Point values are assigned in each section. Points are only added to the total—they cannot be subtracted (unless otherwise specified).

Section I: Easy Problems

Let's start you off with some easy ones… (2 points each)

1) Tickets to raffle off a giant Jefre Jirafa cost $3 apiece while tickets to raffle off a miniature Jefre Jirafa cost $0.50 apiece. If Ali bought 50 total tickets for $50, how many tickets did she buy for the miniature Jefre Jirafa raffle?

2) In my home library, one bookshelf holds either exactly 36 novels or exactly 14 encyclopedia volumes. Therefore, all thirteen bookshelves in my home library can hold all 324 novels I own in addition to a maximum of how many encyclopedia volumes?

3) Henry has four bank accounts. Three of them have a sum total of $1600, while the fourth one has $55 less than the average amount in all four accounts. The fourth bank account contains how much money, in dollars?

4) My really bad watch loses 12 minutes every hour. If I set it correctly this morning at 9 am, and the watch now reads 2 pm, what is the actual time? Be sure to include am or pm in your answer.

5) The drama club has 54 members, each of whom is either a singer or a dancer. Seventeen members of the cast are female dancers, 31 members are singers, and 22 members are male. How many of the drama club's members are male singers?

6) Burt is 61 years old while his daughter, Ernestine, is 37 years old; they also share a birthday, which happens to be today. How many years ago was Ernestine's age one-third that of Burt's?

7) Tracy rows her boat 3 miles downstream in 30 minutes, then turns immediately around and rows upstream 3 miles to her starting point in one hour and 12 minutes. If Tracy rows at a constant speed and the river's current is also a constant speed, what is the speed of the current in miles per hour?

8) Mickey and Donald were the only candidates for mayor in Walt Disney City. Mickey and Donald received all of the votes as there were no write-in candidates. If seven votes that went to Mickey had instead gone to Donald, they would have tied in the voting. If five votes Donald received had instead gone to Mickey, Mickey would have had seven times the number of votes as Donald. How many votes did the winner of the election receive?

9) Quick! I just checked my watch and noticed that one-quarter of the time that has passed since the most recently passed midnight is equal to one-sixth of the time remaining until the next midnight. What time is it?

10) We rotate three drinking cup setups (each consisting of one cup, one lid, and one straw) for my daughter in which she gets her drinks. The three cups are blue, green, and yellow; the lids are orange, red, and yellow; and the straws are blue, red, and yellow. We try to put them together so that no two of the items making up a drinking cup setup share a color. Additionally, being an Auburn family, we always put the orange lid with the blue cup. How many distinct configurations of drinking cup setups are possible, given these conditions? A configuration consists of three groupings to create three drinking cup setups, regardless of the other in which they are mentioned (e.g., the blue cup with the red lid and red straw, the green cup with the orange lid and blue straw, and the yellow cup with the yellow lid and yellow straw is one configuration).

Section II: Hard Problems

These are a little more difficult than the last ones… (5 points each)

1) Find the real solutions for x:

(a) $e^x = 2$

(b) $e^{e^x} = 2$

(c) $e^{e^{e^x}} = 2$

2) Find the sequence of operational signs $(+, -, \times, \text{or } \div)$ that go in the blanks that make the equation true: $18\_\_12\_\_4\_\_5 = 59$.

3) Find the least positive integer (spelled out) that, when using the English alphabet, is spelled with letters in alphabetical order (a would come before b would come before c, etc.).

4) Let $\{a_n\}_{n=1}^{\infty}$ be a sequence of increasing positive integers that satisfies the equation $a_{n+2} = a_{n+1} + a_n$ for all positive integers *n*. If $a_7 = 120$, find the value of a_{10}.

5) If a, b, and c are positive real numbers such that $u = \frac{b}{a-c} = \frac{a+b}{c} = \frac{a}{b}$, what is the numerical value of u?

6) Find the only ordered pair of positive integers (x, y) that satisfies the equation $2^x + 254016 = y^2$.

7) Kristoph traveled at a constant rate of 36 miles per hour in his Sven-pulled sled on his way to work cutting ice. He wanted to travel back home on the same route at a speed under the posted speed limit of 30 miles per hour in such a way that his average speed for the entire round trip was an integer. What is the fastest constant speed, in miles per hour, at which Kristoph and Sven could make the return trip?

