For all questions, answer choice "E. NOTA" means none of the above answers is correct.

Lengths and measurements are given in units unless otherwise specified.
Good luck and have fun!

1. What is the area enclosed by a regular octagon with side length 4?

A) $32\sqrt{2}-32$ B) $32\sqrt{2}+32$ C) $24\sqrt{3}$ D) $32\sqrt{2+\sqrt{2}}$ E) NOTA

2. The area of an equilateral triangle is increasing at a constant rate of 9 units^2/min when the side length is 6 units such that the triangle remains equilateral. What is the rate of increase of the area of the inscribed circle in units^2/min at this instant?

A) $\pi/2$ B) 3π C) $\pi\sqrt{3}$ D) $2\pi\sqrt{3}$ E) NOTA

3. Evaluate the total area bounded by the *x*-axis and the function $y=\sin x$ on the *x*-interval $[0,3\pi]$.

A) 0 B) 2 C) π D) 6 E) NOTA

4. What is the volume of a right circular cone with diameter 12 and height 8?

A) 384π B) 256π C) 96π D) 48π E) NOTA

5. What is the total surface area of a solid right circular cone with diameter 12 and height 8?

A) 240π B) 124π C) 96π D) 84π E) NOTA

6. A right circular cone with diameter 12 and height 8 is deteriorating from its apex to the base such that a right circular frustum is formed at any time. If the vertical rate of deterioration is 0.5 units/min, what is the rate of decrease of the volume after 20 seconds, in units^3/min?

A) $\frac{\pi}{2304}$ B) $\frac{\pi}{384}$ C) $\frac{\pi}{128}$ D) $\frac{225\pi}{2}$ E) NOTA

7. Find the volume of the revolved figure between the curves $y=\sin^2 t$ and $y=\cos^2 t$ between $t \in [0,\frac{\pi}{2}]$ around the y-axis.

A) $\frac{\pi^2}{4}$ B) $\frac{\pi^2}{2}$ C) $\frac{\pi}{2}$ D) π E) NOTA

Use the following information to answer questions 8-10.
The beta distribution is a continuous probability distribution defined on the interval $x \in [0,1]$ with probability density function (pdf) given by $f_X(x;\alpha,\beta)=\frac{1}{B(\alpha,\beta)}x^{\alpha-1}(1-x)^{\beta-1}$, with two shape parameters α and β. The beta function $B(\alpha,\beta)$ is the normalizing constant that allows the pdf to have a total area of 1 defined by probability rules.

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8. If $a = 4$ and $b = 3$, what is $B(a, b)$?

A) $\frac{1}{35}$ B) $\frac{1}{60}$ C) 35 D) 60 E) NOTA

9. The mode of a continuous probability distribution is the value of *x* for which f_X has a maximum value on the appropriate interval. For the beta distribution with $a = 3$ and $b = 6$, the pdf is given as $f_X(x;3,6) = 168x^2(1-x)^5$. What is the mode of this distribution?

A) $\frac{2}{7}$ B) $\frac{1}{3}$ C) $\frac{2}{5}$ D) $\frac{1}{2}$ E) NOTA

10. The mean of a continuous probability distribution can be thought of as the weighted area over the interval, given by the equation $\mu = \int_0^1 x f_X(x; a, b)\,dx$ for the Beta distribution. For the pdf given in #9 with $a = 3$ and $b = 6$, what is the mean of this distribution?

A) $\frac{2}{7}$ B) $\frac{1}{3}$ C) $\frac{2}{5}$ D) $\frac{1}{2}$ E) NOTA

11. Approximate $\int_0^1 \sin x\,dx$ using the first three nonzero terms of its Maclaurin polynomial.

A) $\frac{10}{3}$ B) $\frac{101}{120}$ C) $\frac{331}{720}$ D) $\frac{27}{60}$ E) NOTA

12. The area of a certain figure can be given by the following formula: $A = \int_0^{x^2} t^2 e^t - t\sin t\,dt$. If *x* changes at a rate $\frac{dx}{dt} = 0.5$, what is the rate of change of the area with respect to *t* when x=3?

A) $243e^9 - 27\sin 9$ C) $27e^3 - 9\sin 3$ E) NOTA
B) $81e^9 - 9\sin 9$ D) $9e^3 - 3\sin 3$

13. What is the area of the rectangle with maximum perimeter inscribed in an ellipse with equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, in terms of positive real numbers *a* and *b*?

