Answers:

1. $\boldsymbol{1/5}$
2. $\frac{1}{2}\boldsymbol{tan(x^2)+C}$
3. $\boldsymbol{-6/e^2}$
4. $-\frac{1}{2e^2}$
5. $\boldsymbol{15/2}$
6. $\boldsymbol{-6}$
7. $\boldsymbol{8e^4}$
8. $\boldsymbol{2/3}$
9. $\boldsymbol{2\frac{17}{18}}$ **or** $\boldsymbol{53/18}$
10. $\boldsymbol{36}$
11. $\boldsymbol{2\sqrt{3}/3}$
12. $\boldsymbol{-1/2}$
13. $\boldsymbol{32+64ln(2)}$
14. $\boldsymbol{400}$
15. $\boldsymbol{1/10}$
16. $\boldsymbol{(4-\pi)/4}$
17. $\boldsymbol{61/27}$
18. $\boldsymbol{130}$
19. $\boldsymbol{2/3}$
20. $\boldsymbol{0}$
21. $\boldsymbol{log_{4/3}}$ **2 or equivalent**
22. $\boldsymbol{\pi/2}$
23. $\boldsymbol{100\sqrt{2}-200/3}$
24. $\boldsymbol{\pi\sqrt{3}/9}$
25. $\frac{3\sqrt{3}+1}{8}$

Solutions:

1. $\lim_{x\to 0} x\cot 5x = \lim_{x\to 0}\frac{x\cos 5x}{\sin 5x} = \lim_{x\to 0}\frac{5x\cos 5x}{5\sin 5x} = \frac{1}{5}(1)(1) = \boldsymbol{1/5}$

2. $\int x\sec^2 x^2\, dx = \int \frac{1}{2}\sec^2 u\, du = \frac{1}{2}\tan(u) + C = \boldsymbol{\frac{1}{2}\tan(x^2)+C}$

3. Set the two parts of the function and their derivatives equal. Then solve the system of equations: $e^{-1} = a+b,\ -2e^{-1} = a \to a = -\frac{2}{e},\ b = \frac{3}{e}.\ ab = \boldsymbol{-6/e^2}$

4. You can use L'Hospital's rule or rearrange this limit and recognize it as the definition of a derivative for $f(x) = \ln x$ at $x = e^2$: $\lim_{x\to 0}\frac{2-\ln(x+e^2)}{2x} = -\frac{1}{2}\lim_{x\to 0}\frac{\ln(e^2+x)-\ln(e^2)}{x} = -\boldsymbol{\frac{1}{2e^2}}$

5. $\int_0^2 (2x^2+1)dx \approx \frac{1}{2}\left(\frac{1}{2}\right)\left(1+2\left(\frac{3}{2}\right)+2(3)+2\left(\frac{11}{2}\right)+9\right) = \frac{1}{4}(30) = \boldsymbol{15/2}$

6. For odd functions $\int_a^b f(x)dx = -\int_{-b}^{-a} f(x)dx$. So $\int_{-4}^{0} f(x)dx = -10$. $\int_{-4}^{-2} f(x)dx = \int_{-4}^{0} f(x)dx - \int_{-2}^{0} f(x)dx = -10-(-4) = \boldsymbol{-6}$

7. Velocity is the derivative of position: $\frac{dy}{dx} = \frac{dy}{dt}\left(\frac{dt}{dx}\right) = 2te^{t^2}(t)$. $y'(2) = 8e^4$.

8. $h'(x) = \frac{f(x)f'(g(x))g'(x) - f(g(x))f'(x)}{(f(x))^2}$. $h'(1) = \frac{(3)f'(2)(2)-f(2)(1)}{3^2} = \frac{6}{9} = \boldsymbol{\frac{2}{3}}$

9. $\sqrt[4]{75} \approx \sqrt[4]{81} + \Delta x f'(81) = 3 - 6\left(\frac{1}{4(81)^{3/4}}\right) = 3 - 1/18 = \boldsymbol{2\frac{17}{18}}$ **or** $\boldsymbol{53/54}$

10. Diagonal $= s\sqrt{3}$, so $\frac{ds}{dt}\sqrt{3} = 3 \to \frac{ds}{dt} = \sqrt{3}$. $A = 6s^2 \to \frac{dA}{dt} = 12s\frac{ds}{dt} = 12\sqrt{3}(\sqrt{3}) = \mathbf{36}$

11. The slope between the two points is $\frac{6-0}{2-0} = 3$. $f'(x) = 3x^2 - 1 = 3 \to x = \pm 2\sqrt{3}/3$. The negative solution is out of the range $[0, 2]$, so the answer is $\mathbf{2\sqrt{3}/3}$.

12. Look at the ratio of the coefficients which is $1/2$. The square root on the top makes the top value positive, but the bottom value is negative, giving us $\mathbf{-1/2}$.