8) Find the only two ordered triples of positive integers (m,n,p), where $m<n<p$, such that one more than the product of any two of the coordinates *m*, *n*, or *p* is an integer multiple of the third coordinate.

9) Each of the following answer choices consists of three numbers. Which letter choices consist of three numbers that cannot be the lengths of the three altitudes to the three sides of a specific triangle?

A) $\frac{1}{5}, \frac{1}{4}, \frac{2}{3}$	B) $\frac{2}{7}, \frac{1}{3}, \frac{3}{4}$	C) 2, 3, 4
D) $\frac{1}{3}$, 1, 1	E) $\frac{3}{5}, \frac{2}{3}, \frac{59}{10}$	F) $\frac{20}{13}$, 2, 3
G) 1, 2, 3	H) $\frac{5}{12}, \frac{7}{13}, \frac{11}{5}$	I) $\frac{14}{9}$, 3, $\frac{31}{10}$

10) If x and y are positive real numbers such that $\log_9 x = \log_{15} y = \log_{25}(x+y)$, find the numerical value of $\frac{x}{y}$.

Section III: 1, 2, 3, 4

All of these are related to the sequence 1234… (up to 10 points each plus bonuses)

1) Consider lowest-terms fractions of the form $\frac{a}{b}$, where $0<a<b<400$. There are five such fractions whose decimal representations are all of the form 0.1234______, where any number of digits may fill in the blank. What are these five fractions? Two points for each correct fraction.

2) Reading the decimal representation of π from left to right, the sequence 1234 shows up at least once. In what position after the decimal is the 1 in the first instance of the sequence 1234? The ten guesses closest to the actual answer will receive from 1 to 10 points, depending on the accuracy of the guess. If any team guesses the exact answer, that team will earn an additional 50 points. Hint: the answer is greater than 12345 but less than 123456.

3) Reading the decimal representation of e from left to right, the sequence 1234 shows up at least once. In what position after the decimal is the 1 in the first instance of the sequence 1234? The ten guesses closest to the actual answer will receive from 1 to 10 points, depending on the accuracy of the guess. If any team guesses the exact answer, that team will earn an additional 50 points. Hint: the answer is greater than 123 but less than 1234.

4) Reading the decimal representation of ϕ from left to right, the sequence 1234 shows up at least once. In what position after the decimal is the 1 in the first instance of the sequence 1234? The ten guesses closest to the actual answer will receive from 1 to 10 points, depending on the accuracy of the guess. If any team guesses the exact answer, that team will earn an additional 50 points. Hint: the answer is greater than 123 but less than 1234.

Section IV: I Suspect You Are Irrational!

The first twenty decimal place digits (plus any digits preceding the decimal point) are given for these noteworthy irrational (or suspected irrational) constants. Provide the more common name for the constant. (1-2 are 1 point each, 3 is 2 points, and 4-12 are 4 points each)

1) 3.14159265358979323846
2) 2.71828182845904523536
3) 1.61803398874989484820
4) 0.57721566490153286060
5) 1.32471795724474602596
6) 2.58498175957925321706
7) 3.35988566624317755317
8) 0.83462684167407318628
9) 0.56714329040978387299
10) 2.29558714939263807403
11) 1.60669515241529176378
12) 1.45136923488338105028

Section V: Simple Graphs, Ridiculous Equations

Answer each question, considering only points in the Cartesian plane (points whose coordinates are real). This one seems fairly straight-forward… (1-3 are 2 points each, 4-8 are 5 points each, and 9-10 are 10 points each)

1) The equation $x^2 + y^2 - 6x + 8y + 25 = 0$ describes a single point—which point?

2) The equation $4x^2 + 5y^2 - 24x + 40y + 116 = 0$ also describes a single point—which point?

3) The equation $4x^2+12xy+9y^2+2x+3y-2=0$ describes two parallel lines—what is the slope of these lines?

4) The equation $2x^2+xy-y^2-23x+y+56=0$ describes two non-parallel lines—what is the point common to both of these lines?

5) The equation $x^4+2x^2y^2+y^4-4x^3-18x^2y-4xy^2-18y^3+51x^2-4xy+119y^2+86x-302y+290=0$ describes two points only—which two points?

