"For all questions, answer choice "E. NOTA" means none of the above answers is correct."

1. If a and b are the two closest integers to $\log_5 678$ and $a < b, what\ is$ $b^2 - a^2$?

A. 9 B. 11 C. 51 D. 271 E. NOTA

Answer A. $5^4 = 625$ and $5^5 is$ 3125. Therefore a and b are 4 and 5.

2. What is the sum of the x-intercepts of $g(t) = e^{2t} - e^{t+ln9} + 20$

A. -9 B. $\ln(20)$ C. 9 D. e^{20} E. NOTA

Answer B. This is a factorable quadratic in e^t of the form $(e^t - 4)(e^t - 5)$. Thus the solutions are ln4 and ln5.

3. 2^{50} is between $10^x and$ 10^{x-1} where x is an integer. What is the value of x?

A. 10 B. 15 C. 16 D. 17 E. NOTA

Answer C. $2^{50} = 1024^5 = (1000 + 24)^5$ Expanding using Pascal's Triangle or the Binomial Theorem reveals the first term is 10^{15} and that no other terms will influence that order of magnitude.

4. If $\frac{a}{b}$ represents the solution (in simplest terms) to the equation $243^x = 9$, what is $a - b$?

A. -4 B. -3 C. -2 D. -1 E. NOTA

Answer B. 243 is 3^5 therefore $x = \frac{2}{5}$ is the solution to the equation.

5. How many and what type of solutions are there to the equation $\sqrt{4x^2 - 3x + 2} = -\sqrt{x^2 - 5}$

A. no solutions B. 1 real solution C. 2 positive real solutions D. 2 complex solutions E. NOTA

Answer A. Since you have a positive radical equal to a negative radical the only possibility for a solution would be at the zeroes of the radicands. Since these do not overlap, there are no solutions to the equation.

6. If the points (-1, 5) and (1, 20) are on the function $f(x) = ab^x$, what is a^b?

A. 64 B. 81 C. 625 D. 1024 E. NOTA

Answer C. You can plug in the points to create a system and solve it by substitution (or an advanced elimination technique using division instead of subtraction), but you can also notice that the points are two units away from each other along the x-axis and their y-coordinates differ by a factor of 4. This implies b=2. From there it is clear a=10 and the answer would be 100.

7. Which of the following is a solution to the equation $\log_9(x^2) + [\log_3(9e^2)] = 2$?

A. $\frac{\sqrt{e}}{3}$ B. $\frac{e}{9}$ C. $\frac{1}{e}$ D. $\frac{1}{e^2}$ E. NOTA

Answer D. There are many ways to solve this problem. One way is to use the change of base formula and the power property for logs to rewrite the first term as $\log_3 x$. You can then use the product property to rewrite the left side as $\log_3(9e^2x)$. Then raise 3 to both sides, divide by $9e^2$ and you get answer D.

8. If $f(x) = 3e^{x+2} - 1$ then $f^{-1}(x)$ has which of the following asymptotes?

A. x = -2 B. x = -1 C. y = 1 D. y = 2 E. NOTA

Answer B. This function has an asymptote at y = -1 therefore its inverse will have an asymptote that is a graphical reflection across y=x, which is x=-1. Alternatively, you can find $f^{-1}(x) = \ln\left(\frac{x+1}{3}\right) - 2$ and then it's clear that it has x=-1 as an asymptote (and no other).

9. Solve for x in the following equation: $\log_9(\log_2(\log_3(\log_x e))) = 0$

A. No solution B. $\frac{e}{3^{81}}$ C. $\sqrt[9]{e}$ D. e E. NOTA

Answer C. To isolate x you begin undoing the log operations by raising 9 to both sides, then 2, then 3, then finally x. After this you have $e = x^9$ to which the solution is obvious.

10. What is the perimeter of a triangle with sides of length $\log_{30} 8$, $\log_{30} 27$, and $\log_{30} 125$?

A. 3 B. $\sqrt[3]{30}$ C. $\log_{30} 160$ D. 900 E. NOTA

Answer A. By properties of logs the perimeter is $\log_{30}(8 * 27 * 125) = \log_{30}(30^3) = 3$

11. Which of the following is equivalent to $(\log_4 625)\left(\log_5 \frac{1}{7}\right)(\log_{343} 8)$?

A. $-\frac{8}{3}$ B. -2 C. $\frac{1}{2}$ D. $\frac{8}{3}$ E. NOTA

Answer B. Using the change of base formula and commutativity of multiplication, the expression can be rewritten as $(\log_5 625)\left(\log_{343}\frac{1}{7}\right)(\log_4 8) = 4*\left(-\frac{1}{3}\right)*\frac{3}{2} = -2$

12. What is the tens digit of 6^{578}?

A. 0 B. 1 C. 3 D. 9 E. NOTA

Answer B. After a little experimentation one should notice that the tens digits of powers of 6 (starting at 36) follow a pattern of 3, 1, 9, 7, 5 and then repeat. This is simply because you can multiply the previous tens digit by 6 and then add 3 (mod 10) because of the 36 always generated from multiply 6 by the ones digit (which is always 6). This is a pattern of 5 and the exponent is 3 more than a multiple of five. Accounting for the initial offset (the beginning of the sequence starts with the second power of six), this leaves us at 1.

