1. $\sin 4$
2. $e+3$
3. 0
4. $\frac{7}{3}$
5. -1

6. $\pi - \frac{3\sqrt{3}}{2} = \frac{1}{2}\left(2\pi - 3\sqrt{3}\right)$, or equivalent
7. $\frac{1}{9}$
8. $\frac{p}{4}$
9. $\frac{1}{2}$
10. $\frac{1}{e^6}$

11. $-\frac{5}{9}$
12. $\frac{24p}{5}$
13. 2
14. $\frac{1}{3}$
15. $\frac{\sqrt{2}}{2}$

16. $\frac{p}{2}$
17. 0
18. 3
19. -2
20. 1

21. $-\frac{9}{2}$
22. $3x^2 e^{x^6}$
23. 3
24. $\frac{\sqrt{3}}{2}$
25. 1

1. $\lim_{h\to 0}\frac{1}{h}\int_2^{2+h}\sin\left(x^2\right)\,dx=\lim_{h\to 0}\frac{\int_2^{2+h}\sin\left(x^2\right)\,dx}{h}=\lim_{h\to 0}\sin\left((2+h)^2\right)=\sin 4$

2. $\frac{dy}{dx}=3x^3(y-3)\to y=Ce^{x^2}+3\to y=e^{x^2}+3\to y(1)=e+3$

3. $f(g(x))=x\to\lim_{x\to 0}\frac{f(g(x))-x}{x}=\frac{1-1}{1}=0$

4. Newton's Method: $x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}$

$f(x)=x^2-5,\ f'(x)=2x$

$x_1=1+\frac{4}{2}=3$

$x_2=3-\frac{4}{6}=\frac{7}{3}$

5. $y=x^{x^x}\to y'=x^{x^x}\left(\frac{x^x}{x}+\ln x\left(x^x\right)(\ln x+1)\right)\to y'(1)=1(1+0)=1\to$ normal slope is -1

6. $\frac{1}{2}\int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}}(1+2\cos x)^2\,dx=\pi-\frac{3\sqrt{3}}{2}$

7. $\frac{dx}{dt}=6t^2,\frac{dy}{dx}=4t^3\to\frac{dy}{dx}=\frac{2}{3}t\to\frac{d^2y}{dx^2}=\frac{2}{3\left(6t^2\right)}\to\left.\frac{d^2y}{dx^2}\right|_{t=1}=\frac{1}{9}$

8. $\int_0^1\frac{dx}{x^2-2x+2}=\int_0^1\frac{dx}{(x-1)^2+1}=\int_{-1}^0\frac{du}{d^2+1}=\left[\arctan u\right]_{-1}^0=\frac{\pi}{4}$

9. Maximize $f(x)=x-x^2\to x=\frac{1}{2}$

10. $\lim_{x\to 0}(1+3x)^{-\frac{2}{x}}=\left(\lim_{x\to 0}(1+3x)^{\frac{1}{x}}\right)^{-2}=\frac{1}{e^6}$, using definition of e

11. $\ln y=\ln(t+1)+\ln(t-1)-\ln(t-2)-\ln(t+3)\to y'=-\frac{5}{9}$

12. Shell Method: $V=2\pi\int_a^b xf(x)\,dx=2\pi\int_1^3 x(x-1)(x-3)^2\,dx=\frac{24\pi}{5}$

13. $V=\frac{4}{3}\pi r^3 \rightarrow \frac{dV}{dt}=4\pi r^2\frac{dr}{dt}=8\pi r^2 \rightarrow \frac{dr}{dt}=2$

14. $y=\sqrt{x+y} \rightarrow \frac{dy}{dx}=\frac{1+\frac{dy}{dx}}{2\sqrt{x+y}} \rightarrow \left.\frac{dy}{dx}\right|_{(x,y)=(2,2)}=\frac{1}{3}$

15. $\int_0^{\pi/4}\sqrt{1+\cos 4x}\ dx=\frac{1}{2}\int_0^{\pi/2}\sqrt{1+\cos 2u}\ du=\frac{\sqrt{2}}{2}\int_0^{\pi/2}\cos u\,du=\frac{\sqrt{2}}{2}$

16. $\lim_{n\to\infty}\sum_{i=0}^{n-1}\frac{1}{\sqrt{n^2-i^2}}=\lim_{n\to\infty}\sum_{i=0}^{n-1}\left(\frac{1}{\sqrt{1-\left(\frac{i}{n}\right)^2}}\right)\left(\frac{1}{n}\right)=\int_0^1\frac{1}{\sqrt{1-x^2}}=\left[\arcsin x\right]_0^1=\frac{\pi}{2}$

17. $\lim_{n\to\infty}\left(-\frac{n!}{e^{n^2}}\right)=0$, because denominator blows up faster than numerator

18. $\lim_{n\to\infty}\left|\frac{x^{n+1}}{(n+1)\sqrt{n+1}3^{n+1}}\cdot\frac{n\sqrt{n}3^n}{x^n}\right|<1 \rightarrow \frac{|x|}{3}<1 \rightarrow$ radius of convergence is 3

19. $f'(x)=6x^2+2Ax+B \rightarrow \begin{cases}0=6-2A+B\\ 0=\frac{3}{2}+A+B\end{cases} \rightarrow \begin{matrix}A=\frac{3}{2}\\ B=-3\end{matrix} \rightarrow \frac{B}{A}=-2$

20. $\int_{-2}^{3}\frac{|x|}{x}\,dx=-\int_{-2}^{0}dx+\int_0^3 dx=-2+3=1$

21. $\sin u=u-\frac{u^3}{3!}+\frac{u^5}{5!}-\ldots \rightarrow \sin 3x=3x-\frac{(3x)^3}{3!}+\frac{(3x)^5}{5!}-\ldots \rightarrow$ coefficient of x^3 term is $-\frac{9}{2}$

22. $\frac{d}{dx}\int_4^{x^3}e^{t^2}\,dt=3x^2e^{x^6}$

23. $b=1$ for limit to be finite; $a=\pm 2$ by two rounds of L'Hospital's $\rightarrow 1+2=3$

24. $f(x)=\sin x \rightarrow f'\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}$

25. $\int_1^{\infty}\frac{\ln x}{x^2}dx=\left[-\frac{1}{x}-\frac{\ln x}{x}\right]_1^{\infty}=1$, by parts where $u=\ln x$ and $dv=\frac{dx}{x^2}$