Interschool Test MAO Nationals 2015

1. Two farmers agree that pigs are worth \$300 and that goats are worth \$210. When one farmer owes the other money, he pays the debt in pigs or goats and all "change" received is also in the form of pigs or goats. Using only whole animals, what is the amount of the smallest positive debt that can be resolved in this way?

2. How many even 3-digit integers have the property that their digits, read left to right, are in strictly increasing order?

3. Leap Day on February 29, 2004 occurred on a Sunday. On what day of the week will Leap Day, February 29, 2020 occur?

4. Find the result when the 171^{st} even natural number is subtracted from the 219^{th} odd natural number.

5. Calvin leaves Utah (on his way back to Hawaii), driving at a constant speed. After a while, he passes a mile marker with a two-digit number on it. An hour later, he passes another mile marker with the same two digits on it, but in reverse order. In another hour, he passes a 3^{rd} mile marker again with the same two digits, but separated by a zero. What is the rate of Calvin's car in miles per hour?

6. A metric calendar has 1 metric year equivalent to our calendar year of 365 days. A metric year is divided into 10 equal metric months; a metric month is divided into 10 equal metric weeks; a metric week is divided into 10 equal metric days. To the nearest day of our calendar, how many days are there in 4 metric months, 5 metric weeks and 8 metric days?

7. Two circles and three straight lines lie in the same plane. If neither the circles nor the lines are coincident (meaning they don't lie on top of one another), what is the maximum possible number of points of intersection?

8. There is only one 3-digit decimal numeral which, when expressed as a hexadecimal numeral, simply has its digits reversed. What is this decimal numeral?

9. What is the largest 7-digit number that contains each of the digits 1 through 7 and has the property that the sum of any 2 consecutive digits is a prime number?

10. You have 63 one dollar bills and 6 envelopes. You want to place the bills in the envelopes in such a way that any amount from $1 to $63 could be obtained by selecting a combination of envelopes. If each envelope must have at least $1, how many dollars would you place in the envelope with the greatest number of bills?

Trivia Time:

11. What is the name for the form of government controlled by a privileged, hereditary ruling class, generally resented by the middle and lower classes?

12. Which four stringed instruments typically make up a string quartet?

13. What part of speech is the word "the"?

14. The first Nobel Prize for physics was awarded in 1901 to the German physicist who discovered a short-wave ray called the x-ray. In Europe, they refer to x-rays by his name. Who was he?

15. This five letter word is related to lines; when the last four letters are removed, the word is pronounced the same way. What is it?

16. What two rivals struggled in the Peloponnesian War from 431-404 BC?

17. What does a taxonomist do?

18. Who was the first US President to live in the White House?

19. Between noon and the following midnight, how many times do the hands of a regular clock form a right angle?

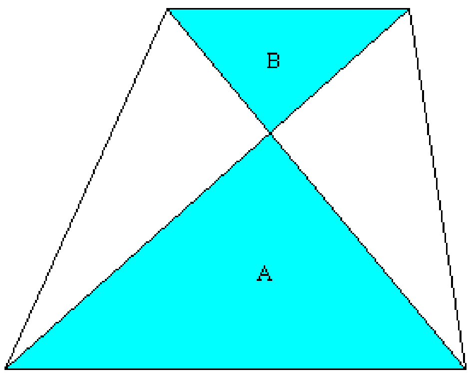
20. This important mathematical concept was invented around the 9th century in India and called Sunya in Sanskrit, and later called zephirum in Medieval Latin, but today we call it what?

Back to Work:

21. Two students play a game based on the total roll of two standard dice. Student A says that a 12 will be rolled first. Student B says that two consecutive 7s will be rolled first. The students keep rolling until one of them wins. What is the probability that A will win?

22. Using only the numbers 1, 3, 4, and 6, together with the operations +, −, ×, and ÷, and unlimited use of brackets, make the number 24. Each number must be used precisely once. Each operation may be used zero or more times. Decimal points are not allowed, nor can you combine 2 or more digits to make a number greater than 10.

23. A trapezoid is divided into four triangles by its diagonals. Let the triangles adjacent to the parallel sides have areas A and B. Find the area of the trapezoid in terms of A and B.



24. A car travels downhill at 72 mph, on the level at 63 mph, and uphill at only 56 mph. The car takes 4 hours to travel from town A to town B. The return trip takes 4 hours and 40 minutes. Find the distance between the two towns.

