1. In order to find the value of $x^3 + y^3$, examine its factors: $(x + y)(x^2 – xy + y^2)$. We know $x + y = 11$ and $x^2 + y^2 = 325$. Therefore $(x + y)^2 = 121 = x^2 + 2xy + y^2$. If we subtract the two equations $(x^2 + y^2 = 325) – (x^2 + 2xy + y^2 = 121)$, we obtain $-2xy = 204$, therefore $xy = -102$. Going back to the factors of $x^3 + y^3$, $(x + y)(x^2 – xy + y^2)$, and substituting, we have 11(325+102) = 11(427) = **4697.**
2. To find the sum of the reciprocals of two numbers a and b, simplify the expression $\frac{1}{a}+\frac{1}{b}$ to $\frac{a+b}{ab}$, which is the quotient of the sum and the product of the two numbers, or in this particular case, $-\mathbf{\frac{4}{3}}$.
3. Transforming the equation into standard form by completing the square, we get $\frac{(x+3)^2}{9}+\frac{(y+4)^2}{4}=1$, which tells us the center is (-3, -4), $a = 3$, and $b = 2$. Therefore $c = \sqrt{5}$ and the distance between the foci is $\mathbf{2\sqrt{5}}$.
4. Consider Kay and Thom as one entity; then we are arranging 7 people in a line, which can be done in 7! ways. Since Kay and Thom can be arranged in 2! ways, the total number of possible line ups is 7!2! , or **10,080.**
5. By synthetic substitution we have

$$\begin{array}{r|cccc} 4 & 2 & -5 & c & 6 \\ & & 8 & 12 & 92 \\ \hline & 2 & 3 & (c+12) & 98 \end{array}$$

, so $4(c + 12) = 92$, and $c =$ **11.**
6. Use the formulas for the sum and product of the roots of a quadratic equation. The sum: = 4; the product $\frac{c}{a} = \left(2+\frac{i\sqrt{3}}{2}\right)\left(2-\frac{i\sqrt{3}}{2}\right) = 4+\frac{3}{4}=\frac{19}{4}$. Getting a common denominator gives us $a = 4$, $b = -16$, and $c = 19$, so the equation is $\mathbf{4x^2 -16x + 19 = 0}$.
7. Converting radicals to rational exponents we have: $\left(5^2\cdot 5^{1/3}\right)^{1/2}\cdot\left(5\cdot 5^{1/2}\right)^{1/3}=5^m$. This simplifies to $\left(5\cdot 5^{1/6}\right)\left(5^{1/3}\cdot 5^{1/6}\right)=5^{10/6}=5^{5/3}$, so $m = \mathbf{\frac{5}{3}}$ **or** $\mathbf{1\frac{2}{3}}$
8. 6th term = $\frac{8!}{3!5!}\left(\frac{x}{2}\right)^3\left(-\frac{2}{x}\right)^5=\frac{8\cdot 7\cdot 6}{6}\cdot\frac{x^3}{8}\cdot\frac{-32}{x^5}=\frac{7(-32)}{x^2}=\mathbf{\frac{-224}{x^2}}$.
9. Using expansion by minors and the second row we have $-x(-2 – 3x) + 1(-4 + x) = 2$. This simplifies to $3x^2 + 3x - 4 = 2$, or $3x^2 + 3x - 6 = 0$, or $x^2 + x - 2 = 0$. Factoring gives us $(x + 2)(x – 1) = 0$, so the solutions are $x = -2$, or 1. The smaller of these is **-2.**
10. Factoring 26,390 we have 13 • 2030 = 13•203 • 10= 13•7•29•2•5. Since the three ages must be greater than 19, we need to separate these prime factors into 3 composite factors, each of which is at least 20. By inspection or trial and error, we get 26•35•29. The sum of these is **90.**
11. The only possible integral roots are -4, -2, -1, 1, 2 , and 4. We can form an arithmetic progression using either -4, -1, and 2 or -2, 1, and 4. Examining the corresponding factors, the correct choice is $(x + 4) (x +1)(x – 2)$ because this gives the constant term -8, so $A =$ **3.**
12. $\frac{13}{52}\bullet\frac{12}{51}+\frac{12}{52}\bullet\frac{11}{51}-\frac{3}{52}\bullet\frac{2}{51}=\frac{1}{4}\bullet\frac{4}{17}+\frac{1}{13}\bullet\frac{11}{17}-\frac{1}{26\bullet 17}=\mathbf{\frac{47}{442}}$
13. The number of zeros depends upon how many factors of 5 are in the factorial. Six factors of five occur in the values of 25! up through 29!; seven factors of 5 occur at 30!. The only prime between 25 and 30 is **29.**
14. Simplifying and rationalizing the denominator: $\frac{1}{2-7i}\bullet\frac{2+7i}{2+7i}=\frac{2+71}{53}=\mathbf{\frac{2}{53}}+\mathbf{\frac{7}{53}i}$.
15. Rewriting without absolute value, we have $2< 3x + 1 \le 7$ or $-2 > 3x + 1 \ge -7$. This then solves to give us $(1/3) < x \le 2$ OR $-1 > x \ge (-8/3)$. The integers in this solution are **1, 2, and -2.**

