

For each question, "E) NOTA" indicates that none of the above answers is correct.

1. If $f(x) = x + 2$, then $f^6(x)$ equals what, where exponentiation represents composition?

- A) $x + 6$ B) $x + 8$ C) $x + 10$ D) $x + 12$ E) NOTA

2. If $f(x + 3) = x^2 + 11x + 2$, find $f(x)$

- A) $x^2 + 11x - 1$ B) $x^2 + 5x - 22$
C) $x^2 + 5x + 22$ D) $x^2 + 11x + 5$ E) NOTA

3. If $f\left(\frac{3}{x-1}\right) = x^2 - 1$, what is/are the root(s) of $f(x)$?

- A) $-1, 1$ B) -1.5 C) -3 D) -4.5 E) NOTA

4. What is the remainder when $f(x) = -6x^{1005} + 3x^{67} - 2x^{31} + x^{20} + x^2 - 20$ is divided by $x + 1$?

- A) -23 B) -13 C) 0 D) -19 E) NOTA

For questions 5 and 6, consider $f: (-\infty, 0) \cup (0, \infty) \rightarrow \mathbb{R}$ such that

$$f(x) + 2f\left(\frac{1}{x}\right) = x^2$$

5. Compute $f(1)$.

- A) $1/3$ B) $1/2$ C) 1 D) 3 E) NOTA

6. Compute $f(2)$.

- A) $-15/4$ B) $-5/4$ C) $-7/6$ D) $31/12$ E) NOTA

7. Find $f(4)$ if $f\left(\frac{1}{x}\right) + 6x + x^2 = (x^2 + x)f(x)$.

- A) $57/21$ B) $95/28$ C) $75/28$ D) $-25/27$ E) NOTA

8. If $f(x)$ satisfies $2f(x) + f(1 - x) = x^2$ for all x , what is $f(x)$?

- A) $\frac{x^2 - 3x + 1}{2}$ B) $\frac{x^2 + 8x - 3}{9}$ C) $\frac{4x^2 + 3x - 2}{6}$ D) $\frac{x^2 + 2x - 1}{3}$ E) NOTA

9. If the perimeter of the region bounded by $f(x) = |x - 3| + 5$ and $g(x) = 7 - |x - 2|$ is expressed in the form $a\sqrt{b}$, where a and b are integers and b is not divisible by the square of any integer, find a^b .

- A) 4 B) 8 C) 9 D) 16 E) NOTA

10. A quadratic polynomial f satisfies $f(x) \geq 0$ for all x , $f(1) = 0$, and $f(3) = 3$. What is $f(5)$?

- A) 9 B) 12 C) 15 D) 18 E) NOTA

11. For any positive integer $x \geq 2$, define $f(x)$ to be the product of the distinct prime factors of x . For example, $f(12) = 2 \cdot 3 = 6$. Compute the number of integers $2 \leq x < 100$ such that $f(x) < 10$.

- A) 27 B) 23 C) 99 D) 8 E) NOTA

12. What is the sum of the ordinates and abscissas of all intersection points between $f(x) = x^3 + 3x^2 + 2x$ and its inverse relation?

- A) $1/2$ B) 6 C) -6 D) 0 E) NOTA

13. For a positive integer a , let $f(a)$ be the average of all positive integers b such that $x^2 + ax + b = 0$ has integer solutions. Compute the unique value of a such that $f(a) = a$.

- A) 5 B) 12 C) 13 D) 24 E) NOTA

14. If $P(x) = 3x^4 - ax^2 + bx - 2$ is divided by $x + 2$, the remainder is 34. When $P(x)$ is divided by $x - 3$ the remainder is 271. What is $2a + b$?

- A) -10 B) -2 C) 2 D) 10 E) NOTA

15. Given a number n in base 10, let $g(n)$ be the base-3 representation of n . Let $f(n)$ be equal to the base-10 number obtained by interpreting $g(n)$ in base 10. Compute the smallest positive integer $k \geq 3$ that divides $f(k)$.

- A) 3 B) 7 C) 15 D) 16 E) NOTA

16. What is the domain of the function $f(x) = \sqrt{\frac{x-5}{x^2-7x+12}}$?

- A) $(-\infty, 3) \cup (5, \infty)$ C) $(3, 4) \cup [5, \infty)$
 B) $(-\infty, 3) \cup (4, 5)$ D) $[3, 4] \cup [5, \infty)$ E) NOTA

17. If f is a monic cubic polynomial with $f(0) = -64$, and all roots of f are non-negative real numbers, what is the largest possible value of $f(-1)$? (A polynomial is monic if it has a leading coefficient of 1.)

