Answers:

0. 66
1. 2
2. 76
3. 12
4. 13
5. $\frac{105}{16}$ (must be a fraction)
6. 0
7. 1002
8. $\frac{453}{1001}$
9. $\frac{2}{3}$
10. $-\frac{9}{7}$
11. 2017
12. $-\frac{216}{35}$

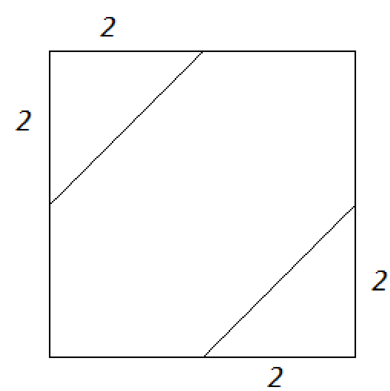
Solutions:

0. The total number of diagonals in a regular n-gon is $\frac{n(n-3)}{2}$. By inspection, since $\frac{65(65-3)}{2}=2015$ and $\frac{66(66-3)}{2}=2079$, the number of sides must be at least 66.

1. $\frac{x^2-2x-15}{x^2+4x+3}\le 7 \Rightarrow \frac{x^2-2x-15}{x^2+4x+3}\le \frac{7x^2+28x+21}{x^2+4x+3} \Rightarrow 0\le \frac{6x^2+30x+36}{x^2+4x+3}=\frac{6(x+2)(x+3)}{(x+1)(x+3)}$.
Sign analysis confirms the solution of the inequality to be $(-\infty,-3)\cup(-3,-2]\cup(-1,\infty)$, so there are only 2 negative integers, namely -3 and -1, that are not solutions.

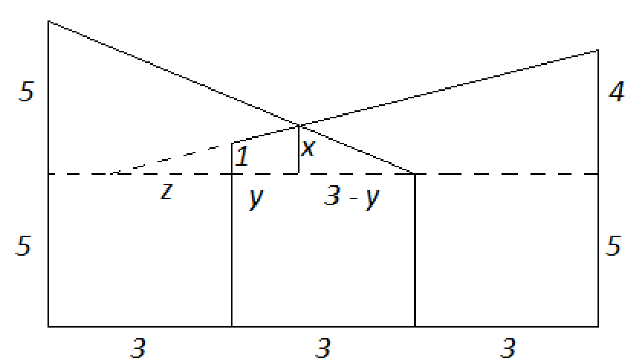
2. f is a parabola that opens downward, so the maximum value of the function occurs at the vertex of the parabola, namely at the y-value of the vertex. Since $f(x)=-6x^2+24x+C=-6(x-2)^2+(C+24)$, we must have $C+24=100 \Rightarrow C=76$.

3. The hexagon is formed as in the picture to the right, eliminating the upper left and lower right corners. Since those two corners form a square whose side length is 2, the area enclosed by the hexagon is $4^2-2^2=12$.



4. The trick is to realize that $(x+4)^3=x^3+12x^2+48x+64$, so $f(x)=(x+4)^3+(C-64)$.
For this function to have an integer root, $C-64$ must be a perfect cube, say $C-64=n^3$ $\Rightarrow C=n^3+64$. For C to fall in the given interval, we can have n be any integer from -3 to 9, inclusive. Since each value of n corresponds to a unique value of C, there are 13 such possibilities.

5. In the diagram, the horizontal dashed line is drawn at the height of the shortest post, x represents the height above this line for the point at which the wires cross, y represents the distance along the horizontal dashed line from the 6-foot post to the point



closest to the intersection along the horizontal dashed line, and z represents the length along the horizontal dashed line where the extended wire connecting the 6- and 9-foot posts would intersect the horizontal dashed line. By similar triangles, we know that $\frac{1}{z}=\frac{4}{6+z}\Rightarrow z=2$. Further, by similar triangles, we now know that $\frac{1}{2}=\frac{x}{y+2}$ $y=2x-2$. Now, using the other right triangle, we have that $\frac{x}{3-y}=\frac{5}{6}\Rightarrow y=3-\frac{6}{5}x$. Solving this system yields $x=\frac{25}{16}$, making the height above ground $5+\frac{25}{16}=\frac{105}{16}$.