16. Using the formula for partial sum of an arithmetic series: -120 = (n/2)(13 + 13 + -3(n-1)); this simplifies to -240 = n(29 – 3n); -240 = $-3n^2 + 29n$; and then $3n^2 - 29n - 240 = 0$. Factoring strategies are helpful here. When the middle term is odd, then pairs of even factors cannot be used. The prime factorization of 240 is $2^4 \bullet 3 \bullet 5$. The only way to factor without a pair of even factors is 15•16. This works perfectly with the coefficient of 3 to yield the middle term of -29n: (3n+16)(n-15). The only feasible solution is n = **15**.
17. Converting to inequalities without absolute value we have $x^2 - 12x + 18 < 18$ and $x^2 + 12x + 18 > -18$, which becomes $x^2 - 12x < 0$ and $x^2 + 12x + 36 > 0$. Further simplifying leads to $x(x - 12) < 0$ and $(x - 6)^2 > 0$. These have solution $0 < x < 12$ and $x \neq 6$. Interval notation: **(0, 6) (6, 12).**
18. Let N = 0.13888…. Then 10N = 1.3888…. Subtracting the two gives us 9N = 1.25, so N = 1.25/9. This simplifies to **5/36**.
19. Subtracting the second equation from the first gives us $y^2 - 2y = 3$, or $y^2 - 2y - 3 = 0$ which factors to $(y - 3)(y + 1) = 0$, so $y = 3$ or $y = -1$. Substituting we find that when $y = 3$, non-real roots of x occur. If $y = -1$, then $x = \pm\sqrt{11}$. Therefore the real solutions of the system are $(\sqrt{11}, -1)$ *and* $\left(-\sqrt{11}, -1\right)$.
20. Multiply by the inverse of $\begin{bmatrix} -7 & 2 \\ 3 & 1 \end{bmatrix}$. This gives us

$$X = -\frac{1}{13} \cdot \begin{bmatrix} 1 & -2 \\ -3 & -7 \end{bmatrix} \cdot \begin{bmatrix} -23 & -18 & -11 \\ 8 & 17 & 1 \end{bmatrix} = -\frac{1}{13} \cdot \begin{bmatrix} -39 & -52 & -13 \\ 13 & -65 & 26 \end{bmatrix} = \begin{bmatrix} \mathbf{3} & \mathbf{4} & \mathbf{1} \\ \mathbf{-1} & \mathbf{5} & \mathbf{-2} \end{bmatrix}.$$

21. First , look at the 2-digit perfect squares: 16, 25, 36, 49, 64, 81. 16 is a factor of 64, and 36 is a factor of 64•81, so we need to determine the LCM of 25, 49, 64, and 81. Each of these numbers is a power of a prime number, so the LCM is the product of 25, 49, 64, and 81 . We want the square root of that product, which is 5•7•8•9 = **2,520**.
22. By multiplying the two polynomials, or by noticing that this is the factored form of the difference of two cubes, we see their product is $x^2 - 10$. Substituting we get 169 - 10 = **159.**
23. Multiply both sides by 5^x, getting $(5^x)^2 - 7(5^x) + 10 = 0$. This can be solved by factoring: $(5^x - 2)(5^x - 5) = 0$, so $5^x = 2$ or $5^x = 5$. The roots are 1 and $\log_5 2$; the irrational root is $\mathbf{\log_5 2}$.
24. The slant asymptote is the linear expression that is the quotient, found by long division:

$$\begin{array}{r} 2x - 11 \\ x^2 + 3x - 1 \overline{\big)\, 2x^3 - 5x^2 + 7} \\ \underline{2x^3 + 6x^2 - 2x} \\ -11x^2 + 2x + 7.... \end{array}$$

. The slant asymptote is $y = 2x - 11$. The y-intercept is -11.

25. $\frac{1}{18} + \frac{1}{30} = \frac{1}{x} \Rightarrow 5x + 3x = 90 \Rightarrow x = \frac{45}{4} = \mathbf{11\frac{1}{4}}$ **minutes.**