13. $y = x^{x^2} \to y = x^2 \ln x \to \frac{y'}{y} = (x + 2x\ln x) \to f'(2) = 2^{2^2}(2 + 4\ln 2) = \mathbf{32 + 64\ln 2}$

14. $\frac{dP}{dt} = kP(2000 - 5P) = \frac{kp}{2000}\left(1 - \frac{P}{400}\right)$. The population stops increasing $\left(\frac{dp}{dt} = 0\right)$ at $P = 400$, so this is the carrying capacity of the population. $\mathbf{400}$

15. $\frac{d}{dx}f^{-1}(4) = \frac{1}{f'(f^{-1}(4))}$. We need to both find $f^{-1}(4)$ and $f'(x)$. $4 = x^3 - 3x^2 + x + 1 \to f^{-1}(4) = 3$. $f'(x) = 3x^2 - 6x + 1 \to f'(f^{-1}(4)) = 10$. $\frac{d}{dx}f^{-1}(4) = \frac{1}{f'(f^{-1}(4))} = \mathbf{\frac{1}{10}}$

16. Add and subtract 1 from the top: $\int_0^1 \frac{x^2}{1+x^2}dx = \int_0^1 1 - \frac{1}{1+x^2}dx = x - \arctan x|_0^1 = \mathbf{1 - \frac{\pi}{4}}$

17. The first point is at $t = 0$ and the second is at $t = 1$. Curve length is given by $\int_0^1 \sqrt{(x'(t))^2 + (y'(t))^2}dt = \int_0^1 \sqrt{(3t^2)^2 + (4t)^2}dt = \int_0^1 t\sqrt{9t^2 + 16}dt$. Use u-sub of $u = 9t^2 + 16$. $\frac{1}{18}\int_{16}^{25}\sqrt{u}\,du = \frac{1}{18}\left(\frac{2}{3}\right)u^{\frac{3}{2}}\Big|_{16}^{25} = \mathbf{61/27}$

18. Before differentiating, take the natural logarithm of both sides and use log laws:

$$\ln y = 2\ln(x+3) + \ln(x+1) - 2\ln(x-3) - \ln(x-1)$$
$$y' = y\left(\frac{2}{x+3} + \frac{1}{x+1} - \frac{2}{x-3} - \frac{1}{x-1}\right) \to f'(2) = f(2)\left(\frac{26}{15}\right) = \mathbf{130}$$

19. The curves intersect at $x = -1, 0, 1$. The first and third quadrant areas are the same, so the area is $2\int_0^1 (x - x^5)dx = 2\left(\frac{x^2}{2} - \frac{x^6}{6}\Big|_0^1\right) = 2\left(\frac{1}{3}\right) = \mathbf{2/3}$

20. You can take the derivative and take the limit, but if you recognize that $f(x)$ approaches $f(x) = e$ as $x \to \infty$, then you can conclude that $\lim_{x\to\infty} f'(x) = \mathbf{0}$

21. We get the differential equation $\frac{dA}{dt} = kA \to A = Ce^{kt}$. We know $A(0) = 10,\ A(1) = 7.5$ which gives us $A = 10e^{(\ln 3/4)t}$. When $A = 5$, we solve to get $t = \mathbf{\log_{3/4} 1/2} = \boldsymbol{log_{4/3}\, 2}$

22. Volume is given by $\pi\int_0^\infty (e^{-x})^2 dx = \lim_{a\to\infty} \pi\left(-\frac{1}{2}e^{-2x}\right)\Big|_0^a = -\frac{\pi}{2}\lim_{a\to\infty}(e^{-2a}-1) = \boldsymbol{\pi/2}$

23. Take the derivative and set equal to zero: $P'(x) = \frac{5}{\sqrt{x}} - \frac{1}{3} = 0 \rightarrow x = 225$. This is greater than the maximum capacity of the shop, so we check end points for the answer: $P(0) = 0$, $P(200) = \mathbf{100\sqrt{2} - 200/3.}$

24. This question deals with the limit definition of an integral:

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{n}{n^2+3i^2} = \frac{1}{\sqrt{3}}\lim_{n\to\infty}\frac{\sqrt{3}}{n}\sum_{i=1}^{n}\frac{1}{1+\left(\frac{i\sqrt{3}}{n}\right)^2} = \frac{1}{\sqrt{3}}\int_0^{\sqrt{3}}\frac{1}{1+x^2}dx$$

$$= \frac{1}{\sqrt{3}}\left(\arctan\sqrt{3} - \arctan 0\right) = \boldsymbol{\frac{\pi\sqrt{3}}{9}}$$

25. Split up the integral: $F(x) = \int_{\cos x}^{\sin x}(1-t^2)\,dt = \int_a^{\sin x}(1-t^2)\,dt - \int_a^{\cos x}(1-t^2)\,dt$ for some constant a. Now apply the 2nd fundamental theorem of calculus: $F'(x) = (1-\sin^2 x)\cos x + (1-\cos^2 x)\sin x = \cos^3 x + \sin^3 x$. $F'\left(\frac{\pi}{6}\right) = \boldsymbol{\frac{3\sqrt{3}+1}{8}}$