6) The equation $2x^4+5x^2y^2+2y^4-8x^3+14x^2y+14xy^2-8y^3+9x^2+244xy+9y^2+86x+86y+1849=0$ also describes two points only—which two points?

7) The equation $-3x^3+4x^2y-3xy^2+4y^3+12x^2-14xy+14y^2-18x+20y+12=0$ describes a line and a point not on that line—what is the distance from that point to that line?

8) The equation $6x^3-13x^2y-14xy^2-3y^3+26x^2+56xy+18y^2-56x-36y+24=0$ describes three lines that intersect in a single point—which point?

9) The equation $x^6+3x^4y^2+3x^2y^4+y^6+8x^5+16x^3y^2+8xy^4+14x^4-32x^3y+32x^2y^2-32xy^3+18y^4-48x^3-124x^2y+112xy^2-44y^3-95x^2-160xy+225y^2+200x-300y+500=0$ describes three points only—what is the area enclosed by the triangle whose vertices are those three points?

10) The equation $x^6+3x^4y^2+3x^2y^4+y^6-18x^5+12x^4y-36x^3y^2+24x^2y^3-18xy^4+12y^5+147x^4-144x^3y+234x^2y^2-144xy^3+87y^4-684x^3+744x^2y-756xy^2+376y^3+1911x^2-1872xy+1131y^2-3042x+2028y+2197=0$ describes a single point—which point?

Section VI: Cryptograms

XUKDBK MKATXEKC GEKBK. DUBF, AFZJCDGNUDGTFZB GF HFN VFC MKATXEKCTZJ GEKBK TZBGCNAGTFZB, DUGEFNJE GEKH DCK ZFG LFCGE XFTZGB. TZ YUDZP ZNWYKC GETCGKKZ, LCTGK "GEDZP HFN". YCTZJ HFNC DZBLKC BEKKG NX GF MC. PNBGFB GF AEKAP, YNG ENCCH—FZUH GEK VTCBG GKZ GKDWB GF MF BF LTUU KDCZ YFZNB XFTZGB… (5 points each)

1) RIRVA UPMNHNIR NYHRKRV LZY FR CVNHHRY ZM HTR MSG PO ZH GPMH OPSV URVORLH MWSZVRM.

2) JZDSFLCDI MKFIU ZT JZDSFLCDI IFPKDSWCU KFC JZDSFLCDI.

3) HR PRF GTLPIPEHXR XA P LCPRT HREX NXREHQZXZG ITQHXRG, RX YXIT EKPR AXZI NXCXIG PIT ITDZHITJ EX NXCXI EKT ITQHXRG GX EKPE RX EOX PJWPNTRE ITQHXRG KPBT EKT GPYT NXCXI.

4) MLMBO GTG-KTGFZIGZ FXGCRM-LIBXINRM YTROGTHXIR DXZJ KTHYRMA KTMWWXKXMGZF JIF IZ RMIFZ TGM KTHYRMA BTTZ.

5) HOV UKOQSOEKEP, LJHY-ZHYEJA REOUQSKO KO H UYKPJA HOA MKEOAJA SOQJLZHY DEPQ HQQHSO H DHCSDED ZHYEJ HOA H DSOSDED ZHYEJ HQ PKDJ FKSOQP KO QIHQ SOQJLZHY.

6) OXPYP PDCAO CFBCFCOPVU TMFU JYCTP FKTHPYA.

7) UI QH UHIUHUXY LYKUYL UL EDHRUXUDHQSSN EDHCYKTYHX, XOYH UXL XYKFL EQH WY KYQKKQHTYR LD XOQX XOY HYZ LYKUYL YUXOYK EDHCYKTYL XD QHN TUCYH CQSVY DK RUCYKTYL.

8) VTVOW VTVF AVONVMU FYJXVO KI ULV AOSCYMU SN B ASRVO SN URS BFC B JVOIVFFV AOKJV.

9) JGW SAE KBP S, PQB ZGXBW KBP GJ S QSK S KPWDHPME YWBSPBW HSWNDASMDPE PQSA PQB HSWNDASMDPE GJ S.

10) J LY LCO N KEWZR YWO ER GYWHUZOWL LY YWO KYXZIY N EQ J JWX N JUO UOIJLEMOIT NUEKO.

11) X VL VNU T BEHO S VL VNU T URHXEO F VL VNU T NXO TL BLOAVAJU ATVUIUD OLEHVALTO MLD X, S, XTZ F QNUT T AO XT ATVUIUD IDUXVUD VNXT LD URHXE VL VNDUU.