25. The towns of Alpha, Beta, and Gamma are equidistant from each other. If a car is three miles from Alpha and four miles from Beta, what is the maximum possible distance of the car from Gamma? Assume the land is flat.

26. In $\triangle$ ABC, sides AB = 20, AC = 11, and BC = 13. Find the diameter of the semicircle inscribed in ABC, whose diameter lies on side AB, and that is tangent to sides AC and BC.

27. Twenty-seven identical white cubes are assembled into a single cube, the outside of which is painted black. The cube is then disassembled and the smaller cubes thoroughly shuffled in a bag. A blindfolded man (who cannot feel the paint) reassembles the pieces into a cube. What is the probability that the outside of this cube is completely black? Write your answer in scientific notation, rounded to two decimal places.

28. A king was trying to determine how to recruit men for his army. However, there were so many distractions around the castle that his thinking became confused. So, in order to change his daze into knights, he took a secluded walk so he could ponder in peace. When he got outside, he noticed that the head gardener had planted lines of high bushes. First, he planted a line running 100 paces east. Then, from the end of that line, he planted a line 100 paces north, then 100 west, 98 south, 98 east, 96 north, 96 west, and so on. This made a square spiral path that was 2 paces wide. If the king only walked down the middle of the path, how long was the total path (in terms of paces)?

29. In a family supper, a grandfather, a grandmother, two fathers, two mothers, four children, three grandchildren, one brother, two sisters, two sons, two daughters, one daughter in law, and one mother in law need to sit at a table. There are only seven chairs available at the place. How many more chairs do they have to borrow so that everyone can sit down together for the suppers if everybody requires a separate chair?

30. We know that the number 7 is a prime number followed by a cube. Which next number is also a prime followed by a cube?

31. Catherine was first to arrive at a 100 seat theater. She forgot her seat number and picks a random seat for herself. After this, every single person who get to the theater sits on his seat if its available, else chooses any available seat at random. Nathan is the last to enter the theater and 99 seats were occupied. What is the probability that Nathan gets to sit in his own seat?

32. Each letter in the long division problem below stands for a single digit of a decimal number. The letter M is not zero and no leading digit is zero. Different letters may be used for the same digit. What is the value of B?

					J	K	8	L	M
A	B	)	C	D	E	F	G	H	I
			N	P	Q				
						R	S		
						T	U		
							V	W	X
							Y	O	Z
									1

33. If $4^x + 4^{-x} = 7$, then what is $8^x + 8^{-x}$?

34. Find all x between 0 and 90 degrees that satisfy the equation: sin(2x) + tan(3x) + cos(4x) = 2.

35. Suppose that A, B, C, and D are real numbers such that the following matrix equation holds true:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} = \begin{bmatrix} B+C & -A \\ B-D & A \end{bmatrix}$$

Determine the value of the following determinant: $\begin{vmatrix} A & B \\ C & D-B \end{vmatrix}$

36. In 1963, mathematicians at the University of Illinois used a computer to show that $2^{11,213}-1$ is a prime number. If this number were written in standard notation, it would contain 3376 digits. What would the units digit be?

37. In the Land of Eternal Darkness, there is no light, so no one can see. All paper money consists of rectangles with triangles cut off some, all or none of the 4 corners of the rectangles. Currency denominations are determined by shape. The dimensions of the rectangles are 2.5 units by 6 units, and the triangles are all right triangles with legs of length 1 unit. Of course citizens cannot tell the front from the back of these bills, but they can recognize the value of the money by feeling which corners have or have not been cut off. How many different distinguishable denominations are possible for the currency in this land?

38. Two regular square pyramids have all edges 12 cm in length. The pyramids have parallel bases and parallel edges, and each has a vertex at the center of the other pyramid's base. What is the total number of cubic centimeters in the volume of the solid of intersection of the 2 pyramids?

39. Three units commonly used to measure angles are degrees (360 degrees in a circle), grads (400 grads in a circle) and mils (6400 mils in a circle). A right angle has an integer value for all 3 units…90 degrees, 100 grads and 1600 mils. Give the number of degrees of the next integer angle which is an integer for all 3 units.

40. Find the principal πth root of the quantity $\sqrt[\pi]{(-1)^{-1}}$.