

“For all questions, answer choice “E. NOTA” means none of the above answers is correct.”

Unless otherwise indicated, you may assume standard notation for a triangle ABC in which capital letters represent angles and lowercase letters represent sides and angle A is opposite side a, angle B is opposite side b, and angle C is opposite side c. For right triangles, C is always 90 degrees and c is always the hypotenuse.

1. Right triangle ABC has  $A = \tan^{-1} \frac{7}{24}$  and  $a = 14$ . Find c.

- A. 25      B. 50      C. 625      D. 1250      E. NOTA

2. Right triangle ABC has sides such that b and c are two consecutive integers and  $a=13$ . What is  $b+c$ ?

- A. 29      B. 39      C. 149      D. 159      E. NOTA

3. If triangle ABC has  $a = 4$  and  $B = 60^\circ$ , for which value of b can a second triangle with these same three measurements (a, B, and b), but not congruent to ABC, be formed?

- A. 2      B.  $2\sqrt{3}$       C. 3.7      D. 4      E. NOTA

4. If triangle ABC has  $a = 4$  and  $B = 60^\circ$ , which value of b is NOT possible?

- A. 2      B.  $2\sqrt{3}$       C. 3.7      D. 4      E. NOTA

5. Which of the following is NOT equivalent to the other three? (If all are equal, choose E)

- A.  $\sin^{-1} \frac{9}{41}$       B.  $\cos^{-1} \frac{40}{41}$       C.  $\cot^{-1} \frac{40}{9}$       D.  $\csc^{-1} \frac{41}{40}$       E. NOTA

6. A right triangle can be used to prove the following property of an ellipse:  $c^2 = a^2 - b^2$ . Which of the following points (relative to the ellipse) is NOT a vertex of that right triangle? (If more than one is not a vertex of the triangle, choose E.)

- A. co-vertex      B. focus      C. center      D. vertex      E. NOTA

7. If a tangent line is drawn to a parabola at a point other than the vertex, let the point where the tangent line and axis of symmetry of the parabola intersect be A, the point of tangency be B, and the focus be C. What kind of triangle must ABC be (be as specific as possible)?

- A. equilateral      B. isosceles      C. right isosceles      D. scalene      E. NOTA

8. Given a triangle ABC, which of the following is NOT equivalent to the other three? (If all are equal, choose E)

- A.  $a^2 + b^2 - 2ab \cos C$       B.  $a^2 - b^2 + 2bc \cos A$       C.  $b^2 - a^2 + 2ac \cos B$   
D.  $\frac{\sin^2 C}{[\sin(A)]^2}$       E. NOTA

9. In triangle ABC,  $a = 67$  and  $b = 99$ . Which of the following is NOT a possible value for  $c$ ?

- A. 33      B. 83      C. 133      D. 183      E. NOTA

10. An equilateral triangle is divided into 4 smaller triangles by its 3 mid-segments. One of these triangles is shaded black and the other three are each further subdivided in the same way. One-quarter of the smallest new triangles produced this way are shaded black and all others are again subdivided. If this pattern continues infinitely many times, how much of the triangle will be shaded black?

- A.  $\frac{1}{3}$       B.  $\frac{2}{5}$       C.  $\frac{7}{16}$       D.  $\frac{1}{2}$       E. NOTA

Use the following for questions 11-13:  $\triangle XYZ$  has  $XY = 6$ ,  $m\angle ZXY = 45$ , and  $XY \perp YZ$ . Points  $A(2,1)$  and  $B(2,7)$  are on  $\triangle ABC$ , which lies entirely in the first quadrant, and  $\triangle ABC \cong \triangle XYZ$ .

11. What are the coordinates of C?

- A. (-6,7)      B. (5,4)      C. (8,1)      D. (8,7)      E. NOTA

12. What are the coordinates of the centroid of ABC?

- A. (4, 4)      B. (4,5)      C. (5,4)      D. (5,5)      E. NOTA

13. What is the radius of the circle that circumscribes ABC?

- A.  $2\sqrt{3}$       B.  $3\sqrt{2}$       C.  $2\sqrt{5}$       D.  $3\sqrt{3}$       E. NOTA

Use the following information for questions 14-16: Triangle DEF has an inscribed circle and a circumscribed circle with the same center C. The circumscribed circle has circumference  $24\pi$ .

