

- 1. **B** Completing the square and transforming the equation yields  $(x+4)^2 + (y-3)^2 = 40$ , so  $r^2 = 40$  and  $r = 2\sqrt{10}$ .
- 2. **D** The equation transforms to  $\frac{x^2}{x^2}$  $\frac{x^2}{9} + \frac{y^2}{36}$  $\frac{y}{36}$  = 1, so *a* = 6 and the length of the major axis is **12.**
- 3. **D**  $y^2 6y = -32x + 23 \Rightarrow y^2 6y + 9 = -32x + 23 + 9 \Rightarrow (y 3)^2 = -32(x 1)$ . The vertex is (1, 3).
- becomes  $(x+7)^2 + (y-1)^2 = 144$ . The center is (-7, 1). 4. A Completing the square and transforming the equation gives us  $x^2 + 14x + 49 + y^2 - 2y + 1 = 94 + 49 + 1$ , which
- Checking the numerical values after completing the square, we have  $9x^2 36x +$  \_\_\_ ( $y^2 6y +$  \_\_\_)= -27  $\Rightarrow$  $9(x^2-4x+4) - (y^2-6y+9) = -27 + 36 - 9 \Rightarrow 9(x-2)^2 - (y-3)^2 = 0$ . This is the degenerate case of a hyperbola, 5. **E** The equation in general form has quadratic terms with opposite signs, which indicates a hyperbola.
- which simplifies to  $\pm 3(x-2) = y 3$ , so the equation represents *two intersecting lines.*
- 6. **E** The given coordinates for the vertex and focus tell us that the parabola is a function that opens upward, with  $p = 3$ . The equation is  $12(y - 1) = (x - 2)^2$ , which can be expanded and simplified to  $12y = x^2 - 4x + 16$ , or  $y = \frac{1}{x}$  $\frac{1}{12}$  $\left(x^2 - 4x + 16\right)$ .
- 7. **C** This parabola opens to the left, with  $p = -4$ . The directrix will be the vertical line located 4 units to the right of the vertex  $(1, -3)$ , so it is  $x = 5$ .
- 8. **B** First, we can determine the slope of the tangent line  $5x 3y = 7$ ;  $m = 5/3$ . Therefore the radius to the point of tangency has  $m = -3/5$ . Using the coordinates of the center, this radius lies on the line  $y - 4 = (-3/5)(x + 3)$ , or  $3x + 5y = 11$ . Now we can determine the point of tangency by finding the intersection point of the radius and the tangent line by solving the linear system consisting of  $5x - 3y = 7$  and  $3x + 5y = 11$ . Using linear combination the intersection (the point of tangency) is (2, 1). Now we can use the distance formula to find the

length of the radius from the center (-3, 4) to the point of tangency  $(2, 1)$ :  $r = \sqrt{(-3-2)^2 + (4-1)^2}$ , so  $r = \sqrt{34}$ . 9. **A** We can transform the equation to standard form:  $3(x^2 - 2x + 1) + 4(y^2 + 4y + 4) = -7 + 3 + 16$ , or

- ajor axis is norizonial,  $a = 2$  $(x-1)^2$ 4  $+\frac{(y+2)^2}{2}$  $\frac{2f}{3}$  = 1. This tells us that the center is (1, -2), the major axis is horizontal,  $a = 2$  and  $b = \sqrt{3}$ , and therefore  $c = 1$ . So the foci are points 1 unit to the left and right of the center, or  $(2, -2)$  and  $(0, -2)$ .
- standard form:  $9(x^2 10x + 25) 16(y^2 4y + 4) = -17 + -64 + 225$ , or  $9(x 5)^2 16(y 2)^2 = 144$ , so we have 10. **B** The length of the transverse axis will be the value of 2*a*. To find that, we will transform the equation into  $(x-5)^2$  $-\frac{(y-2)^2}{2}$  $\frac{27}{9}$  = 1, which tells us that  $a = 4$  and  $2a = 8$ .
- 16 11. A The two quadratic terms  $x^2$  and  $y^2$  have equal coefficients, so that implies that this is a circle. However, we must verify that  $r > 0$  and that this equation is not a degenerate case of a circle. Completing the square will give us  $(x - 3)^2 + (y - 1)^2 = 15$ , so  $r > 0$ , and this is indeed a **circle.**
- 12. C The length of the diameter is found with the distance formula:  $d = \sqrt{(-2-6)^2 + (3-5)^2} = \sqrt{64+4} = \sqrt{68} = 2\sqrt{17}$ and the radius is half that, or  $\sqrt{17}$ . Therefore the area of the circle is  $17\pi$ .
- 13. **D** The equation transforms to  $(y 2)^2 = 4(x + 1)$ , which tells us that the parabola opens to the right, has vertex at  $(-1, 2)$  and  $p = 1$ . Therefore the focus is  $(0, 2)$ .
- 14. **C** Transforming the equation gives us  $4(x^2 6x + 9) 9(y^2 + 4y + 4) = 36 36 + 36$ , or  $4(x-3)^2 9(y+2)^2 = 36$ , so we have  $\frac{(x-3)^2}{2}$  $\frac{(-3)^2}{9} - \frac{(y+2)^2}{4}$  $\frac{12}{4}$  = 1. The slopes of the asymptotes are -2/3 and 2/3, and the center is (3, -2). So the equations of the asymptotes are  $y + 2 = (-2/3)(x - 3)$  and  $y + 2 = (2/3)(x - 3)$ . These simplify to  $2x + 3y = 0$  and

