

- *NOTA means None of These Answers*

1. The number e is usually referred to as which mathematician's number?

- A. Euclid B. Euler C. Eratosthenes D. Erdős E. NOTA

2. Which mathematician discovered e while exploring compounded interest?

- A. Fermat B. Napier C. Leibniz D. Bernoulli E. NOTA

3. Which of the following is **not** equal to e^x for all $x \in \mathbb{R}$?

- A. $\lim_{n \rightarrow \infty} \left(1 + \frac{x}{n}\right)^n$ B. $\lim_{n \rightarrow 0} (1 + n)^{\frac{x}{n}}$
 C. $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ D. $\cosh x + \sinh x$ E. NOTA

4. What is the sum of the solutions to: $e^x + 12e^{-x} = 7$?

- A. $\ln 7$ B. $\ln 4$ C. $\ln 3$ D. $\ln 12$ E. NOTA

5. This statement describes which function? If a certain area is inputted, this function outputs the b such that the area between the curve $y = \frac{1}{x}$ and the x -axis from 1 to b is the inputted value.

- A. $\ln x$ B. $\frac{1}{x}$ C. e^x D. $-\frac{1}{x^2}$ E. NOTA

6. How many of the following series converge?

- i. $\sum_{n=0}^{\infty} \frac{e^n}{n!}$ iv. $\sum_{n=0}^{\infty} e^{-n}$
 ii. $\sum_{n=0}^{\infty} \frac{e^n}{2^n}$ v. $\sum_{n=1}^{\infty} \frac{e^n}{n^e}$
 iii. $\sum_{n=1}^{\infty} \frac{(-1)^n e}{n}$ vi. $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

- A. 1 B. 2 C. 3 D. 4 E. NOTA

7. Find the radius of convergence of $\sum_{n=1}^{\infty} \frac{x^n (2n)^n}{3^n n!}$

- A. $2e/3$ B. $3e/2$ C. $2/(3e)$ D. $3/(2e)$ E. NOTA

8. Find the global maximum of $f(x) = \sqrt[x]{x}$. The function's domain is all positive reals.

- A. e B. e^e C. $\frac{1}{e}$ D. $e^{\frac{1}{e}}$ E. NOTA

9. I invest a sum of money in an account with $100R\%$ interest compounded continuously. If I make no withdrawals or deposits, how long does it take for my money to triple?

- A. $(\ln 3)/R$ B. $\ln\left(\frac{3}{R}\right)$ C. $\ln(3R)$ D. $\ln 3$ E. NOTA

10. Evaluate: $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{3}{n+i}$

- A. $3 \ln 2$ B. $3 \ln 3$ C. $\ln 3$ D. Does not converge E. NOTA

11. Evaluate: $\int_{e^e}^{\infty} \frac{dx}{x \ln x (\ln(\ln x))^{3/2}}$

- A. $2\sqrt{e}$ B. $2e$ C. e^2 D. $2e^2$ E. NOTA

12. Given $\cosh t = 2$, Find the value of $|\sinh 2t|$.

- A. $\sqrt{3}$ B. $2\sqrt{3}$ C. $4\sqrt{3}$ D. 6 E. NOTA

13. Evaluate: $\lim_{n \rightarrow \infty} \ln \left(\frac{n!}{(2n)^n} \right)^{\frac{1}{n}}$

- A. $-1 - \ln 2$ B. $-\frac{1}{2}(1 + \ln 2)$ C. $\frac{1}{2}(1 + \ln 2)$ D. $-\ln 2$ E. NOTA

14. You have 100g of a radioactive substance. After 60 minutes, 80g remains. What is the half-life of the substance in hours?

- A. $\log_5(.8)$ B. $\log_{1.25}(2)$ C. $2 \ln(4/5)$ D. $\frac{1}{2 \ln(4/5)}$ E. NOTA

15. How many functions that pass through the origin are there such that $f'(x) = f(x)$?

- A. 0 B. 1 C. 2 D. infinitely many E. NOTA

16. The function $f(x) = e^x$ is rotated about the x axis. Find the volume of the resulting solid that lies to the left of the y axis.