12) EKM RVL EYVYWQ AMKID A, WBQ KMHQM KE QPQML CIJAMKID KE A HYPYHQC WBQ KMHQM KE A.

Section VII: X-treme Sudoku

Fill out the Sudoku below in the standard way (each row, column, and smaller, bold outlined 4×4 square must contain each of the integers from 1 to 16), with the added criterion that each of the two diagonals must also contain each of the integers from 1 to 16 (those diagonals are shaded for your convenience). 1 point for each correct white cell, 2 points for each correct shaded cell. For each incorrect cell, you will lose twice that number of points (-2 for white cells, -4 for shaded cells).

	12			11	14			6			16	2			
		5	7			1	3			10		12	8		
4	1			9			10		2	5				15	
11		15			5			7	14		8			16	10
	11	9			13		16	15	4				2		
3	8			10		2		14				1		6	
			15		6		5			9			16		4
	14	2		1					6		11	13			7
8		1			7	14			15	4		9	10	11	
	10		2	3			12			13			14		
6						10	9	1			3		12		8
	4		9			13		16				5		7	
	5	4				16	8		1					3	12
7			13			15				11	9			10	2
		3	8	2	4			5		7			6		
		16		13			11	12			6	14			

Section VIII: You Must Be Joking, My Head Hurts Already

We saved the best for last… (point values are given for each question).

1) Imagine a standard, 4×4 Sudoku, where each row and column contain the integers from 1 to 4, inclusive, as does each 2×2 block (4 total, each containing one of the corner squares)—just as with normal Sudoku rules. Call two Sudoku solution arrangements unique if they differ in at least one individual square. How many unique Sudoku solution arrangements are there? This one is worth 10 points.

2) Imagine a standard, 9×9 Sudoku, where each row and column contain the integers from 1 to 9, inclusive, as does each 3×3 block (9 total, the center squares of which are simultaneously in columns 2, 5, or 8 and in rows 2, 5, or 8)—just as with normal Sudoku rules. Call two Sudoku solution arrangements unique if they differ in at least one individual square. How many unique Sudoku solution arrangements are there? The ten guesses closest to the actual answer will receive from 1 to 10 points, depending on the accuracy of the guess. If any team guesses the exact answer, that team will earn an additional 50 points.

3) There are eight right triangles whose legs have lengths that are consecutive integers such that the shorter leg is less than 1,000,000. You will earn $n(n+1)$ points for identifying *n* of these eight triangles (write your answers as ordered triples in increasing length order), and you will lose 1 point for each answer you include that is incorrect. Hint: these triples can be found recursively.

4) There are ten triangles whose sides have lengths that are consecutive integers that also have an enclosed area that is an integer such that the shortest side is less than 1,000,000. You will earn $n(n+1)$ points for identifying *n* of these ten triangles (write your answers as ordered triples in increasing length order), and you will lose 1 point for each answer you include that is incorrect. Hint: these triangles middle-length side can be found recursively.

5) Find the negative solution to the equation $x^3+60x^2-150x+125=0$. This one is worth 10 points, and you need not rationalize the answer.

6) Lagrange's Theorem says that each positive integer can be written as the sum of no more than four positive perfect square real numbers (which can include repeated perfect squares). There are 32 positive integers less than 200 that, when written as the sum of four or fewer positive perfect squares, require four positive perfect squares. Find these 32 positive integers. You will earn $\left\lfloor \frac{n(n+1)}{20} \right\rfloor$ for identifying *n* of these integers, and you will lose 1 point for each answer you include that is incorrect.

7) For the following polynomial, find all of its complex roots. If a root occurs with multiplicity more than one, list the root each time it appears (for example, if 5 is a root of multiplicity three, make sure to list 5 three times). You will earn $\frac{n(n+1)}{2}$ points for identifying *n* of these roots.

$$y = x^{16} - 6x^{15} - 26x^{14} + 216x^{13} + 42x^{12} - 2582x^{11} + 3475x^{10} + 10650x^{9} - 29407x^{8} + 3942x^{7}$$
$$+62088x^{6} - 84388x^{5} + 24604x^{4} + 48246x^{3} - 57537x^{2} + 23922x - 3240$$