A) $2ab$ B) $2\sqrt{a^2 + b^2}$ C) $4\sqrt{a^2 + b^2}$ D) $\frac{(ab)^2}{a^2 + b^2}$ E) NOTA

14. What is the area of the figure enclosed by the parabola $y^2 - 12x - 6y - 3 = 0$ and its latus rectum?

A) 4.5π B) 18 C) 24 D) 27 E) NOTA

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15. Using Simpson's rule, estimate $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec x\, dx$ using four equal subintervals of the *x*-axis.

A) $\frac{\pi}{24}\left(2+2\sqrt{2}+8\sqrt{2\left(2-\sqrt{2}\right)}\right)$

B) $\frac{\pi}{24}\left(2+2\sqrt{2}+8\sqrt{2\left(2+\sqrt{2}\right)}\right)$

C) $\frac{\pi}{24}\left(4+2\sqrt{2}+4\sqrt{2\left(2-\sqrt{2}\right)}\right)$

D) $\frac{\pi}{24}\left(4+2\sqrt{2}+4\sqrt{2\left(2+\sqrt{2}\right)}\right)$

E) NOTA

16. The base of a solid lies in the xy-plane, bounded by the the graphs of $f(x)=\sqrt{x}$, the *x*-axis, and $x=9$. Cross-sections perpendicular to the x-axis are isosceles triangles such that the legs of the triangle are twice the length of the base of the triangle, and the base of the triangle is in the base of the solid. What is the volume of this solid?

A) $\frac{81}{2}$ B) $\frac{81}{4}$ C) $\frac{81\sqrt{15}}{4}$ D) $\frac{81\sqrt{15}}{8}$ E) NOTA

17. What is the area bounded by the curves $y=x^2+6x-22$ and $y=4x-7$?

A) $\frac{106}{3}$ B) $\frac{160}{3}$ C) $\frac{256}{3}$ D) $\frac{310}{3}$ E) NOTA

18. An ellipsoid centered at the origin in 3-dimensional space is projected onto the xy-plane and the xz-plane (you can think of these as cross-sections through the missing axis). The ellipse in the xy-plane has equation $9x^2+4y^2=36$. The ellipse in the xz-plane has its major axis in the z-direction with eccentricity $\frac{2\sqrt{5}}{5}$. What is the volume of the ellipsoid?

A) $12\pi\sqrt{5}$ B) $16\pi\sqrt{5}$ C) 60π D) 80π E) NOTA

19. A convex quadrilateral in the Cartesian coordinate plane has coordinates (2,3), (5,-4), (-1,-2) and (6,6). What is its area?

A) 12.5 B) 18 C) 33.5 D) 36.5 E) NOTA

20. The centroid of the quadrilateral in #19 has coordinates $(\bar{x},\bar{y})$. Evaluate $\bar{y}$.

A) 3 B) $\frac{77}{219}$ C) $\frac{3}{4}$ D) $\frac{159}{73}$ E) NOTA

21. A semicircle described by the equation $y=\sqrt{r^2-x^2}$ is revolved about the x-axis, generating a sphere. Given this information, or using any other method, calculate the y-coordinate of the centroid in terms of positive value *r*.

A) $\frac{r}{\pi}$ B) $\frac{2r}{\pi}$ C) $\frac{2r}{3\pi}$ D) $\frac{4r}{3\pi}$ E) NOTA

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22. A solid is generated by revolving the area bounded by the curves $y=\tan x$ and $y=\sec x$ between $x=-\frac{\pi}{4}$ and $x=\frac{\pi}{4}$ about the line $y=-1$. Which of the following integrals will generate the volume of this solid?

A) $\pi\int_{-\pi/4}^{\pi/4} 1+2\sec x-2\tan x\,dx$ C) $\pi\int_{-\pi/4}^{\pi/4} dx$ E) NOTA

B) $\pi\int_{-\pi/4}^{\pi/4} 1-2\sec x+2\tan x\,dx$ D) $2\pi\int_{-\pi/4}^{\pi/4} \tan^2 x-\tan x\sec x\,dx$

23. Using a right endpoint Riemann sum approximation and the true area under the curve, calculate the absolute value of the difference between the two values for the function $f(x)=\ln x$ on $x\in[1,5]$. For the Riemann sum, divide the interval into four equal subintervals on the *x*-axis.

