The answer choice E. NOTA denotes “None of the Above Answers is Correct”.

1. Find the volume of the solid made by revolving the region bounded by the y-axis and the functions $f(x) = x - 2$ and $g(x) = \sqrt{4 - x^2}$ about the y-axis.

A. $\frac{16\pi}{3}$ B. $\frac{20\pi}{3}$ C. $\frac{40\pi}{3}$ D. 8π E. NOTA

For problems 2-3, Use the following information: On the unit circle there is a sector with positive angle measure θ < $\frac{\pi}{2}$. There are also two triangles in the first quadrant, both of which share vertices at the origin and at (1, 0) and both of which have the same angle θ as one interior angle. One of the triangles has $(cos\theta, sin\theta)$ as a vertex and the other has $(1, tan\theta)$ as a vertex.

2. Which of the following is NOT an expression for the area of one of the three shapes described above (i.e. the 2 triangles and the sector)?

A. $\frac{1}{2}$ B. $\frac{\theta}{2}$ C. $\frac{tan\theta}{2}$ D. $\frac{sin\theta}{2}$ E. NOTA

3. Writing an inequality with the expressions for the area of each of the three shapes, manipulating the expressions, and applying the Squeeze Theorem are all steps in proving which of the following limits:

A. $\lim_{x\to 0} \frac{sinx}{x} = 0$ B. $\lim_{x\to 0} \frac{sinx}{x} = 1$ C. $\lim_{x\to 0} \frac{1-cosx}{x} = 1$

D. Both A and C E. NOTA

4. Find the volume of the solid whose cross-sections are rectangles with a height determined by the function $f(x) = x^2$ on the interval [0, 3] and a length that is 5 times their height.

A. 45 B. 125 C. 135 D. 243 E. NOTA

5. Evaluate the following $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\pi\cos\left(\frac{\pi i}{n} - \frac{\pi}{2}\right)}{n}$

A. 0 B. 1 C. 2 D. $\frac{1}{2}$ E. NOTA

6. If a sphere of radius r is inscribed in a cylinder of radius r and height 2r, and both shapes are changing such that the volume of the sphere is increasing at a constant rate of 36 in.^{3}/min, at what rate (in in.^{2}/min) is the surface area of the cylinder increasing?

A. 54 B. 24 C. 36 D. cannot be determined E. NOTA

7. Let R be the region in the first quadrant bound by the x-axis, the y-axis, and a line with a negative slope passing through the point (2, 4). Find the minimum possible area of R.

A. 16 B. 8 C. 4 D. there is no minimum area E. NOTA

8. Let R be the region bounded by the x-axis and f(x) to the right of the line x = 1. If when R is rotated about the x-axis, the resulting solid is to have an infinite surface area but a finite volume, which of the following functions can f(x) be?

A. $\frac{1}{x^3}$ B. $\frac{1}{x^2}$ C. $\frac{1}{x}$ D. Both B and C E. NOTA

9. The phenomenon described in the preceding problem (#8) is also known by which name?

A. The Leibnitz Funnel B. Gabriel's Horn C. Euler's Paradox D. The Wizard's Hat E. NOTA

10. What is the total area enclosed by the polar function $r(\theta) = 3\sin(5\theta)$

A. $\frac{3\pi}{4}$ B. $\frac{9\pi}{4}$ C. $\frac{3\pi}{2}$ D. $\frac{\pi}{2}$ E. NOTA

11. What is the area enclosed by the curve given by the parametric function $x(t) = 2\cos(t)$, $y(t) = \pi\sin(t)$

A. 4π B. 2π C. $4\pi^2$ D. $2\pi^2$ E. NOTA

12. Find the volume of the solid made by revolving the region bounded by the function $f(x) = e^x$ and the lines y = 1 and x = 1 about the line x = 1.

A. $\pi(2e-5)$ B. $\pi(e-3)$ C. $\pi(2e-1)$ D. $\pi(2e+1)$ E. NOTA

13. If the area of a circular puddle is increasing at a constant rate $\frac{dA}{dt}$ and the puddle initially has a radius of .01m, then the rate at which the radius of the puddle is increasing is:

A. initially less than $\frac{dA}{dt}$ but then greater B. initially greater than $\frac{dA}{dt}$ but then less C. always greater than $\frac{dA}{dt}$ D. always less than $\frac{dA}{dt}$ E. NOTA

14. Which of the following is NOT equivalent to the area bounded by y=f(x) and the x-axis on the interval [a,b] for any function f(x) which is positive on the interval:

A. $\lim_{n\to\infty} \sum_{i=0}^{n-1} \left(\frac{b-a}{n}\right) f\left(a + i\left(\frac{b-a}{n}\right)\right)$ B. $\lim_{n\to\infty} \sum_{i=0}^{n-1} \left(\frac{b-a}{n}\right) f\left(a + (2i+1)\left(\frac{b-a}{2n}\right)\right)$

C. $\lim_{n\to\infty} \sum_{i=0}^{n-1} \left(\frac{b-a}{n}\right) f\left(a + (5i+3)\left(\frac{b-a}{5n}\right)\right)$ D. $\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{b-a}{n}\right) f\left(a + i\left(\frac{b-a}{n}\right)\right)$

E. NOTA

15. A regular N-gon (polygon with N sides) with sides of length S and an apothem of length A is inscribed in a circle of radius R. If S and A change accordingly as N increases so that the polygon remains inscribed in the same circle and $p = \lim_{N\to\infty} SN$ and $q = \lim_{N\to\infty} \frac{1}{2} ASN$, what is $\frac{p}{q}$?