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2x-3y=12.
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- 15. **C** The center is (3, 1),  $c = 3$  and  $b = 4$ , so  $a = 5$ . Area =  $ab\pi = 20\pi$ .
- 16. **A** We can create a system of 3 equations in 3 variables by substituting the values of *x* and *y* from the known ordered pairs into the general form for the equation of a circle  $(x - h)^2 + (y - k)^2 = r^2$ . This gives us the following:

 $(-2, 3)$ :  $(-2 - h)^2 + (3 - k)^2 = r^2$  which becomes  $13 + 4h + h^2 - 6k + k^2 = r^2$  $(6, -5): (6-h)^2 + (-5-k)^2 = r^2$  which becomes  $61 - 12h + h^2 + 10k + k^2 = r^2$  $(0, 7)$ :  $49 + h^2 - 14k + k^2 = r^2$ 

Combining the first and third equations gives us  $-36 + 4h + 8k = 0$  or  $h + 2k = 9$ . Combining the first and second equations gives us  $12 - 12h + 24k = 0$ , or  $-\hbar + 2\hbar = -1$ . The two linear equations combine to give us  $(5, 2)$  for  $(h, k)$ , which is the center of the circle. We can now find the length of the radius by using the distance formula between any one of the points on the circle and the center:  $r = \sqrt{50}$ , so  $A = 50\pi$ .

- 17. **C** The figure described is an ellipse, with foci  $(5, -1)$  and  $(5, 3)$ . Therefore the center is  $(5, 1)$ , and  $c = 2$ . The sum of the distances from the point P to the two foci is 2*a*, so  $a = 4$ . Knowing  $a^2 - c^2 = b^2$ , we can conclude
	- that  $b^2 = 12$ . Thus the equation of this ellipse is  $\frac{(x-5)^2}{12} + \frac{(y-1)^2}{12} = 1$ , or  $4(x-5)$ 12  $+\frac{(y-1)^2}{4}$  $\frac{(-1)}{16}$  = 1, or  $4(x-5)^2 + 3(y-1)^2 = 48$ .
- axis is horizontal and  $a = 2$ , so the vertices are two units to the left and right of the center, or (0, 3) and (-4, 3). 18. **A** This equation of a hyperbola transforms to  $\frac{(x+2)^2}{4}$ 4  $+\frac{(y-3)^2}{25}$  $\frac{25}{25}$  = 1, which has center (-2, 3). The transverse
- 19. **B**  $a = 5$ ,  $b = 2$ , therefore  $c = \sqrt{21}$ . Eccentricity =  $c/a = \sqrt{21/5}$ .
- 20. **A** At first glance it looks like an ellipse, because the coefficients of the quadratic terms are different numbers standard form:  $3(x^2 - 4x + 4) + 4(y^2 - 2y + 1) = -16 + 12 + 4 = 0$ . This is indeed the degenerate case of an that have the same sign, but we must verify that this is not a degenerate case by transforming the equation to ellipse, and therefore represents a **point.**
- 21. **B** We are considering a parabola with vertex (0,0) and opening upward. Using the reflective property of parabolas, with axis of symmetr*y x*= 0, we can set equal the distances between the focus and the point of tangency, and the *y*-intercept  $(0, b)$  of the tangent line and the point of tangency. Given  $x^2 = 2y$  we have  $p = \frac{1}{2}$ , so the focus is the point  $(0, \frac{1}{2})$ .  $d_1 = \sqrt{(-3-0)^2 + (4.5-0.5)^2} = \sqrt{25} = 5$ .  $d_2 = \frac{1}{2} + |b|$ . Setting  $d_1 = d_2$  gives us  $5 = \frac{1}{2} + |b|$ , so  $b = -4.5$ . Now considering the tangent line, we can determine the slope *m*:  $\frac{4.5 - -4.5}{2.8}$  $\frac{3^{2}-4.5}{-3-0}$  = -3, so
	- $\ddot{\phantom{0}}$ using slope-intercept form, we have the equation of the tangent line is  $y = -3x - 4.5$ , and the *x*-intercept is  $-3/2$ .
- 22. **D** From the given information we have  $a = 9$  and  $b = 3$ . The equation can be written  $\frac{(x+1)^2}{9}$ 9  $+\frac{(y-8)^2}{x}$  $\frac{91}{81}$  = 1