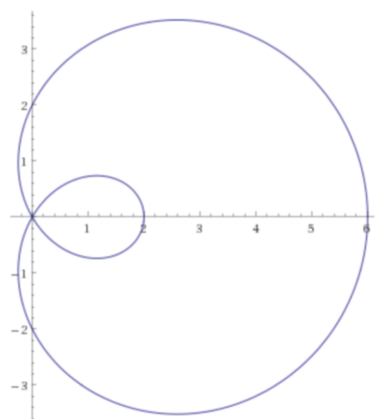
A) $\ln\frac{24e^4}{625}$ B) $\ln\frac{625}{24e^4}$ C) $\ln\frac{24e^5}{625}$ D) $\ln\frac{625}{24e^5}$ E) NOTA

24. What is the surface area of the parallelepiped formed by the vectors <3,2,1>, <-1,3,0>, and <2,2,5> as the edges emanating from one vertex?

A) 43 C) $\sqrt{131}+\sqrt{237}+\sqrt{314}$ E) NOTA

B) 47 D) $2\sqrt{131}+2\sqrt{237}+2\sqrt{314}$

25. The figure to the right depicts the graph of the inner-loop limaçon with equation $r=2+4\cos\theta$. What is the area enclosed by the inner loop?



A) π B) $2\pi-\sqrt{3}$

C) $4\pi-6\sqrt{3}$ D) $\frac{4\pi}{3}-4\sqrt{3}$

E) NOTA

26. Evaluate $\lim_{n\to\infty}\sum_{i=1}^{n}\left[2\left(1+\frac{3i}{n}\right)+1\right]\frac{3}{n}$.

A) 9 B) 12 C) 15 D) 18 E) NOTA

27. Given a sphere of radius R changing over time, both the volume and surface area will change with a respective rate. What is the ratio of the rate of change in volume to the rate of change in surface area, both with respect to time, in terms of R?

A) R B) 2R C) $\frac{R}{2}$ D) $\frac{2}{R}$ E) NOTA

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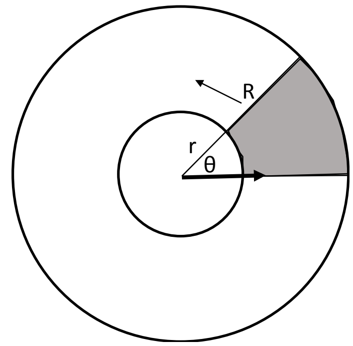
28. An annulus between two concentric circles with radii of lengths *r* and *R* (*R*>*r*) is formed, intersected by two radii to form an annular sector with angle θ, as depicted in the figure to the right. The outer radius is fixed at 10 units while the inner radius is growing at a rate of $\frac{dr}{dt} = \sqrt{t}$ units/sec, t≥0, initially at 0 units. Meanwhile, the angle θ is increasing at a rate of $\frac{\pi}{50}$ rad/sec, also initially at 0 radians. Rounding your answer to the nearest second, after how many seconds will the area change from increasing to decreasing?

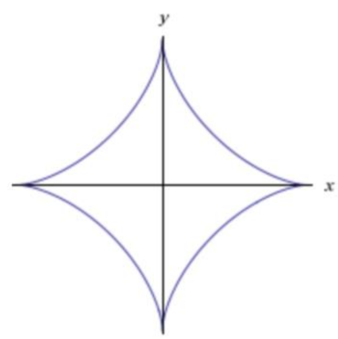


A) 4 B) 8 C) 19 D) 56 E) NOTA

29. What is the volume of Gabriel's Horn, defined as the volume of revolution about the x-axis of the function $f(x) = \frac{1}{x}$ for $x \geq 1$?

A) 1 B) 2 C) π D) π^2 E) NOTA

30. An astroid is a hypocycloid with four cusps that has the appearance of a four-pointed star, as shown to the right. The astroid has the equation $x^{2/3} + y^{2/3} = a^{2/3}$, or can be represented by the parametric equations $x = a\cos^3 t, y = a\sin^3 t$ for $t \in [0,2\pi]$. Green's theorem states that the area of the astroid can be calculated as $A = \frac{1}{2}\oint_{\partial C} x\,dy - y\,dx$, but when parametrized by *t* becomes $A = \frac{1}{2}\int_0^{2\pi} x(t)d[y(t)] - y(t)d[x(t)]$. Given this information, what is the area of the astroid in terms of *a*?



A) $\frac{3a^2\pi}{4}$ B) $\frac{3a^2\pi}{8}$ C) $\frac{3a^2\pi}{16}$ D) $\frac{a^2\pi}{2}$ E) NOTA