A. 2πR B. πR^2 C. $2R^{-1}$ D. 2R E. NOTA

16. What is the volume of a parallelepiped with one vertex at the origin and three adjacent vertices at the points (-3, -5, -1), (-6, -10, -3), and (5, -3, 0)?

A. 70 B. 44 C. 116 D. 34 E. NOTA

17. $\int_{-9}^{7} x^3 + 3x^2 + 3x + 1\, dx =$

A. $\frac{121}{4}$ B. 30 C. $\frac{59}{2}$ D. $\frac{119}{4}$ E. NOTA

18. A triangle has two sides of length 6 in. and 15 in. If the angle between these two sides is allowed to vary, what is the maximum area enclosed by the triangle, in square inches?

A. 45 B. 90 C. 72 D. 36 E. NOTA

19. In the triangle from the previous problem, if the angle starts at 0 and begins to increase at a constant rate of 2°/sec, at what rate is the area enclosed by the triangle changing when the area is at a maximum?

A. 45 in.^{2}/sec. B. 90 in.^{2}/sec. C. -4150 in.^{2}/sec. D. -45 in.^{2}/sec. E. NOTA

20. A square of area 9 ft.^{2} is split into 9 separate squares of equal area and the four corner squares are shaded. If this process is repeated with the center square, and then again with the square in the center of that one and so on without end, what is the total shaded area?

A. $\frac{9}{2}$ B. $\frac{9}{8}$ C. $\frac{40}{9}$ D. $\frac{81}{16}$ E. NOTA

21. A right triangle has each of its three sides (a, b, c in increasing order) being used as the side of a square, forming three squares, each of which is adjacent to the triangle on one of its sides. Which of the following represents the sum of the areas of the three squares?

I. $2c^2$ II. $2a^2 + 2b^2$ III. $a^2 + b^2 + c^2$

A. I only B. III only C. I and III only D. I, II, and III E. NOTA

22. The base of a solid is given by the region bounded by $y = \sin(x)$ and the x-axis for $0 \leq x \leq \frac{\pi}{2}$. If the cross-sections of this solid perpendicular to the x-axis are rectangles with height given by $h(x) = \cos(x)$, what is the volume of the solid?

A. $\frac{1}{4}$ B. $\frac{1}{2}$ C. 1 D. 2 E. NOTA

23. If a triangle has the vertices (3, 4), (-4, 3), and (0, -5), what is the area enclosed by the circle that circumscribes the triangle?

A. 29π B. 5π C. 10π D. 25π E. NOTA

24. Let R be the region between f(x) and the x-axis on the interval [0, 10]. For which of the following functions, f(x), would you be unable to calculate the area of R, but be able to calculate the volume of the solid generated by revolving R about the y-axis by using the "Shell Method"?

A. $\frac{1}{x-5}$ B. e^{x^2} C. $\sin(\sqrt{x})$ D. $\cos(x)$ E. NOTA

25. The two diagonals of parallelogram ROFL intersect at point P, forming 4 triangles inside ROFL. Inside ROFL is a smaller similar parallelogram ABCD, such that AB is the midsegment of triangle RPO, BC is the midsegment of OPF, CD is the midsegment of FPL, and DA is the midsegment of LPR. What is the ratio of the area enclosed by ROFL to the area enclosed by ABCD?

A. 2:1 B. 1:2 C. 4:1 D. 1:4 E. NOTA

26. The tangent lines at the pole to the function $r(\theta) = \theta^2 - \frac{7\pi}{6}\theta + \frac{\pi^2}{3}$ divide the unit circle ($r(\theta) = 1$) into 4 sectors. Let A, B, C, and D represent the areas of each of these four sectors in non-decreasing order, respectively. Find $\frac{A+B}{C+D}$

A. $\frac{1}{3}$ B. $\frac{1}{4}$ C. $\frac{1}{6}$ D. $\frac{1}{12}$ E. NOTA

27. What is the area of the xz-trace of solid formed by $(x+2)^2 + (y+1)^2 + (z+5)^2 = 64$

A. 35π B. 63π C. 65π D. 93π E. NOTA

28. If the radius of a sphere is increasing at a rate of $\frac{dr}{dt}$ at what rate is the volume changing when the volume and surface area are equal and non-zero?

A. $24\frac{dr}{dt}$ B. $36\frac{dr}{dt}$ C. $24\pi\frac{dr}{dt}$ D. $36\pi\frac{dr}{dt}$ E. NOTA

29. What is the area of the region bounded by $f(x) = \frac{1}{\sqrt{x^2+1}}$ and the x-axis on the interval [0, 1]?

A. $\frac{1}{\sqrt{2}}$ B. $\frac{1}{2}\ln(2)$ C. $\ln(1+\sqrt{2})$ D. $\frac{\pi}{4}$ E. NOTA

30. The sum of the areas of an infinite series of rectangles yields a finite result. If the height of the n^{th} rectangle in the series is $n!$ which of the following can be the width of the n^{th} rectangle?

I. $\frac{1}{n^n}$ II. $\frac{1}{n^{2015}}$ III. $\frac{1}{2015^n}$

A. I only B. I and III only C. III only D. I, II, and III E. NOTA