13. If $a = 3^{63}$, $b = 6^{42}$ and $c = 12^{31}$ which of the following represents the order of these numbers from least to greatest?

A. a-b-c B. b-c-a C. c-a-b D. cannot be determined E. NOTA

Answer A. An easy way to compare is to divide two of the numbers and notice whether the quotient is greater or less than 1. In the case of $\frac{b}{a}$, it reduces to $\frac{2^{42}}{3^{21}}$ which simplifies to $\frac{4^{21}}{3^{21}} > 1$ therefore b>a. In the case of $\frac{c}{b}$, it reduces to $\frac{2^{31}}{6^{11}}$ which simplifies to $\frac{2^{20}}{3^{11}} = \frac{4^{10}}{3^{11}}$. At first it may seem difficult to tell if this is greater than 1, but after comparing $4^4 = 256$ with $3^5 = 243$ it becomes obvious that it is and therefore c>b.

14. What is $\log(.2) - \log(2) + \log(20)$

A. $-\log(2)$ B. 0 C. $\log(2)$ D. $\log(8)$ E. NOTA

Answer C. There are many ways to do this using product and quotient properties of logs. One way is to rewrite it as such: $\log(2) - \log(10) - \log(2) + \log(2) + \log(10)$

15. If a type of bacteria has a continuous exponential rate of growth such that, after 2 hours, an initial population of 891 has increased to 1584, how long will it take (in hours) to reach a population of 999,999?

A. $\log_{\frac{3}{4}} 10{,}101$ B. $\log_{\frac{4}{3}} 10{,}101$ C. $\log_{\frac{3}{4}} 101{,}010$ D. $\log_{\frac{4}{3}} 101{,}010$
E. NOTA

Answer E. The model is $y=891\left(\frac{4}{3}\right)^t$, so $999999=891\left(\frac{4}{3}\right)^t \rightarrow \frac{3367}{3}=\left(\frac{4}{3}\right)^t$

$\rightarrow t=\log_{\frac{4}{3}}\frac{3367}{3}$

16. A game company wants to distribute 5,000 numerical game keys (with no restriction on the digits used or repeat digits) to their new game. What is the minimum number of digits these keys should have so that a blind guesser has less than a 1 in a trillion chance of guessing any of the game keys?

A. 9 B. 10 C. 12 D. 13 E. NOTA

Answer E. The following inequality represents the problem: $\frac{5{,}000}{10^x}<\frac{1}{10^{12}}$ which means that $5*10^{15}<10^x$ therefore x=16

17. $\sum_{i=1}^{n}\log_b\frac{i^n}{\sqrt[n]{n!}}$ is equivalent to which of the following expressions?

A. 0 B. $\log_b(n!)$ C. $\log_b[(n!)^{n-1}]$ D. $\log_b[(n!)^{\frac{n^2-1}{n}}]$ E. NOTA

Answer C. The sum of these logs would result in a single log of the product of all the arguments. Now the only part of the argument to the log that changes is the numerator. Therefore, the denominator of the product, if we have n terms in the sum, is simply n!. The numerator is going to be of the form $1^n*2^n*3^n*\ldots(n-1)^n*n^n=(n!)^n$. Dividing these two leaves you with answer C.

18. If a, b, and c are integer side lengths of a right triangle (with c being the hypotenuse) and $\log_5 c=2$, which of the following is a possible value for $a+b$?

A. 7 B. 14 C. 17 D. 31 E. NOTA

Answer D. Raising 5 to both sides of the equation gives us c=25. From here, some trial and error (or good knowledge of Pythagorean Triples!) leaves you with 7 and 24 for a and b, thus giving you answer D (15 and 20 are also possible for the sides, but the problem states which of the following is a possible value for a+b, which answer D satisfies).

19. If a, b, and c are side lengths of a right triangle (with c being the hypotenuse) and $\log_7 a=8$, what is $\log_{49}(b+c)+\log_{49}(c-b)$?

A. 4 B. 8 C. 16 D. cannot be determined E. NOTA

Answer B. The expression can be collapsed to $\log_{49} c^2 - b^2 = \log_{49} a^2 = \frac{\log_7 a^2}{\log_7 49} = \log_7 a = 8$

20. $\sum_{i=3}^{98} \log \frac{n+4}{n+1}$ is equivalent to which of the following?

