Answers:

0. y=2x

1. $\frac{3}{8\pi}$

2. 4

3. $4\pi - 6\sqrt{3}$

4. $10\sqrt{2}+4$ by $\frac{25\sqrt{2}+10}{2}$

5. $\frac{52\pi}{3}$

6. $\frac{25}{2}$ or 12.5

7. 24π

8. $\frac{33}{2}$ or 16.5

9. 36

10. $\frac{1}{49}$

11. $\frac{4}{3}$

12. 160

0. $y' = \cos x + 1 \to y' = 2 \to y = 2x$

1. $V = \frac{\pi r^2 h}{3} = \left(\frac{\pi}{3}\right)\left(\frac{h}{3}\right)^2 h = \frac{\pi h^3}{27} \to dV = \frac{\pi h^2 dh}{9} = \frac{2}{3}$

$dh = \frac{6}{\pi h^2} = \frac{3}{8\pi}$

2. $m = \frac{k}{2\sqrt{x}} = 1 \to x + 4 = k\sqrt{x} \to k = \frac{x+4}{\sqrt{x}}$

$\frac{\frac{x+4}{\sqrt{x}}}{2\sqrt{x}} = 1 \to \frac{x+4}{\sqrt{x}} = 2\sqrt{x} \to x + 4 = 2x \to x = 4 \to k = 4$

3. $2\frac{1}{2}\int_0^{\frac{\pi}{3}} (2 - 4\cos\theta)^2 \, d\theta = \int_0^{\frac{\pi}{3}} \left(4 - 16\cos\theta + 16\cos^2\theta\right) d\theta$

$\int_0^{\frac{\pi}{3}} \left(4 - 16\cos\theta + 8 + 8\cos 2\theta\right) d\theta = 4\theta - 16\sin\theta + 8\theta + 4\sin 2\theta \Big|_0^{\frac{\pi}{3}}$

$4\pi - 6\sqrt{3}$

4. $(x+4)(y+5) = \min imum \to xy = 250 \to y = \frac{250}{x}$

$(x+4)\left(\frac{250}{x} + 5\right) = \min = 250 + 5x + \frac{1000}{x} + 20$

$5 - \frac{1000}{x^2} = 0 \to x = 10\sqrt{2} \to 10\sqrt{2} + 4 \; and \; \frac{25\sqrt{2}}{2} + 5$

5.Washer method, but solved for *x*: $x^2 = -4x + 12 \to x^2 + 4x - 12 = 0 \to \text{intersect at } (2, 4)$

$\pi\int_0^4 \left(\left(3 - \frac{1}{4}y\right)^2 - y\right) dx = \pi\left(9y - \frac{5}{4}y^2 + \frac{1}{48}y^3\right)\Big|_0^4$

$\pi\left(36 - 20 + \frac{4}{3}\right) = \frac{52\pi}{3}$

6. $\tan\theta = \frac{50}{x} \rightarrow \sec^2\theta d\theta = \frac{-50}{x^2}dx \rightarrow \theta = \frac{\pi}{4} \rightarrow x = 50$

$50\sec^2\theta d\theta = dx \rightarrow 50\left(\frac{2}{\sqrt{2}}\right)^2\left(\frac{1}{8}\right) = dx = \frac{25}{2}$

7. $x = \sqrt{9-\frac{9y^2}{16}} \rightarrow 2\bullet\frac{1}{2}\int_0^4\left(\sqrt{9-\frac{9y^2}{16}}\right)^2 dy = \pi\int_0^4\left(9-\frac{9y^2}{16}\right)dy$

$\pi\left(9y-\frac{3y^3}{16}\right)\Big|_0^4 = \pi(36-12) = 24\pi$

8. $\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{18n-3i}{n^2}\right) = \lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{18n-3i}{n}\right)\left(\frac{1}{n}\right) = \int_0^1(18-3x)\,dx$

$18x-\frac{3x^2}{2}\Big|_0^1 = 18-\frac{3}{2} = \frac{33}{2}$

$8-y^2 = 2y \rightarrow y^2+2y-8 = 0 \rightarrow (y+4)(y-2) = 0$

9. $\int_{-4}^{2}(8-y^2-2y)\,dy = 8y-\frac{y^3}{3}-y^2\Big|_{-4}^{2} = 16-\frac{8}{3}-4-\left(-32+\frac{64}{3}-16\right)$

$\frac{28}{3}--\frac{80}{3} = 36$

$u = 1+\sqrt{y} \rightarrow du = \frac{1}{2\sqrt{y}}dy \rightarrow 2du = \frac{dy}{\sqrt{y}}$

10. $\int\frac{2}{u^2}du = \frac{-2}{u} = \frac{-2}{1+\sqrt{x}}\Big|_k^1 = \frac{3}{4} \rightarrow -1+\frac{2}{1+\sqrt{k}} = \frac{3}{4}$

$-1+\frac{2}{1+\sqrt{k}} = \frac{3}{4} \rightarrow \frac{2}{1+\sqrt{k}} = \frac{7}{4} \rightarrow 8 = 7+7\sqrt{k} \rightarrow k = \frac{1}{49}$

11. $\ln(1+x)=\int\frac{1}{1+x}dx=\int\left(\sum_{n=0}^{\infty}(-1)^n x^n\right)dx=C+\sum_{n=0}^{\infty}(-1)^n\frac{x^{n+1}}{n+1}=C+\sum_{n=1}^{\infty}(-1)^{n+1}\frac{x^n}{n}$, provided that $-1<x<1$. Plugging in $x=0$ yields $C=0$, so $\ln(1+x)=\sum_{n=1}^{\infty}(-1)^{n+1}\frac{x^n}{n}$. Plugging in $x=\frac{1}{3}$ gives the series in question, so the sum is $\ln\left(1+\frac{1}{3}\right)=\ln\frac{4}{3}\Rightarrow A=\frac{4}{3}$.

12. $\ln\ln y=\ln\left(x^x\ln x\right)=x\ln x+\ln\ln x\Rightarrow\frac{1}{y\ln y}\frac{dy}{dx}=1+\ln x+\frac{1}{x\ln x}$

$\Rightarrow\frac{dy}{dx}=y\ln y\left(1+\ln x+\frac{1}{x\ln x}\right)=x^{x^x}x^x\ln x\left(1+\ln x+\frac{1}{x\ln x}\right)\Rightarrow\left.\frac{dy}{dx}\right|_{x=2}=64\ln 2+64(\ln 2)^2$ $+32$, so $A+B+C=32+64+64=160$.