Hustle – Trig
2016
Mu Alpha Theta National Convention

1. $\frac{4p}{3}$
2. $-\frac{1}{2}$
3. $-\frac{5\sqrt{26}}{26}$
4. 30
5. 11
6. $-\frac{1}{8}i$
7. 20p
8. $x = \frac{31}{8}\pi$
9. $\frac{1}{4}$
10. 180
11. none (0)
12. $\frac{\sqrt{5}}{3}$
13. 37.5 (degrees)
14. 2
15. 2
16. $-\frac{1}{5}$
17. 8
18. 4p
19. $\frac{53}{12}p$
20. $72\sqrt{2}$
21. $\frac{240}{161}$
22. $\frac{2}{3}$
23. 8
24. 120°
25. $\frac{6k}{k^2+9}$

1. $4\sin x\cos x=\sqrt{3}\rightarrow 2\cdot 2\sin x\cos x=\sqrt{3}\rightarrow 2\sin 2x=\sqrt{3}\rightarrow \sin 2x=\frac{\sqrt{3}}{2}$

   $2x=\frac{\pi}{3},\frac{2\pi}{3},\frac{7\pi}{3},\frac{8\pi}{3}\rightarrow x=\frac{\pi}{6},\frac{\pi}{3},\frac{7\pi}{6},\frac{4\pi}{3}\Rightarrow \frac{4\pi}{3}.$

2. $\tan\frac{\pi}{4}\sin\frac{11\pi}{4}\cot\frac{18\pi}{4}+\sec 5\pi\cos\frac{\pi}{6}\tan\frac{7\pi}{6}\rightarrow (1)\left(\frac{\sqrt{2}}{2}\right)(0)+(-1)\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{3}\right)\Rightarrow -\frac{1}{2}.$

3. Based on the given information, the terminal side of A is in quadrant IV, so the terminal side of $\frac{1}{2}A$ is in quadrant II. $\cos\frac{1}{2}A=-\sqrt{\frac{1+\cos A}{2}}\rightarrow -\sqrt{\frac{1+\frac{12}{13}}{2}}\Rightarrow -\sqrt{\frac{25}{25}}\rightarrow -\frac{5\sqrt{26}}{26}.$

4. Arc length is found by $s=r\theta$, so we have $s=(6)(3)=18$. Adding the two radii gives a total of 30.

5. The two angles will be complementary, so $3x+10+4x+3=90\rightarrow x=11.$

6. $\left[2\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)\right]^{-3}\rightarrow 2^{-3}\left[\cos\left(-3\cdot\frac{\pi}{6}\right)+i\sin\left(-3\cdot\frac{\pi}{6}\right)\right]\rightarrow \frac{1}{8}(0-i)=-\frac{1}{8}i.$

7. $2\sin^2 x-2\sin^2 x\cos x-\sin x\cos x+\sin x=0\rightarrow 2\sin^2 x(1-\cos x)-\sin x(\cos x-1)=0\rightarrow$ $2\sin^2 x(1-\cos x)+\sin x(1-\cos x)=0\rightarrow (1-\cos x)(\sin x)(2\sin x+1)=0.$ Setting these three factors equal to 0, we get $0, 2\pi, 4\pi; 0, \pi, 2\pi, 3\pi, 4\pi; \frac{7}{6}\pi, \frac{11}{6}\pi, \frac{19}{6}\pi, \frac{23}{6}\pi.$ The sum of the distinct solutions is 20π.

8. One asymptote will occure when $3x+\frac{7}{8}\pi=\frac{\pi}{2}\rightarrow x=-\frac{\pi}{8}$. The period of the function is $\frac{\pi}{3}$ so the asymptotes are $\frac{\pi}{3}$ apart. We want to get close to 4π without going past it.

   $-\frac{\pi}{8}+\frac{\pi}{3}n=4\pi\rightarrow n=12\tfrac{7}{8}.\quad x=-\frac{\pi}{8}+12\left(\frac{\pi}{3}\right)=\frac{31}{8}\pi.$

9. $\lim_{\theta\to 0}\frac{1-\cos\theta}{2\sin^2\theta}\rightarrow \lim_{\theta\to 0}\frac{1-\cos\theta}{2\left(1-\cos^2\theta\right)}\rightarrow \lim_{\theta\to 0}\frac{1}{2(1+\cos\theta)}\rightarrow \frac{1}{2(1+1)}=\frac{1}{4}.$

10. $\sin 1^\circ=\cos 89^\circ$. Using this idea and $\sin^2 x+\cos^2 x=1$, we see that there are 44 pairs of 1 in each quadrant, giving a total of 176. The "45° angles" in each quadrant give $\frac{1}{2}$ each, for a total of 2. The four quadrantal angles give a total of 2. The grand total is 180.

11. The graphs of $y=x^2+1$ and $y=\sin x$ never intersect, so 0.

12. The given information sends us to Quadrant II where the opposite side has length $\sqrt{5}$. The sine value is $\frac{\sqrt{5}}{3}$.

