = 18$ which by inspection has a root at x=3 (the other two are imaginary). This means that $t=x^6=3^6=729$ hours of sunlight

6) $\frac{88 + 72 + x}{3} = 86 \Rightarrow x = 98$

7) His expected value is the sum of the probabilities times their output. Since the doubling portions of the wheel are half sized they take up $\frac{4}{2 \times 28 + 4}$ of the wheel and the halving sections take up $\frac{2 \times 28}{2 \times 28 + 4}$. Thus his expected value is

$$\left(\frac{4}{60}\right)2M + \left(\frac{56}{60}\right)\frac{M}{2} = \frac{3}{5}M$$

8) There is a leap year every 4 years in this time interval. Thus the number of Days =

$$100 \times 365 + \frac{100}{4} = 36{,}525$$

9) We need to choose a and b such that a+b=5 and $\binom{5}{a}\left(\frac{2}{x^2}\right)^a\left(-3x^3\right)^b$ has no x term.

a=3 and b=2 works so the term is $\binom{5}{3}\left(\frac{2}{x^2}\right)^3\left(-3x^3\right)^2=720$

10) Flip the point (2,4) over the y axis and find the distance from (-2,4) to (7,1).
$d=\sqrt{(7-(-2))^2+(4-1)^2}=3\sqrt{10}$

11) A cube inside a unit sphere has long diagonal 2, so the side length would be $2/\sqrt{3}$.

A sphere inside this cube has a radius of half the side length or $\frac{\sqrt{3}}{3}$

12) Either list the combinations or think about it in a combinatorial context. The 6 orders are MJBS, MBJS, MBSJ, BMJS, BMSJ, and BSMJ. OR Think of Mary as the shortest. There are 6 ways to arrange the remaining three in line but half of them break the condition that Billy is shorter than Steve. The only other person who can be the shortest is Billy, and the same logic applies for 3+3 total arrangements.

13) It only takes one draw. The correct draw is to draw from the urn labeled BW. If he draws Black it is the BB urn, then the BB urn must be WW and BB must be BW. This is only true because all of the urns are labeled "incorrectly." Had they been randomly rearranged instead, it would take two or more draws.

14) Count the number of powers of 17 in 291. $\left\lfloor\frac{291}{289}\right\rfloor+\left\lfloor\frac{291}{17}\right\rfloor=1+17=18$

15) She travels 3000 miles in 26 hours, so the average is $\frac{3000}{26}=115.3...$ mph.

16) We're solving for r here $501=500\left(1+\frac{r}{12}\right)^1\Rightarrow r=\frac{12}{500}=2.4\%$

17) The maximum amount of words per book is 390k, times 10 books divided by 3k words per week = 1300 weeks = 25 years

18) The function given tells how many ADDITIONAL fans Matt gains per book.

$$f_0=250$$
$$\Delta f_1=\lfloor 1.1f_0\rfloor=275\Rightarrow f_1=525$$
$$\Delta f_2=\lfloor 1.1f_1\rfloor=577\Rightarrow f_2=1102$$
$$\Delta f_3=\lfloor 1.1f_2\rfloor=1212\Rightarrow f_3=2314$$
$$\Delta f_4=\lfloor 1.1f_3\rfloor=2545\Rightarrow f_4=4859$$

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The trick for multiplying by 1.1 is just multiply by 11 and ignore the last digit of the result (this is the floor function in action)

19) 3*f_0+423=1173 which falls in to the bracket of $8 per copy so 1173*8=9384

20) $0.\overline{13A}_{16} = \frac{13A_{16}}{FFF_{16}} = \frac{1(16^2)+3(16)+13}{15(16^2)+15(16)+15} = \frac{317}{4095}$

21) We use the "chicken mcnugget theorem" here which states for coprime *m* and *n*, the largest unattainable number only using positive integer multiples of *m* and *n* is *mn-m-n*. Plugging in we have 7*12-12-7=65

22) The probability here is two instances of hitting a letter that is not T then one instance of T so we have $\left(\frac{25}{26}\right)\left(\frac{25}{26}\right)\left(\frac{1}{26}\right) = \frac{625}{17{,}576}$

23) $\frac{1-2a-b}{2a+2b} = \frac{\log 10 - \log 4 - \log 3}{2(\log 2 + \log 3)} = \frac{\log 5 - \log 6}{2\log 6} = -\frac{1}{2} + \log_6 \sqrt{5}$

24) We just need to shoelace the points (0,0) (1,1) and (x,x2). This gives us $\frac{x^2-x}{2} = 6$ so x (must be positive) is 4 and y is 16

25) The midpoint of the parabola is the average of the zeroes. The average is the sum over 2. Therefore the axis of symmetry is at x=7/2

26) We are given that $\begin{array}{l} x+y+z=28 \\ x-2z=4 \\ -2y+3z=11 \end{array}$. Solving this system with elimination gives $(x,y,z) = \left(\frac{154}{9}, \frac{13}{3}, \frac{59}{9}\right)$, the largest of which is $x = \frac{154}{9}$

27) In a single elimination tournament of n teams there will be n-1 games (because n-1 teams have too lose to crown a champion). By the same logic, if each team must lose twice to be eliminated we would play 2(n-1) games. But the Winner's bracket finalist can lose once in the Grand Finals and reset the bracket (because he has to lost twice to be eliminated) so the max number of games is 2n-1. Here we're paying $500 for each of the 2(64)-1=127 games which is a net payout of $63,500.

28) The given info leads you to the Venn Diagram below. We need to find x+y+z. We know from the given info that 14+4+x+z=28, 7+4+y+x=24 and 2+4+y+z=25. Simplifying these you get x+z=10, x+y=13, and y+z=19. Add these three equations together and divide by two to get x+y+z=21. Now we add the rest of the numbers in and get a total poll size of 60 people.

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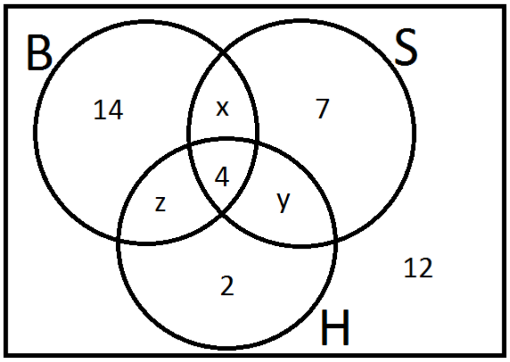

29) Write the middle term as *a*. Then the product of the sequence is $ar^{-2} \cdot ar^{-1} \cdot a \cdot ar \cdot ar^2 = a^5 = 32 \Rightarrow a = 2$. We know that $ar^2 = \frac{338}{49} \Rightarrow r = \frac{13}{7}$. Now we need to find $a - ar^{-2} = 2\left(1 - \frac{49}{169}\right) = \frac{240}{169}$

30) The corners of his expanded hexagon fence become circles (equidistant from the corner points), which is where the 6pi comes in. The remaining 120 meters of material is actually the same amount as his original enclosure, so the side length of his original enclosure is 20 and the area is $6\frac{20^2\sqrt{3}}{4} = 600\sqrt{3} = 1038$