which transforms to  $9(x^2 + 2x + 1) + y^2 - 16y + 64 = 81$ , with simplifies to  $9x^2 + 18x + y^2 - 16y - 8 = 0$ . 23. A First ransform the equation:  $9(x^2 + 8x + 16) - 25(y^2 - 4y + 4) = -269 + 144 - 100$ , which becomes

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\mathbf{u}_{\mathcal{E}}(y-2)^29
-\frac{(x+4)^2}{x+4}\frac{11}{25} = 1. The hyperbola opens up and down; the length of the conjugate axis is 10.
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- 24. **D** The fact that the parabola has two *x*-intercepts tells us that this is a function, so it will have an equation of the form  $y = ax^2 + bx + c$ . We can substitute for  $(x, y)$  using the coordinates of the three given points as follows:  $(2, 0) \rightarrow 0 = 4a + 2b + c$ 
	- $(4, 0) \rightarrow 0 = 16a + 4b + c$
	- $(0, -8) \rightarrow -8 = c$

This leads to the linear system  $8 = 4a + 2b$  and  $8 = 16a + 4b$ , which simplifies to  $4 = 2a + b$  and  $2 = 4a + b$ . Solving we get  $a = -1$ , and therefore  $b = 6$ . The equation of the function is  $y = -x^2 + 6x - 8$ , so the vertex is (3,1) and  $p = -\frac{1}{4}$ ; therefore the focus is  $(3, \frac{3}{4})$ .

- 25. **C** The center is (0,0), so the slope of the radius to (-1, 3) is  $m_r = -3$ . The slope of the tangent line to (-1,3) must then be the opposite reciprocal of  $m_r$ , or  $\frac{1}{3}$ . The equation of the tangent line is  $y - 3 = \frac{1}{3}(x + 1)$ . Solving for the *x*-intercept:  $-3 = \frac{1}{3}(x + 1)$ , so  $x = -10$ .
- 26. **D** This parabola opens to the left and has 2 *y*-intercepts, which are symmetric points across the axis of symmetry of the parabola. The axis of symmetry is the horizontal line that passes through the vertex,  $y = 1$ . The arithmetic mean of the two *y*-intercepts is the *y* coordinate of the vertex, **1.**
- 27. **E** From the given equation we can discern that we have a hyperbola with center at (2, 3), with the slopes of the asymptotes being  $\pm$ 5/4. These asymptotes then have the following equations:  $y - 3 = (5/4)(x - 2)$ , which simplifies to  $y = (5/4)x + 4/2$ ; and  $y - 3 = (-5/4)(x - 2)$ , which simplifies to  $y = (-5/4)x + 11/2$ . The sum of these two *y*-intercepts is **6.**
- 28. **B** The equation transforms to  $(y 1)^2 = 12(x 2)$ , which indicates opening to the right, with vertex at (2,1), and  $p = 3$ . Therefore, the equation of the directrix is  $x = -1$ .
- 29. **D** With a horizontal directrix of  $y = 6$ , and focus at (4, -3), we conclude that the value of  $|p|$  is 9/2. The length of the latus rectum is 4*p*, which equals **18.**
- 30. **B** This is the definition of a **parabola.**