A. $\log \frac{18}{99!}$ B. $\log \frac{102!}{7!} - \log \frac{99!}{4!}$ C. $\log \frac{51}{2}$ D. $2 + \log 10{,}302$ E. NOTA

Answer E. The easiest way would be to recognize that this is a telescoping series in which all the terms will cancel except for parts of the first and last 3 terms (rewriting the argument to the summation as $\log(n+4) - \log(n+1)$ makes this much more clear). So the whole summation can be reduced to $\log 102 + \log 101 + \log 100 - \log 6 - \log 5 - \log 4 = \log \frac{102*101*100}{6*5*4} = \log 3535$ which is not equivalent to any of the given answers (the closest is B, but it replaces the 6*5*4 on the denominator with 7*6*5).

21. What is $\log_{a^3} \sqrt{a}$?

A. $\frac{1}{a}$ B. $\frac{1}{5}$ C. 5 D. a E. NOTA

Answer E. $(a^3)^{\frac{1}{6}} = \sqrt{a}$, therefore the answer is 1/6.

22. Solve for x in the following equation: $\log(x^5 + 10x^4 + 40x^3 + 80x^2 + 80x + 32) = 10$

A. 8 B. 48 C. 90 D. 98 E. NOTA

Answer D. The argument in the log is a perfect fifth power (as hinted by the leading coefficient and constant term). The equation is $\log((x+2)^5) = 5\log(x+2) = 10$. From there we get that $\log(x+2) = 2$ which implies answer D.

23. $-(i^i)^i$ is equivalent to which of the following?

A. $-i$ B. $\frac{1}{i}$ C. 1 D. i E. NOTA

Answer D. By rules of exponents the expression simplifies to $-i^{-1} = -\frac{1}{i} = i$

24. Simplify $i^{1528} + 2i^{560} - 6i^{303}$

A. $6 - 3i$ B. $6 + 3i$ C. $3 - 6i$ D. $3 + 6i$ E. NOTA

Answer D. Powers of i form a cycle of 4 terms; starting with i^0 we have 1, i, $-i$, and -1. Since the first two terms have exponents that are multiples of 4, the land on the first term of the cycle and therefore add up to 3. The third term has a remainder of 3 when divided by 4, so it lands on the third term of the cycle and is – i, thus producing answer D.

25. Find the sum of all the solutions to the following equation:
$((2\mathrm{x}+\ 5)^{\mathrm{x}+7})^{\mathrm{x}-3} =\ (24\mathrm{x}-72)^{0}$

A. -9 B. -7 C. -6 D. -4 E. NOTA

Answer E. The right hand side is equivalent to 1 so long as $x \neq 3$ (because 0^0 is undefined). So we are looking for all values of x that make the left hand side equal 1, except $x = 3$. There are 3 ways a power can equal 1: a)either the exponent is zero and the base is not or b) the base is 1 or c) the base is negative 1 and the power is even. Each of these can be the case when $x =\ -7, -2, or - 3$ respectively (recall we have already removed $x = 3$ from consideration). Therefore the sum is -12.

26. What is the positive difference between the two solutions to the following equation:
$8\mathrm{x}^{\mathrm{y}-7} =\ 2\mathrm{x}^{\mathrm{y}-8}$, provided y > 8

A. $\frac{1}{4}$ B. $\frac{15}{4}$ C. 4 D. 8 E. NOTA

Answer A. Since $y > 8$, $x = 0$ is one solution. The other we can find by considering that there is 1 more x on the left than on the right, so the equation can be simplified (since we have already considered $x = 0$) to $8x = 2$ which has a solution of $\frac{1}{4}$.

27. Solve for x in the following: $16 * 2^{\mathrm{x}-2} =\ 8^{\mathrm{x}-6}$

A. -10 B. -4 C. $\frac{2}{3}$ D. 10 E. NOTA

Answer D. If one rewrites all powers present with a base of 2 you get $2^4 * 2^{x-2} = 2^{3x-18}$ This yields the equation $x + 2 = 3x - 18$ which has the solution $x = 10$.

Use the following information for problems 28-30: Let a, b, and c be integers such that $a^{bc} < -1 \; and \; 0 < b^{ac} < 1$

28. Based on the above, which of the following MUST be true:

A. c^{ab} is an integer B. c^{ab} < 0 C. $0 < c^{ab} < 1$ D. c^{ab} > 1 E. NOTA

Answer C. The first condition implies that a<0 and that b and c are both odd and the same sign (positive or negative). The second implies b and c are positive (because c must be positive in order for ac to be negative and produce a power in that range). This further implies that ab is negative and therefore answer C must be true.

29. Based on the above, which of the following MUST be false:

A. a is odd B. b is even C. $c > 0$ D. two or more of the above E. NOTA

Answer B. See answer to #28.

30. Based on the above, a^a MUST be:

A. even B. odd C. positive D. A and C E. NOTA

Answer E. Since the information cannot determine if a is even or odd (because with $b>0$, that is enough for the second condition without necessitating that a be even) none of these MUST be true. If a were even, both A and C would be true, and otherwise B would be true.