Answers:

1. D
2. B
3. B
4. B
5. A
6. C
7. C
8. D
9. D
10. B
11. B
12. C
13. A
14. E
15. B
16. C
17. A
18. B
19. C
20. C
21. D
22. D
23. B
24. C
25. D
26. B
27. A
28. A
29. D
30. B

Solutions:

1. Beginning with the definition of a function raised to a power: $f^6(x) = f(f(f(f(f(f(x)= ((((((x+2)+2)+2)+2)+2)+2) = x + 12$. **D**

2. We need to find a value to plug in so that the function simplifies to $f(x)$. Let $a = x + 3$. Then $x = a - 3$. Thus, $f(a) = (a-3)^2 + 11(a-3) + 2 = a^2 + 5a - 22 \implies f(x) = x^2 + 5x - 22$. **B**

3. Using the same strategy to begin as problem 2, Let $a = \frac{3}{x-1}$. Then $x = \frac{3+a}{a}$. Then $f(a) = \frac{9+6a}{a^2}$ so $f(x) = \frac{9+6x}{x^2}$, which equals 0 when $9 + 6x = 0 \implies x = -1.5$. **B**

4. The way to solve this problem will be to make use of the remainder theorem. Plug in $x = -1$ and the result is your remainder. $f(-1) = 6 - 3 + 2 + 1 + 1 - 20 = -13$. **B**

5. Simply plug in 1 to the following functional equation. $f(1) + 2f(1) = 1 \rightarrow 3f(1) = 1 \rightarrow f(1) = 1/3$. **A**

6. Plugging in $x = 2$ and $1/2$ gives use two equations of two variables. $f(2) + 2f\left(\frac{1}{2}\right) = 4$, $f(1/2) + 2f(2) = 1/4$. Solving system of equations gives $f(2) = -7/6$. **C**

7. Given the functional equation, we can make use of the same strategy as in problems 5 and 6. $f\left(\frac{1}{4}\right) + 24 + 16 = (16+4)f(4)$, $f(4) + 6/4 + 1/16 = (1/16 + 1/4)f(1/4)$. Solving the system yields: $f\left(\frac{1}{4}\right) - 20f(4) = -40$, $-5f(1/4) + 16f(4) = -25$ gives $f(4) = 225/84 = 75/28$. **C**

8. We can generate two equations with two unknowns by substituting in $x = x - 1$ in the given functional equation. $2f(x) + f(1-x) = x^2$, $f(x) + 2f(1-x) = (1-x)^2$. Solving the system gives $f(x) = \frac{x^2+2x-1}{3}$. **D**

9. f and g form a rectangle with vertices $(1.5, 6.5), (2, 7), (3, 5),$ and $(3.5, 5.5)$, which has sides of length $\sqrt{2}/2$ and $3\sqrt{2}/2$, giving perimeter $4\sqrt{2}$, so $a = 4$ and $b = 2$, so $a^b = 4^2 = 16$. **D**

10. Since f satisfies $f(x) \geq 0$ for all x and $f(1) = 0$, $f(x)$ has a double root at $x = 1$. Otherwise, the function would cross into the negative region of the xy-plane. $f(x) = k(x-1)^2$ for some constant k. $f(3) = k(3-1)^2 = 4k = 3 \rightarrow k = 3/4$. $f(5) = \left(\frac{3}{4}\right)(5-1)^2 = 12$. **B**

11. Clearly, $f(x) \leq x$, so we can start with $2, 3, 4, 5, 6, 7, 8, 9$. Then, any other x with $f(x) < 10$ will be one of these numbers multiplied by a prime (or multiple primes) that already divides it. Otherwise, if we multiply by a new prime, then that prime will contribute to f(x) and make it ≥ 10. So we can investigate each of our starting numbers by utilizing casework counting.

$$2 \rightarrow 2n \rightarrow 4, 8, 16, 32, 64$$
$$3 \rightarrow 3n \rightarrow 9, 27, 81$$
$$4 \rightarrow 2n \rightarrow 8, 16, 32, 64$$
$$5 \rightarrow 5n \rightarrow 25$$
$$6 \rightarrow 2m \cdot 3n \rightarrow 12, 18, 24, 36, 48, 54, 72, 96$$
$$7 \rightarrow 7n \rightarrow 49$$
$$8 \rightarrow 2n \rightarrow 16, 32, 64$$
$$9 \rightarrow 3n \rightarrow 27, 81$$

So we have $2, 3, 4, 5, 6, 7, 8, 9, 12, 16, 18, 24, 25, 27, 32, 36, 48, 49, 54, 64, 72, 81, 96$, for a total of 23 numbers that work. **B**

12. The function intersects with its inverse when $f(x) = x$. So we need to solve $x^3 + 3x^2 + 2x = x \rightarrow x^3 + 3x^2 + x = 0$. Since there are three real roots, the sum of the ordinates and abscissas will just be the sum of the roots times 2 $= \left(-\frac{3}{1}\right) * 2 = -6$. **C**

13. Note that we want the average of all cd such that c and d are positive integers and $(x + c)(x + d) = x^2 + ax + b$ (so $a = c + d$ and $b = cd$). For $a = 5$, this is the average of $1 \cdot 4 = 4$ and $2 \cdot 3 = 6$. For smaller a, the average is smaller than a. For larger a, the average is larger than a. **A**