- A) -125 B) -101 C) -5 D) -66 E) NOTA

18. Given that $f(x+1)f(x) = x$ for $x > 0$, what is $f(2014)f(2017)$ rounded to the nearest integer?

- A) 2014 B) 2015 C) 2016 D) 2017 E) NOTA

19. If r, s, t , and u denote the roots of the polynomial $f(x) = x^4 + 4x^3 + 8x^2 + 9x + 10$, find

$$\frac{r^2}{s^2 + t^2 + u^2} + \frac{s^2}{r^2 + t^2 + u^2} + \frac{t^2}{r^2 + s^2 + u^2} + \frac{u^2}{r^2 + s^2 + t^2}$$

- A) -9 B) -8 C) -4 D) 0 E) NOTA

20. Consider the even function $f(x)$. If $f(-3) = 7$, $f(x)$ is symmetric with respect to the _____ and $f(3)$ is equal to _____. Fill in the respective blanks.

- A) origin; 7 B) origin; -7 C) y-axis; 7 D) y-axis; -7 E) NOTA

21. The roots of a cubic polynomial $f(x)$ are integers that form an arithmetic progression with common difference 2. The sum of the pairwise products of the roots is 8. Given that $f(1) = 30$ and $f(-1) = -6$, what is $f(3)$?

- A) -30 B) -6 C) 6 D) 210 E) NOTA

22. Let $f(x) = x + 2$ and $g(x) = x^2 - 4$. Also, let p be the statement " $f(1) = 3$ " and let q be the statement " $g(-1) = 3$." Which of the following is false?

- A) $p \rightarrow \sim q$ B) $p \vee q$ C) $p \wedge \sim q$ D) $p \rightarrow q$ E) NOTA

23. Consider $f(x) = Ax^4 + Bx^3 + Cx^2 + Dx + E$ such that A, B, C, D , and E are positive integers. If $f(-1) = 6$, $f(1) = 10$, and $f(0) = 2$, how many different polynomials could $f(x)$ be?

- A) 1 B) 5 C) 6 D) 7 E) NOTA

24. If $f(x) = \frac{3}{x-2}$ and $g(x) = x + \frac{1}{x}$, find the domain of $(f \circ g)(x)$.

- A) $\{x \in \mathbb{R} \mid x \neq 0\}$ B) $\{x \in \mathbb{R} \mid x \neq 2\}$
C) $\{x \in \mathbb{R} \mid x \neq 0, 1\}$ D) $\{x \in \mathbb{R} \mid x \neq 0, 2\}$ E) NOTA

25. Find the value(s) of b such that $f(x) = -x^2 + bx - 16$ has a maximum value of 48.

- A) $\pm 4\sqrt{2}$ B) $\pm 8\sqrt{\frac{3}{5}}$ C) ± 8 D) ± 16 E) NOTA

26. Let $f(x) = x^3 - 12x + 16$. If the three roots of $f(x)$ are a, b , and c , what is the maximum value of $a - b + c$?

- A) 12 B) 8 C) 4 D) 0 E) NOTA

27. Define a sequence of functions $f_1(x), f_2(x), f_3(x), \dots$ such that $f_1(x) = x$ and $f_{n+1}(x) = \frac{1}{1 - f_n(x)}$ for integers $n \geq 1$. Find an expression for $f_{2017}(x)$.

- A) x B) $\frac{1}{1-x}$ C) $\frac{x-1}{x}$ D) $\frac{1}{x}$ E) NOTA

28. Find the sum of all the value(s) of k where $x = k$ is a vertical asymptote for the graph of $g(x) = \frac{x-2}{x^2-5x+6}$.

- A) 3 B) ∞ C) 6 D) 5 E) NOTA

29. The domain of f , written in interval form in the set of real numbers, of $f(x) = \log(x^2 + 6x - 16)$ is:

- A) $(0, \infty)$ B) $(-\infty, 0) \cup (0, \infty)$
C) $(-8, -2)$ D) $(-\infty, -8) \cup (2, \infty)$ E) NOTA

30. If $f(x) = 5$, what is $f(10)$?

- A) 10 B) 5 C) 0 D) 525,600 E) NOTA