13. Using $\left|\frac{60H-11M}{2}\right|,\left|\frac{60(4)-11(15)}{2}\right|=37.5.$

14. $\lim\limits_{\theta\to 0}\frac{\sin 2\theta}{\theta}\to\lim\limits_{\theta\to 0}\frac{2\sin\theta\cos\theta}{\theta}\to 2\left(\lim\limits_{\theta\to 0}\frac{\sin\theta}{\theta}\right)\left(\lim\limits_{\theta\to 0}\cos\theta\right)\to 2(1)(1)=2.$

15. $y=\sqrt{1-\cos 2x}+\sqrt{1+\cos 2x}=\sqrt{1-\left(1-2\sin^2 x\right)}+\sqrt{1+\left(2\cos^2 x-1\right)}=\sqrt{2}\left|\sin x\right|+\sqrt{2}\left|\cos x\right|$

$=\sqrt{2}\left(\left|\sin x\right|+\left|\cos x\right|\right)$. The maximum value occurs when $x=\frac{\pi}{4}$, so the maximum value is $\sqrt{2}\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}\right)=2.$

16. $\operatorname{Tan}^{-1}\frac{5}{12}+2\operatorname{Tan}^{-1}a=0\to\operatorname{Tan}^{-1}\frac{5}{12}=-2\operatorname{Tan}^{-1}a.$ If $\operatorname{Tan}^{-1}\frac{5}{12}=\theta$, then $\tan\theta=\frac{5}{12}$. We know that $\theta>0$ since we are using principal values. Substituting, we have $\theta=-2\operatorname{Tan}^{-1}a\to$ $\operatorname{Tan}^{-1}a<0\to a<0$. If we let $\operatorname{Tan}^{-1}a=x$, then $\tan x=a$, and then $\operatorname{Tan}^{-1}\frac{5}{12}=\theta-2x\to$ $\tan(-2x)=\frac{5}{12}\to-\tan(2x)=\frac{5}{12}$. $-\frac{2\tan x}{1-\tan^2 x}=-\frac{2a}{1-a^2}=\frac{5}{12}\to-24a=5-5a^2$. Factoring, we get $(5a+1)(a-5)=0$. Since we know that $a<0, a=-\frac{1}{5}$.

17. There is no work to do here. If $P(x)=8$, Vieta's formulas tell us that the product is 8.

18. By definition of logarithmic functions, $\sin 2\left(x-\frac{\pi}{4}\right)>0$. $\sin 2\left(x-\frac{\pi}{4}\right)=0$ at $x=\frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{4},\frac{7\pi}{4}$. Thinking about the graph of $y=\sin 2\left(x-\frac{\pi}{4}\right)$, we know that the graph is above the x-axis (producing positive y-values) between the first two values and the last two values: $\left(\frac{\pi}{4},\frac{3\pi}{4}\right)\cup\left(\frac{5\pi}{4},\frac{7\pi}{4}\right)$. $A+B+C+D=4\pi$.

19. $f(x)=3\cot\frac{2}{3}x\to\frac{\pi}{\frac{2}{3}}=\frac{3}{2}\pi$, $g(x)=\left|\sin 4x\right|\to\frac{2\pi}{4}\to\frac{\pi}{2}$, but due to the x-axis reflection, the period is cut in half, so the period is $\frac{\pi}{4}$; $h(x)=\frac{1}{2}\sec\frac{3}{4}x\to\frac{2\pi}{\frac{3}{4}}=\frac{8}{3}\pi$. $\frac{3}{2}\pi+\frac{\pi}{4}+\frac{8}{3}\pi=\frac{53}{12}\pi$.

20. Area is found by $\frac{1}{2}ab\sin C\to\frac{1}{2}(12)(24)\left(\frac{\sqrt{2}}{2}\right)=72\sqrt{2}$

21. The given information sends us to Quadrant IV and the adjacent side has length 8.

$$\tan 2\theta=\frac{2\tan\theta}{1-\tan^2\theta}\to\frac{2\left(-\frac{15}{8}\right)}{1-\left(-\frac{15}{8}\right)^2}=\frac{-\frac{15}{4}}{-\frac{161}{64}}=\frac{240}{161}.$$

22. $\sin x < \frac{1}{2}$ over $\left[0, \frac{\pi}{6}\right)$ and $\left(\frac{5}{6}\pi, 2\pi\right]$. This is $\frac{2}{3}$ of the total interval.

23. $2^{\sin^2 x} = 4^{\cos x} \rightarrow 2^{\sin^2 x} = 2^{2\cos x} \rightarrow \sin^2 x = 2\cos x \rightarrow 1 - \cos^2 x - 2\cos x = 0 \rightarrow$ $\cos^2 x + 2\cos x - 1 = 0$. $\cos x = -1 + \sqrt{2}$. $\cos x$ will take on this value twice in each interval of $\left[0, 2\pi\right]$, so there are 8 solutions overall.

24. Since the given expression is a root of the equation, $x = \frac{\cos\theta}{1+\sin\theta}$. We can divide the quadratic equation by x to get $x + 4 + \frac{1}{x} = 0$. Now we have $\frac{\cos\theta}{1+\sin\theta} + 4 + \frac{1+\sin\theta}{\cos\theta} = 0$.

This leads to $\frac{\cos^2\theta + (1+\sin\theta)^2}{(1+\sin\theta)(\cos\theta)} = -4 \rightarrow \frac{\cos^2\theta + \sin^2\theta + 2\sin\theta + 1}{(1+\sin\theta)(\cos\theta)} \rightarrow \frac{2+2\sin\theta}{(1+\sin\theta)(\cos\theta)} = -4$
$\rightarrow \frac{2}{\cos\theta} = -4 \rightarrow \cos\theta = -\frac{1}{2} \rightarrow \theta = 120°$.

25. $\sin 2\theta = 2\sin\theta\cos\theta$. Since $\tan\theta = \frac{k}{3}$, then $\sin\theta = \frac{k}{\sqrt{k^2+9}}$ and $\cos\theta = \frac{3}{\sqrt{k^2+9}}$. Using these values, $2\sin\theta\cos\theta = \frac{6k}{k^2+9}$.