14. This problem will again make use of the remainder theorem. $P(-2) = 48 - 4a - 2b - 2 = 34 \rightarrow 2a + b = 6$. We can actually stop here because this equation gives $2a + b$. **E**

15. Let's convert integers to base 3 starting with 3 and working up until the condition is satisfied. $3 \rightarrow 10$, $4 \rightarrow 11, 5 \rightarrow 12$, $6 \rightarrow 20$, $7 \rightarrow 21$, and 7 divides 21. **B**

16. We must have $\frac{x-5}{x^2-7x+12} \geq 0$ and $x^2 - 7x + 12 \neq 0$. $x^2 - 7x + 12$ factors to $(x - 3)(x - 4)$, so 3 and 4 are not allowed in the domain of f. From the first condition, x must either be between 3 and 4 or greater than 5, giving the domain $(3, 4)U[5, \infty)$. **C**

17. $f(x) = x^3 + ax^2 + bx - 64$. $f(-1) = a - b - 65$. So we need to maximize $a - b$ (minimize the sum of the roots of f and the sum of the roots two at a time of f), which happens when all the roots are the equal ($r_1 = r_2 = r_3 = 4$). So $f(-1) - 12 - 48 - 65 = -125$. **A**

18. $f(2014)*f(2015)=2014,\ f(2015)*f(2016)=2015,\ f(2016)*f(2017)=2016.\ f(2016)=\frac{2015}{f(2015)},\ f(2015)=\frac{2014}{f(2014)}\rightarrow f(2016)=\left(\frac{2015}{2014}\right)*f(2014)\rightarrow \left(\frac{2015}{2014}\right)f(2014)f(2017)=2016\rightarrow\ f(2014)f(2017)=\ 2016*2014/2015\ =$ $(2015-\ 1)/2015\ =\ 2015-\ (1/2015)$, which rounds down to 2015. **B**

19. First notice that the expression we wish to find is equivalent to: $\frac{r^2}{s^2+t^2+u^2+r^2-r^2}+\frac{s^2}{r^2+t^2+u^2+s^2-s^2}+\frac{t^2}{r^2+s^2+u^2+t^2-t^2}+\frac{u^2}{r^2+s^2+t^2+u^2-u^2}$. Hence, we can find the sum of the squares of the roots: $\frac{b^2-2ac}{2a}=0$. So the expression becomes: $\frac{r^2}{-r^2}+\frac{s^2}{-s^2}+\frac{t^2}{-t^2}+\frac{u^2}{-u^2}=-4$. **C**

20. Even functions are symmetric with respect to the y- axis, and have the property that $f(-x)=f(x)$. **C**

21. Let's call the roots $r-2,\ r,\ r\ +\ 2$, so $f(x)=a(x-r+2)(x-r)(x-r-2)$. Then the sum of the roots two at a time is $3r^2-4=8\rightarrow r=\pm 2$. Plugging in to f(x), we see that only r = -2 works, and a = 2, so f(3) = 2(3 + 2 + 2)(3 + 2)(3 + 2 – 2) = 210 **D**

22. p is true. q is false. So a, b, and c are true and d is false. **D**

23. We see from $f(0)=2$ that $E=2$. The other two conditions give $A-B\ +\ C-D+2=6$ and $A+B+C+D+2=10$, which gives that $A+C=6$ and $B+D=2$. B and D must both be 1 and the possible combinations of (A,C) are $(1,5),(2,4),(3,3),(4,2),$ and $(5,1)$, giving 5 combinations. **B**

24. First begin by inserting g into function f. $f\big(g(x)\big)=\frac{3}{x+\frac{1}{x}-2}=\frac{3x}{(x-1)^2}$. The composition of functions has domain $x\neq 1$. $g(x)$ has domain $x\neq 0$. The domain of $(f\circ g)(x)$ is $\{x\in\mathbb{R}\ |x\neq 0,1\}$. **C**

25. The x coordinate of the max value is at $x=-\frac{b}{2a}=\frac{b}{2}$. Plugging in $b/2$ to f gives $\frac{b^2-64}{4}$, so $\frac{b^2-64}{4}=48$. Solving for b gives $b=\pm 16$. **D**

26. f factors to $(x+4)(x-2)^2$, so the roots are 2, 2, and -4. To maximize $a-b+c$, we need to make b the smallest root, hence $a-b+c=2-(-4)+2=8$. **B**

27. It is easiest to see the pattern if the first few terms are simplified out: $f_1(x) = x,\ f_2(x) = \frac{1}{1-x},\ f_3(x) = \frac{x-1}{x},\ f_4(x) = x$. This sequence clearly repeats, with $f_a(x) = x$ whenever $a = 3n + 1$, where n is an integer. $2017 = 3 * 672 + 1$, so $f_{2017}(x) = x$. **A**

28. $g(x) = \frac{x-2}{(x-3)(x-2)}$, so there is a hole at $x = 2$ and a vertical asymptote $x = 3$. **A**

29. We need to solve the inequality $x^2 + 6x - 16 > 0 \rightarrow (x + 8)(x - 2) > 0$. Expression is negative between -8 and 2 and 0 at -8 and 2 so the domain is $(-\infty, -8)U(2, \infty)$. **D**

30. Plugging in is not even necessary. 5. **B**