14. What is  $m\angle DCE + m\angle CFE$ , in degrees?

- A. 60      B. 90      C. 120      D. 150      E. NOTA

15. What is the altitude of DEF?

- A. 12      B. 16      C. 18      D. 24      E. NOTA

16. What is the perimeter of DEF?

- A.  $9\sqrt{3}$       B.  $12\sqrt{3}$       C.  $27\sqrt{3}$       D.  $36\sqrt{3}$       E. NOTA

17. What are the interior angles, in degrees, of the triangular region bounded by the y-axis and the lines  $y = \sqrt{3}x - 5$  and  $y = -\sqrt{3}x + 3$ ?

- A. 30-30-120      B. 30-60-90      C. 45-45-90      D. 60-60-60      E. NOTA

18. What are the coordinates of the other vertex (in the first quadrant) of an equilateral triangle with one vertex at the origin and another at (0, 4)?

- A.  $(\frac{2\sqrt{3}}{3}, 2)$       B.  $(\sqrt{3}, 2)$       C. (3, 2)      D.  $(2\sqrt{3}, 2)$       E. NOTA

19. A farmer wants to build a fence surrounding a portion of field in the shape of a right triangle with one of its sides bordered by a river (so that no fencing is required on that side). If he only has 100 feet of fence what is the largest such area he can enclose (in  $ft.^2$ )?

- A. 1152      B. 1250      C. 1825      D. 2500      E. NOTA

20. If triangle ABC has an enclosed area of  $6\sqrt{3}$ , a perimeter of 18, and a side of length 8, what is the product of the other two side lengths?

- A. 9      B. 16      C. 21      D. 25      E. NOTA

21. If an acute triangle has sides of length 9 and 12 and an area of  $27\sqrt{3}$ , what is the measure of the angle (in degrees) formed by the two given sides?

- A. 30      B. 45      C. 60      D. 90      E. NOTA

22. Which of the following is NOT a triangle congruence shortcut?

- A. AAS      B. ASA      C. SSA      D. SAS      E. NOTA

23. A triangle has vertices at (0, 6), (3, 0) and the origin. What is the area of the largest rectangle (with one vertex at the origin) inscribed in such a triangle?

- A. 1.5      B. 3      C. 4.5      D. 5      E. NOTA

24. In Square ABCD, let E and F be the midpoints of two opposite sides and P be the midpoint of EF. If a line segment was drawn from point P to each of the other six points mentioned and line segments between any two points on one side of the square, what are the angle measures (in degrees) of the smallest triangles (by area) formed by doing this?

- A. 20-70-90    B. 30-60-90    C. 40-50-90    D. 45-45-90    E. NOTA

25. Which of the following expresses the area of an isosceles right triangle in terms of its perimeter?

- A.  $\frac{P^2(3-2\sqrt{2})}{2}$     B.  $\frac{P^2(3+2\sqrt{2})}{2}$     C.  $P^2(3-2\sqrt{2})$     D.  $P^2(3+2\sqrt{2})$     E. NOTA

26. Find  $\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{4}{5}$ , in degrees.

- A. 90    B. 180    C.  $\sin^{-1}\frac{3}{4}$     D.  $\sin^{-1}\frac{7}{5}$     E. NOTA

27. Given the following system, find  $\frac{5bc}{26}$  (all variables are positive):

$$\begin{aligned} a^2 + b^2 &= c^2 \\ b^2 &= c^2 + a^2 - \frac{10ac}{13} \\ a^2 &= c^2 + b^2 - \frac{24bc}{13} \end{aligned}$$

- A.  $\frac{25}{2}$     B. 25    C. 30    D. 60    E. NOTA

28. Line segment AF is subdivided into 5 smaller segments by points B, C, D, and E (in that order from A to F) such that AB = EF and BC=DE. If B, C, D, and E were allowed to act as pivots so that the line segments could rotate about them arbitrarily, and they were rotated in such a way as to produce three congruent equilateral triangles then what is the ratio of AB:BC:CD?

- A. 1:1:1    B. 1:2:3    C. 2:1:3    D. 3:1:2    E. NOTA

29. Which of the following represent the side lengths of a right triangle?

- A. 37-679-680    B. 39-755-756    C. 41-840-841    D. 43-934-935    E. NOTA

30. Triangle XYZ is inscribed in circle C. Arc XY measures 50 degrees and angle YCZ measures 100 degrees. What is the measure of angle XYZ, in degrees?

- A. 30    B. 75    C. 105    D. 120    E. NOTA