- A. π B. $\pi/2$ C. $e\pi$ D. π/e E. NOTA

17. What is the 10th digit after the decimal place in the decimal equivalent of e ?

- A. 1 B. 2 C. 4 D. 8 E. NOTA

Use the following information for questions 18-19:

A population $P(t)$ grows at a rate directly proportional to $P(t)(10,000 - 2P(t))$. There are initially 1000 members of the population, and after two years there are 3000 members.

18. Which of the following is $P(4)$ closest to?

- A. 3500 B. 4000 C. 4500 D. 5000 E. NOTA

19. When is the population growing the fastest?

- A. $P = 1000$ B. $P = 2500$ C. $P = 5000$ D. $P = 10,000$ E. NOTA

20. Evaluate: $\lim_{x \rightarrow \infty} (1 + 2e^x)^{1/x}$

- A. 1 B. e C. $2e$ D. e^2 E. NOTA

21. A ball with mass 5kg falls through water and experiences both the force of gravity and a drag force directly proportional to its velocity. Find the velocity of the ball at $t = \ln 5$. Use Newton's second law of motion, $mg - F_{DRAG} = ma$, $g = 10 \frac{m}{s^2}$, $v(0) = 0$, $v(\ln 2) = 5$.

- A. 8 B. $48/5$ C. 7 D. 6 E. NOTA

22. Find the product of all real k such that $y = e^{kx}$ is a solution to the differential equation

$$\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0.$$

- A. $-\sqrt{2}$ B. $\sqrt{2}$ C. -2 D. 2 E. NOTA

23. The complex number $2 + 2i$ can be expressed in the form e^{a+bi} , where a and b are real and b is the least possible positive radian measure angle. What is ab ?

- A. $\frac{\pi}{2} \ln 2$ B. $\frac{\pi}{2} \ln \sqrt{2}$ C. $\frac{3\pi}{4} \ln 2$ D. $\frac{3\pi}{4} \ln \sqrt{2}$ E. NOTA

24. $f(x) = \frac{e^{x^2} - x^2 - 1}{4x^4}$. Find $\lim_{x \rightarrow 0} f(x)$.

- A. $1/8$ B. $1/24$ C. $1/64$ D. $1/96$ E. NOTA

25. $e^{\begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}}$ = ? (Hint: use power series)

- A. e^7 B. e^{12} C. $\begin{bmatrix} 3e & 0 \\ 0 & 4e \end{bmatrix}$ D. $\begin{bmatrix} e^3 & 0 \\ 0 & e^4 \end{bmatrix}$ E. NOTA

26. Evaluate: $\int_0^{\infty} \frac{dx}{e^{1+2x} + e^{1-2x}}$

- A. $\frac{\pi}{4e}$ B. $\frac{\pi}{8e}$ C. $\frac{\pi}{4}$ D. $\frac{\pi}{8}$ E. NOTA

27. $f(x) = (x^2 + 1)e^x$. Evaluate $f^{(10)}(0)$.

- A. 1 B. 90 C. 91 D. 180 E. NOTA

28. Suppose you play a slot machine that pays out with probability $1/1,000,000$. If you play $1,000,000$ times, the probability you will lose every time is closest to which of the following?

- A. $1/2$ B. $1/e$ C. $1/2e$ D. $1/e^2$ E. NOTA

29. Evaluate: $\sum_{n=0}^{\infty} \frac{\cos(n\theta)}{3^n}$ where $\cos(\theta) = 1/3$. Hint: $\cos(n\theta) = \operatorname{Re}(e^{in\theta})$

- A. $3/5$ B. $6/5$ C. $9/5$ D. $6/7$ E. NOTA

30. Evaluate: i^{i^3}

- A. 1 B. -1 C. $e^{\frac{\pi}{2}}$ D. $e^{-\frac{\pi}{2}}$ E. NOTA