6. $A=\begin{bmatrix}2 & 1\\ -2 & 0\\ 3 & -4\end{bmatrix}\cdot\begin{bmatrix}3 & 2 & -5\\ -1 & 1 & -2\end{bmatrix}=\begin{bmatrix}5 & 5 & -12\\ -6 & -4 & 10\\ 13 & 2 & -7\end{bmatrix}\Rightarrow |A|=0$

7. The number of degrees in an exterior angle is $\frac{360}{n}$ and the total number of diagonals is $\frac{n(n-3)}{2}$, so this product is $180(n-3)$. The number of degrees in the sum of the interior angles is $180(n-2)$, so dividing by this we get $\frac{n-3}{n-2}=1-\frac{1}{n-2}$. Setting this equal to $0.999=1-\frac{1}{1000}$, we get that $n=1002$.

8. All five cards need to show a different digit in order for them to be sequenced in a strictly increasing order. The denominator of this probability is $\binom{14}{5}=2002$. To get the numerator, consider that five digits appear only once each (1, 3, 4, 6, and 9), 3 digits appear twice each (2, 7, and 8), and 1 digit appears thrice (5). It is possible that all five digits that appear once are chosen, of which there is $\binom{5}{5}=1$ way. You may also choose four of those non-repeated digits and one other, of which there are $\binom{5}{4}\binom{9}{1}=45$ ways. You may also choose three of those non-repeated digits and two of the others, which is a little more complicated to find because you must consider choosing two different of the singly-repeated digits or one of the singly-repeated digits and one of the 5s: $\binom{5}{3}\left(\binom{3}{2}\binom{2}{1}^2+\binom{3}{1}\binom{2}{1}\binom{3}{1}\right)=300$ ways to do that. You may also choose two of those non-repeated digits and three of the others, which could be all three singly-repeated

digits or two of the singly-repeated digits and one of the 5s:
$\binom{5}{2}\left(\binom{3}{3}\binom{2}{1}^3+\binom{3}{2}\binom{2}{1}^2\binom{3}{1}\right)=440$ ways to do that. Finally, you may choose one of the non-repeated digits and one of each of the others: $\binom{5}{1}\left(\binom{3}{3}\binom{2}{1}^3\binom{3}{1}\right)=120$ ways to do that. Therefore, the probability is $\dfrac{1+45+300+440+120}{2002}=\dfrac{906}{2002}=\dfrac{453}{1001}$.

9. Because the two asymptotes' slopes have the same magnitude, the transverse axis of the hyperbola is either horizontal or vertical, and because the two *y*-intercepts of the hyperbola are on the same branch, the transverse axis must be horizontal. This means that, using the traditional conic section nomenclature, $\dfrac{b}{a}=\dfrac{3}{2}\Rightarrow b=\dfrac{3}{2}a$. Further, since the two asymptotes intersect at the point $(-2,3)$, the equation of the hyperbola is $1=\dfrac{(x+2)^2}{a^2}-\dfrac{(y-3)^2}{b^2}=\dfrac{(x+2)^2}{a^2}-\dfrac{(y-3)^2}{\frac{9}{4}a^2}=\dfrac{9(x+2)^2-4(y-3)^2}{9a^2}\Rightarrow 9a^2=9(x+2)^2$ $-4(y-3)^2$. Plugging in $x=0$ and solving for *y* yields $y=3\pm\dfrac{3}{2}\sqrt{4-a^2}$, so the distance between the *y*-intercepts is $3\sqrt{4-a^2}$. Therefore, $3\sqrt{4-a^2}=\sqrt{35}\Rightarrow a=\dfrac{1}{3}$ (since $a>0$), and the distance between the two vertices is $2a=\dfrac{2}{3}$.

10. The harmonic mean of *n* numbers is given by $\dfrac{n\cdot(\text{product of numbers})}{\text{sum of all products taking } n-1 \text{ of the numbers at a time}}$ (verify that this equals the reciprocal of the average of the reciprocals of the numbers, which is the definition of harmonic mean). Therefore, the harmonic mean is $\dfrac{3\cdot -6}{14}=-\dfrac{9}{7}$.

11. Since 2017 is odd, any line through the center and a vertex of the 2017-gon must pass through the midpoint of the side opposite the vertex. Therefore, there is one line of symmetry through each vertex, or 2017 lines of symmetry.

12. $-\dfrac{1}{180}=\dfrac{6}{5}r^3\Rightarrow r=-\dfrac{1}{6}$ and $a_1=\dfrac{6/5}{-1/6}=-\dfrac{36}{5}$, so the sum is $S=\dfrac{-36/5}{1-\left(-1/6\right)}=-\dfrac{216}{35}$